

# Boundary Value Problem and its application in I-Function of Multivariable

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**Abstract**

In this research paper, we make a model of a boundary value problem and its application in I-function of multivariable. Some particular cases have also been derived at the end of the paper.

**Keywords:** Boundary value problem, I-Function of multivariable, General class of polynomials.

**Mathematics subject classification:** 2011, 33c, secondary 33c50.

**1. Introduction:** Y.N. Prasad [5] is defined. I-Function of multivariable are as follows:

$$\sum [Z_1, \dots, Z_r] = \sum_{\substack{o_1, n_2, o_3, \dots, o_r \\ p_2, q_2, p_3, q_3, \dots, p_r, q_r}} (m, n; \dots; (m^{(r)}, n^{(r)}); (p, q); \dots; (p^{(r)}, q^{(r)})) \\ \left[ Z_1, Z_2, \dots, Z_r \left( \begin{matrix} (a_{2j}, \alpha'_{2j}; \alpha''_{2j})_1, p_2, \dots, (a_{rj}, \alpha'_{rj}; \alpha''_{rj})_1, p_r, (a'_j; \alpha'_j)_1, p, \dots, (a_j^{(r)}; \alpha_j^{(r)})_1, p^{(r)} \\ (b_{2j}, \beta'_{2j}; \beta''_{2j})_1, q_2, \dots, (b_{rj}, \beta'_{rj}; \beta''_{rj})_1, q_r, (b'_j; \beta'_j)_1, q, \dots, (b_j^{(r)}; \beta_j^{(r)})_1, q^{(r)} \end{matrix} \right) \right] \\ = \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \phi_1(s_1) \dots \phi_r(s_r) \psi(s_1, \dots, s_r) Z_1^{s_1} \dots Z_r^{s_r} ds_1 \dots ds_r \quad (1.1)$$

Where

$$\phi_i s_i = \frac{\prod_{j=1}^{m^{(i)}} (b_j^{(i)} - \beta_j^{(i)} s_i) \prod_{j=1}^{n^{(i)}} (1 - (a_j^{(i)} + \alpha_j^{(i)} s_i))}{\prod_{j=m^{(i)+1}}^{q^{(i)}} (1 - b_j^{(i)} - \beta_j^{(i)} s_i) \prod_{j=n^{(i)+1}}^{p^{(i)}} (a_j^{(i)} - \alpha_j^{(i)} s_i)} \quad \forall i.e. \{1, 2, \dots, r\}$$

$$\text{and } \psi(s_1, s_2, \dots, s_r) = \frac{\prod_{j=1}^{n_2} (1 - a_{2j} + \sum_{i=1}^2) (\alpha_{2j}^{(i)} s_i)}{\prod_{j=n_2+1}^{p_2} (a_{2j} - \sum_{i=1}^2) (\alpha_{2j}^{(i)} s_i)} \times$$

$$\frac{1}{\prod_{j=1}^{q_2} (1 - b_{2j} + \sum_{i=1}^2) (\beta_{2j}^{(i)} s_i) \dots \prod_{j=1}^{q_r} (1 - b_{rj} + \sum_{i=1}^r) (\beta_{rj}^{(i)} s_i)}$$

The operational techniques are important tools to compute various problem in various fields of sciences. Which are used in works of Kumar [3] to find out several results in various problem in different field of sciences and thus motivating by this work, we construct a model problem for temperature distribution in a rectangular plate under prescribed boundary conditions and then evaluate its solution involving I-Function of multivariable with products of general class of polynomials

$$S_{n_1, \dots, n_r}^{m_1, \dots, m_r} (x_1, \dots, x_r) = \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1} k_1}{k_1!} \dots \frac{(-n_r)_{m_r} k_r}{k_r!} F[n_1, k_1, \dots, n_r, k_r] x_1^{k_1} \dots x_r^{k_r} \quad (1.2)$$

where,  $m_1, \dots, m_r$  are arbitrary positive integers and the coefficients  $F[n_1, k_1, \dots, n_r, k_r]$  are arbitrary constants real or complex. Finally, we derive some new particular cases & find their applications also.

**2.A Boundary value problem the boundary value conditions are,**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = a, \quad 0 < x < \frac{a}{2}, \quad 0 < y < \frac{b}{2} \tag{2.1}$$

$$\left. \frac{\partial u}{\partial x} \right|_{x=0} = \left. \frac{\partial u}{\partial x} \right|_{x=\frac{a}{2}} = 0, \quad 0 < y < \frac{b}{2} \tag{2.2}$$

$$U(x, 0) = 0, \quad 0 < x < \frac{a}{2} \tag{2.3}$$

$$U(x, \frac{b}{2}) = f(x) = \left( \cos \frac{\pi x}{a} \right) S_{n_1, \dots, n_r}^{m_1, \dots, m_r} \left[ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1}, \dots, y_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right] \sum_{A, c: [B', D']; \dots; [B^{(n)}, D^{(n)}]}^{0, \lambda: [U', V']; \dots; [U^{(n)}, V^{(n)}]} \left[ (a): \theta', \dots, \theta^{(n)}; [(b'); \phi']; \dots; [(b^{(n)}); \phi^{(n)}]; Z_1 \left( \cos \frac{\pi x}{a} \right)^{26_1}, \dots, Z_n \left( \cos \frac{\pi x}{a} \right)^{26_n} \right. \\ \left. [(c): \psi', \dots, \psi^{(n)}]; [(d'); s^1]; \dots; [(d^{(n)}); \delta^{(n)}]; \right]$$

where,  $0 < x < \frac{a}{2}$  Provided that  $\text{Re}(\eta) - 1$ , all  $\sigma_i (i = 1, 2, \dots, n)$  are positive real numbers,  $S_{n_1, \dots, n_r}^{m_1, \dots, m_r} [x]$  is due to (1.1) and I-Function of multivariable.  $u(U(x, y))$  is the temperature distribution in the rectangular plate at point.

**3.Main Integral:**In our investigations, we make an appeal to the modified formula

$$\int_0^{a/2} \left( \cos \frac{\pi x}{a} \right)^\eta \cos \frac{2m\pi x}{a} dx = \frac{a \sqrt{(\eta+1)}}{2^{\eta+1} \left( \frac{\eta}{2} + m + 1 \right) \left( \frac{\eta}{2} - m + 1 \right)} \tag{3.1}$$

$$\int_0^{a/2} \left( \cos \frac{\pi x}{a} \right)^\eta \cos \frac{2m\pi x}{a} S_{n_1, \dots, n_r}^{m_1, \dots, m_r} \left[ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1}, \dots, y_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right] \sum_{A, c: [B', D']; \dots; [B^{(n)}, D^{(n)}]}^{0, \lambda: [U', V']; \dots; [U^{(n)}, V^{(n)}]} \left[ (a): \theta', \dots, \theta^{(n)}; [(b'); \phi']; \dots; [(b^{(n)}); \phi^{(n)}]; \right. \\ \left. [(c): \psi', \dots, \psi^{(n)}]; [(d'); \delta']; \dots; [(d^{(n)}); \delta^{(n)}]; \right] Z_1 \left( \cos \frac{\pi x}{a} \right)^{26_1}, \dots, Z_n \left( \cos \frac{\pi x}{a} \right)^{26_n} dx \\ \frac{a}{2^{n+1}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1) m_1 k_1}{k_1!} \dots \frac{(-n_r) m_r k_r}{k_r!} F[n_1, k_1, \dots, n_r, k_r] \times \\ \sum (k_1, \dots, k_r) \left( \frac{y_1}{4p_1} \right)^{k_1} \dots \left( \frac{y_r}{4p_r} \right)^{k_r} \tag{3.2}$$

where

$$\sum (k_1, \dots, k_r) = \sum_{A+1, c+2[B', D']; \dots; [D^{(n)} D^{(n)}]}^{0, \lambda+1[U', V']; \dots; [U^{(n)}, V^{(n)}]} \left[ (a): \theta', \dots, \theta^{(n)}, [-\eta - 2p_1 k_1 \dots - 2p_r k_r; 2\sigma_1, \dots, 2\sigma_n], [ \dots ]; \right. \\ \left. [(c): \psi', \dots, \psi^{(n)}], [-\eta/2 - m - s_1 k_1 \dots - s_r k_r; \sigma_1, \dots, \sigma_n], [-\eta/2 + m - s_1 k_1 \dots - s_r k_r; \sigma_1, \dots, \sigma_n] \right. \\ \left. \frac{[(b'); \phi']; \dots; [(b^{(n)}): \phi^{(n)}]_{-q_0}}{[(d'); \delta']; \dots; [(d^{(n)}): \delta^{(n)}]}; Z_1, \dots, Z_n \right] \frac{Z_n}{4^{\sigma_n}} \tag{3.3}$$

Provided that  $F(n_1, k_1, \dots, n_r, k_r)$  are arbitrary functions of  $n_1 k_1, \dots, n_r k_r$ , real or complex independent of  $x, y_j, p_j, j = 1, 2, \dots, r$ , the condition of (2.4) and (3.1) are satisfied and,

$$R_e \left( \eta + \sum_{i=1}^n \sigma_i d_i^{(i)} / S_j^{(j)} \right) > -1, \lambda, A, C, U^{(i)}, V^{(i)}, B^{(i)}, D^{(i)} \text{ are such that,}$$

$$A \geq \lambda \geq 0, C \geq 0, D^{(i)} \geq U^{(i)} \geq 0, B^{(i)} \geq V^{(i)} \geq 0 \text{ and}$$

$$\theta_j^{(i)}, j = 1, 2, \dots, A; \phi_j^{(i)}, j = 1, 2, \dots, B^{(i)}, \psi_j^{(i)}, j = 1, 2, \dots, C, S_j^{(i)}, j = 1, 2, \dots, D^{(i)} \text{ are positive real}$$

$$\text{numbers, } \arg(z_i) \leq \Delta_i, y_2 \text{ and } \Delta_i = - \sum_{j=\lambda+1}^A \phi_j^{(i)} + \sum_{j=1}^{V^{(i)}} \phi_k^{(i)} - \sum_{j=V^{(i)+1}^{B^{(i)}}} \phi_j^{(i)} - \sum_{j=1}^C \psi_j^{(i)} + \sum_{j=1}^{U^{(i)}} S_j^{(i)} - \sum_{j=U^{(i)+1}^{D^{(i)}}} S_j^{(i)} > 0$$

$$i = 1, 2, \dots, n$$

**4. Solution of Boundary Value Problem:**

In this section, we obtain the solution of the boundary value problem stated in the section (2) as using (2.1), (2.2) and (2.3) with the help of the techniques referred to Zill [1] as,

$$U(x, y) - A_0 y + \sum_{p=1}^{\infty} A_p \sinh \frac{2p\pi y}{a} \cos \frac{2p\pi x}{a}, 0 < x < \frac{a}{2}, 0 < y < \frac{b}{2} \tag{4.1}$$

for  $y = \frac{b}{2}$ , we find that

$$U \left( x, \frac{b}{2} \right) = f(x) = \frac{A_0 b}{2} + \sum_{p=1}^{\infty} A_p \sinh \frac{p\pi b}{a} \cos \frac{2p\pi x}{a}, 0 < x < \frac{a}{2} \tag{4.2}$$

Now making an appeal to (2.4) & (4.2) and then integrating both sides with respect to  $x$  from 0 to  $a/2$ , We derive,

$$A_0 = \frac{2}{b\sqrt{\pi}} \sum_{k_1=0}^{(n_1/m_1)} \dots \sum_{k_r=0}^{(n_r/m_r)} (-n_1)_{m_1 k_1} (-n_r)_{m_r k_r} F(n_1, k_1, \dots, n_r, k_r) \sum (k_1, \dots, k_r) \frac{y_1^{k_1}}{k_1!} \dots \frac{y_r^{k_r}}{k_r!} \tag{4.3}$$

where

$$\sum (k_1, \dots, k_r) = \sum_{A+1, C+1 \{B, D\}; \dots; \{B^{(n)}, D^{(n)}\}}^{0, \lambda+1 \{U, V\}; \dots; \{U^{(n)}, V^{(n)}\}} \left[ [(a) : \theta^1, \dots, \theta^{(n)}], [1/2 - n/2 - s_1 k_1, \dots - s_r k_r : \sigma_1, \dots, \sigma_n] [(b^1) : \phi^1], \dots, [(b^{(n)}) : \phi^{(n)}], \right. \\ \left. [(c) : \psi^1, \dots, \psi^{(n)}], [-\eta/2 - s_1 k_1, \dots - s_r k_r : \sigma_1, \dots, \sigma_n] [(d^1) : \delta^1], \dots, [(d^{(n)}) : \delta^{(n)}] \right] z_1, \dots, z_n \tag{4.4}$$

Where all conditions of (2.4), (3.1) and (3.3) are satisfied. Again making an appeal to (2.4) and (4.2) and then multiplying by  $\cos 2m\pi x/a$  both sides and integrate w.r.t  $x$  from 0 to  $a/2$ , we find,

$$A_m = \frac{1}{2^{\eta-1} \sinh \frac{p\pi b}{a}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 k_1}}{k_1!} \dots \frac{(-n_r)_{m_r k_r}}{k_r!} F[n_1 k_1, \dots, n_r k_r] \\ \sum (k_1, \dots, k_r) \left( \frac{y_1}{4p_1} \right)^{k_1} \dots \left( \frac{y_r}{4s_r} \right)^{k_r} \tag{4.5}$$

Provided that all condition of (2.4), (3.1), and (3.3) are satisfied. finally making an appeal to the result (4.1), (4.3), and (4.5), we drive the required solution of the boundary value problem,

$$U(x, y) = \frac{2y}{b\sqrt{\pi}} \sum_{k_1=0}^{(n_1/m_1)} \dots \sum_{k_r=0}^{(n_r/m_r)} \left[ \prod_{s=1}^r \left( (-n_s)_{m_s k_s} \frac{y_s^{k_s}}{k_s!} \right) \right] \tag{4.6}$$

$$F[n_1, k_1, \dots, n_r, k_r] \cdot \sum(k_1, \dots, k_r) + \sum_{m=1}^r \left( (-n_j) m_j k_m \left( \frac{y_j}{4s_j} \right)^{k_j} \frac{1}{k_j!} \right) F[n_1, k_1, \dots, n_r, k_r] \sum(k_1, \dots, k_r)$$

Where,  $0 < x < a/2 < 0 < y < b/2$ , provided that all conditions f(2.4), (3.1) and (3.3) are satisfied.

**5. Expansion Formula:**

With the aid of (2.4) and (4.6) and then setting  $y = (b/2)$  we evaluate the expansion formula,

$$\begin{aligned} & \left( \cos \frac{\pi x}{a} \right)^n S_{n_1, \dots, n_r}^{m_1, \dots, m_r} \left[ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1}, \dots, y_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right] \\ & \sum_{\substack{o, \lambda: [u', v']; \dots; [u^{(n)}, v^{(n)}] \\ A, C: [B', D']; \dots; [B^{(n)}, D^{(n)}]}} \left[ [(a): \theta', \dots, \theta^{(n)}]: (b'): \phi'; \dots; [(b)^{(n)}: \phi^{(n)}] \right. \\ & \left. [(c): \phi', \dots, \phi^{(n)}]: (d'): \delta'; \dots; [(d)^{(n)}: \delta^{(n)}] \right] Z_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1}, \dots, Z_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \\ & = \frac{1}{\sqrt{\pi}} \sum_{k_1=0}^{(n_1/m_1)} \dots \sum_{k_r=0}^{(n_r/m_r)} \left[ \prod_{j=1}^r \left( (-n_j) m_j k_j \frac{y_j^{k_j}}{k_j} \right) \right] F[n_1, k_1, \dots, n_r, k_r]. \\ & \sum(k_1, \dots, k_r) + \sum_{m=1}^{\infty} \frac{\cos 2m\pi x}{2^{n-1}} \sum_{k_1=0}^{(n_1/m_1)} \dots \sum_{k_r=0}^{(n_r/m_r)} \left[ \prod_{j=1}^r \left( (-n_j) m_j k_j \left( \frac{y_j}{4p_i} \right)^{k_j} \frac{1}{k_j} \right) \right] \\ & F[n_1, k_1, \dots, n_r, k_r] \cdot \sum(k_1, \dots, k_r). \end{aligned}$$

Where  $0 < x < (a/2)$ , provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.

**6. Particular cases and Applications:**

In this section, we do some setting of different parameters of our results and then drive some particular cases as stated here as taking  $m_1 = m_r = \gamma$  and,

$$F[n_1, k_1, \dots, n_r, k_r] = \left[ \frac{h}{(-v)^\gamma} \right]^{k_1 + \dots + k_r} \frac{1}{(1 + p - n_1 \dots n_r) / (k_1 + \dots + k_r)} \text{ in (1.1)}$$

We get,

$$S_{n_1, \dots, n_r}^{\gamma_1, \dots, \gamma_r} [x_1, \dots, x_r] = \frac{(-V)^{-n_1, \dots, n_r}}{(-P)_{n_1 + \dots + n_r}} (x_1)^{n_1/\gamma} \dots (x_r)^{n_r/\gamma}, \sum_{n_1, \dots, n_r}^{(h, \gamma, v, p)} \left[ (x_1)^{-1/\gamma}, \dots, (x_r)^{-1/\gamma} \right] \quad (6.1)$$

and thus, we obtain an integral for product of a class of polynomials of several variables

$$\begin{aligned} & \int_0^{a/2} \left( \cos \frac{\pi x}{a} \right)^n \cos \frac{2\pi x}{a} \frac{(-V)^{n_1, \dots, n_r}}{(-P)_{n_1 + \dots + n_r}} \left[ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1} \right]^{n_1/\gamma} \dots \\ & \left[ \gamma_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right]^{n_r/\gamma} \sum_{n_1, \dots, n_r}^{(h, \gamma, v, p)} \left\{ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1} \right\}^{1/\gamma}, \dots, \left\{ y_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right\}^{1/\gamma} \\ & \sum_{\substack{o, \lambda: [u', v']; \dots; [u^{(n)}, v^{(n)}] \\ A, C: [B', D']; \dots; [B^{(n)}, D^{(n)}]}} \left[ [(a): \theta', \dots, \theta^{(n)}]: (b'): (\phi'); \dots; [(b)^{(n)}: \phi^{(n)}] \right. \\ & \left. [(c): \phi', \dots, \phi^{(n)}]: (d'): (\delta'); \dots; [(d)^{(n)}: \delta^{(n)}] \right] \\ & Z_1 \left( \cos \frac{\pi x}{a} \right)^{2\sigma_1}, \dots, Z_n \left( \cos \frac{\pi x}{a} \right)^{2\sigma_n} dx \\ & = \frac{a}{2^{n+1}} \sum_{k_1=0}^{(n_1/\gamma)} \dots \sum_{k_r=0}^{(n_r/\gamma)} (-n_1)\gamma k_1 \dots (-n_r)\gamma k_r \left[ \frac{h}{(-V)^\gamma} \right]^{k_1 + \dots + k_r} \frac{1}{k_1!} \dots \frac{1}{k_r!} \frac{1}{(1 + P - n_1, \dots, n_r)\gamma_{(k_1 + \dots + k_r)}} \sum(k_1, \dots, k_r) \end{aligned}$$

$$\left(\frac{y_1}{4s_1}\right)^{k_1} \dots \left(\frac{y_r}{4s_r}\right)^{k_r} \tag{6.2}$$

Provided that all condition one of (2.4), (3.1) and (3.2) are satisfied. The solution of the given problem is:

$$U(x, y) = \frac{2y}{b\sqrt{\pi}} \sum_{k_1=0}^{[n_1/\gamma]} \dots \sum_{k_r=0}^{[n_r/\gamma]} \left[ \prod_{j=1}^r \left\{ (-n_j)\gamma k_j \left(\frac{hy_j}{(-v)^\gamma}\right)^{k_j} \frac{1}{k_j!} \right\} \right] \frac{1}{(1+p-n_1-\dots-n_r)\gamma_{(k_1+\dots+k_r)}} \sum_{m=1}^{\infty} \frac{\sinh \frac{2m\pi y}{a} \cos \frac{2m\pi x}{a}}{2^{\eta-1} \sinh \frac{m\pi b}{a}} \sum_{k_1=0}^{[n_1/\gamma]} \dots \sum_{k_r=0}^{[n_r/\gamma]} \left[ \prod_{j=1}^r \left\{ (-n_j)\gamma k_j \left(\frac{hy_j}{(-v)\gamma 4s_j}\right)^{k_j} \frac{1}{k_j!} \right\} \right] \frac{1}{(1+p-n_1-\dots-n_r)\gamma_{(k_1+\dots+k_r)}} \cdot \sum(k_1, \dots, k_r) \tag{6.3}$$

When  $0 < x < (a/2), 0 < y < (b/2)$ , provided that all condition of (2.4), (3.1) and (3.3) are satisfied, The expansion formula is,

$$\frac{(-V)^{-n_1 \dots n_r}}{(-P)_{n_1+\dots+n_r}} \left( \cos \frac{\pi x}{a} \right)^\eta \left[ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1} \right]^{n_1/\gamma} \dots \left[ y_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right]^{n_r/\gamma} \sum_{n_1, \dots, n_r}^{(h, \gamma, v, p)} \left\{ y_1 \left( \cos \frac{\pi x}{a} \right)^{2s_1} \right\}^{-1/\gamma}, \dots, \left\{ y_r \left( \cos \frac{\pi x}{a} \right)^{2s_r} \right\}^{-1/\gamma} \sum_{A, C, [B', D], \dots, [B^{(n)}, D^{(n)}]}^{0, \lambda, [u', v], \dots, [u^{(n)}, v^{(n)}]} \left[ [(a); \theta', \dots, \theta^{(n)}] : [(b'); \phi']; \dots; [(b^{(n)}); \phi^{(n)}] \right] \left[ [(c); \phi', \dots, \phi^{(n)}] : [(d'); \delta']; \dots; [(d^{(n)}); \delta^{(n)}] \right] Z_1 \left( \cos \frac{\pi x}{a} \right)^{2\sigma_1}, \dots, Z_n \left( \cos \frac{\pi x}{a} \right)^{2\sigma_n} = \frac{1}{\sqrt{\pi}} \sum_{k_1=0}^0 \dots \sum_{k_r=0}^0 \left( \prod_{j=1}^0 \left[ (-n_j)\gamma k_j \left(\frac{hy_j}{[-v]\gamma 4}\right)^{k_j} \frac{1}{k_j!} \right] \right) \frac{1}{(1+p-n_1-\dots-n_r)\gamma_{(k_1+\dots+k_r)}} \sum_{m=1}^{\infty} \frac{\cos \frac{2m\pi x}{a}}{2^{\eta-1}} \sum_{k_1=0}^{[n_1/\gamma]} \dots \sum_{k_r=0}^{[n_r/\gamma]} \left[ \prod_{j=1}^r \left\{ (-n_j)\gamma k_j \left(\frac{hy_j}{(-v)^\gamma 4s_j}\right)^{k_j} \frac{1}{k_j!} \right\} \right] \frac{1}{(1+p-n_1-\dots-n_r)\gamma_{(k_1+\dots+k_r)}} \sum(k_1, \dots, k_r) \tag{6.4}$$

When  $0 < x < a/2$ , provided that all conditions of (2.4), (3.1) and (3.3) are satisfied

$$\sum_{n_1, \dots, n_r}^{(h, \gamma, 1, p)} \left( \frac{x_1}{p}, \dots, \frac{x_r}{p} \right) = \sum_{n_1, \dots, n_r}^{(h, \gamma, 1/p, p)} (x_1, \dots, x_r) = g_{n_1}^\gamma(x_1, h) \dots g_{n_r}^\gamma(x_r, h) \tag{6.5}$$

to the results, we evaluate the another results of Hermite polynomials by same techniques.

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