Common Fixed Point Theorems for Four Mappings in Fuzzy 2-Metric Spaces

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Abstract
In this paper some common fixed point theorem have been proved as a generalization of result of Seong Hoon Cho [1] the conditions for continuous self mappings S, T of complete fuzzy 2-metric space (X, M, •) have been characterised to have a unique common fixed point in X.

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1. Introduction
The concept of fuzzy sets was developed extensively by many authors[2, 3, 6] and used in various fields. Recently sushil Sharma [9], Urmila Mishra and et. al.[8 ] proved fixed point theorems in fuzzy metric space and fuzzy 2-metric space.
Bijendra Singh and M. S. Chauhan [10] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sense of George and Veeramani [4 ]. Recently Seong Hoon Cho[1]
generalized this results and characterized the conditions for two continuous self mappings of complete fuzzy metric space.

In this paper, we have a generalization of the result obtained in [1] in fuzzy 2-metric space including a continuous function Φ:0, 1 → [0, 1].

2. Preliminaries
In this section, we give some definitions and lemmas. A binary operation •: [0, 1] × [0, 1] → [0, 1] is called a continuous t-norm on [0, 1] if ([0, 1], •) is an abelian topological monoid with 1 such that a • b ≤ c • d, whenever a ≤ c, b ≤ d for all a, b, c, d ∈ [0, 1]. Examples of t-norm are a • b = ab and a • b = min {a, b}.

Definition 2.1: The 3-tuple (X, M, •) is called a fuzzy 2-metric space if X is an arbitrary set, • is a continuous t-norm and M is a fuzzy set on X 3 × (0, ∞) satisfying the following conditions:
(1) M(x, y, z, t) > 0,
(2) M(x, y, z, t) = 1 if and only if at least two out of three points are equal,
(3) M(x, y, z, t) = M(y, z, x, t)
(4) M(x, y, u, t1) • M(x, u, z, t2) • M(u, y, z, t3) ≤ M(x, y, z, t1 + t2 + t3),
(5) M(x, y, z) : (0, ∞) → [0, 1] is continuous, for all x, y, z X and t, s > 0.

A sequence {x_n} in a fuzzy 2-metric space (X, M, •) is said to be convergent to a point x X if for each > 0 and each t>0, there exists n_0 ≥N such that M(x_n, x, c, t)>1− ε for all c X for all n≥n_0. Equivalently, a sequence {x_n} in a fuzzy 2-metric space (X, M, •) converges to a point x X if
lim M(x_n, x, c, t) = 1, for all c X and t>0. n→∞

A sequence {x_n} in a fuzzy metric space (X, M, •) is called Cauchy sequence if for each > 0 and each t>0, there exists n_0 ≥N such that M(x_n, x_n, c, t) >1− ε for all n, m≥n_0 and for all c X. A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Self mappings A and B of a fuzzy 2-metric space (X, M, •) is said to be compatible if
limM(ABx_n, BAx_n, c, t) = 1, for all c X ,for all t>0, n→∞
Whenever \( \{x_n\} \) is a sequence in \( X \) such that 
\[
\lim_{n \to \infty} A_{x_n} = \lim_{n \to \infty} B_{x_n} = z, \text{ for some } z \in X.
\]
Fuzzy 2-metric space version of the following results will be used to prove our main results.

Lemma 2.1 [5]. Let \((X, M, \cdot)\) be a fuzzy metric space. Then for all \( x, y \in X \), \( M(x, y, \cdot) \) is non-decreasing.

Lemma 2.2[1]. Let \((X, M, \cdot)\) be a fuzzy metric space with \( \lim_{r \to \infty} M(x, y, t) = 1 \) for all \( x, y \in X \) and \( r \cdot t \geq r \) for all \( r \in [0, 1] \). If there exists \( 0 < q < 1 \) such that for all \( x, y \in X \) and \( t > 0 \), \( M(x, y, q t) \geq M(x, y, t) \), then \( x = y \).

Lemma 2.3 [1]. Let \((X, M, \cdot)\) be a fuzzy metric space with \( \lim_{t \to \infty} M(x, y, t) = 1 \)
for all \( x, y \in X \) and \( r \cdot t \geq r \) for all \( r \in [0, 1] \) and let \( A \) and \( B \) be continuous self mappings of \( X \) and the pair \([A, S]\) be compatible. Let \( \{x_n\} \) be a sequence in \( X \) such that \( A_{x_n} \to z \) and \( S_{x_n} \to z \). Then \( A_{S_{x_n}} \to S_z \).

Lemma 2.4[7]. The only t-norm \( * \) satisfying \( r \cdot t \geq r \) for all \( r \in [0, 1] \) is the minimum t-norm, that is, \( a \ast b = \min \{a, b\} \) for all \( a, b \in [0, 1] \).

From now on, let \((X, M, \cdot)\) be a fuzzy 2-metric space such that \( \lim_{t \to \infty} M(x, y, z, t) = 1 \)
for all \( x, y \in X \), \( r \cdot t \geq r \) for all \( r \in [0, 1] \) and \( \Phi:[0,1] \to [0,1] \). A continuous function
Such that \( \Phi(t) > t \), \( 0 < t < 1 \)

3. Main Results:

In this section, we prove some common fixed point theorems. To prove our next result we will use the next Proposition which is a generalization of the result of [1].

Proposition 3.1: Let \( A, B, S \) and \( T \) be self maps on a complete fuzzy 2-metric space \((X, M, \cdot)\) where \( * \) is a continuous t-norm defined by \( a \ast b = \min \{a, b\} \) such that the following conditions are satisfied:

(i) \( A X \), \( TX, SX \), \( SX \),
(ii) \( S \) and \( T \) are continuous,
(iii) the pairs \([A, S]\) and \([B, T]\) are compatible,
(iv) there exists q \(\in (0,1)\) such that for every \( x, y \in X \) and \( t > 0 \),
\[
M(Ax, By, c, qt) \geq M\{M(Sx, Ty, c, t) - M(Ax, Sx, c, t) - M(By, Ty, c, t) - M(Ax, Ty, c, t)\}. \]

Then \( A, B, S \) and \( T \) have a unique common fixed point in \( X \).

Proof. Let \( x_0 \in X \). From (i), there exists \( x_1 \) such that \( A_{x_0} = Tx_1 \) and for this \( x_1 \), \( x \in X \) from (i), there exists \( x_2 \) such that \( Bx_1 = Sx_2 \). Inductively, we can find a sequence \( \{y_n\} \) in \( X \) as follows:
\[
y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \text{ and } y_{2n} = Sx_{2n} = Bx_{2n-1} \text{ for } n = 1, 2, \ldots.
\]

From (iv), we have
\[
M(y_{2n+1}, y_{2n+2}, c, q t) = M(Ax_{2n+1}, Bx_{2n+1}, c, qt) \\
\geq \Phi\{M(Sx_{2n+1}, Tx_{2n+1}, c, t) - M(Ax_{2n}, Sx_{2n}, c, t)\} \\
= \Phi\{M(y_{2n+2}, y_{2n+2}, c, t) - M(y_{2n+1}, y_{2n+1}, c, t)\} \\
\geq \Phi\{M(y_{2n+2}, y_{2n+2}, c, t) - M(y_{2n+1}, y_{2n+1}, c, t)\}.
\]

From Lemma 2.1 and 2.4, we have
\[
M(y_{2n+1}, y_{2n+2}, c, q t) \geq \Phi\{M(y_{2n+1}, y_{2n+1}, c, t)\} > M(y_{2n+1}, y_{2n+1}, c, t).
\]

Similarly, we have also \( M(y_{2n+2}, y_{2n+2}, c, q t) > M(y_{2n+1}, y_{2n+1}, c, t) \).

Thus we have \( M(y_{2n+1}, y_{2n+1}, c, q t) > M(y_{2n+1}, y_{2n+1}, c, t) \) for \( n = 1, 2, \ldots \), and so
\[
M(y_{2n+1}, y_{2n+1}, c, t) > M(y_{2n+1}, y_{2n+1}, c, t/q^k) \\
> M(y_{2n}, y_{2n}, c, t/q^{k^2}) \\
> \cdots > M(y_{2}, y_{2}, c, t/q^{n}) \to 1, \text{ as } n \to \infty,
\]

and hence \( M(y_{2n+1}, y_{2n+1}, c, t) \to 1 \) as \( n \to \infty \) for any \( t > 0 \) and \( c \).

For each \( n \), \( 0 \leq t \leq 0 \), we can choose \( n_0 \) \( N \) such that
\[
M(y_{2n+1}, y_{2n+1}, c, t) \geq 1 - \epsilon \text{ for all } n > n_0.
\]

For \( m, n \), \( N \), we suppose \( m \geq n \). Then we have that
\[
M(y_{2m+1}, y_{2m+1}, c, t/m-n) \geq \Phi\{M(y_{2m+1}, y_{2m+1}, c, t/m-n) - M(y_{2m+1}, y_{2m+1}, c, t/m-n) \}
\geq (1-\epsilon) \ast (1-\epsilon) \ast \cdots \ast (1-\epsilon) \geq 1 - \epsilon.
\]
And hence \( \{y_n\} \) is a Cauchy sequence in \( X \).

Since \((X, M, *, t)\) is complete, \( \{y_n\} \) converges to some point \( z \in X \), and so \( \{Ax_{2n-2}\}, \{Sx_{2n}\}, \{Bx_{2n-1}\} \) and \( \{Tx_{2n-1}\} \) also converges to \( z \). From Lemma 2.3 and (iii),

\[
\text{ASx}_{2n} \rightarrow Sz \quad \text{(3.1)}
\]

and

\[
\text{BTx}_{2n-1} \rightarrow Tz. \quad \text{(3.2)}
\]

From (iv),

\[
M(\text{ASx}_{2n}, \text{BTx}_{2n-1}, c, qt) \geq \Phi \left\{ M(\text{SSx}_{2n}, \text{TTx}_{2n-1}, c, t) \cdot M(\text{ASx}_{2n}, \text{SSx}_{2n}, c, t) \cdot M(\text{BTx}_{2n-1}, \text{TTx}_{2n-1}, c, t) \cdot M(\text{ASx}_{2n}, \text{TTx}_{2n-1}, c, t) \right\}
\]

Taking limit as \( n \rightarrow \infty \), and using (3.1) and (3.2),

\[
M(Sz, Tz, c, qt) \geq \Phi \left\{ M(Sz, Sz, c, t) \cdot M(Sz, Tz, c, t) \cdot M(Tz, Tz, c, t) \cdot M(Sz, Tz, c, t) \right\}
\]

and hence,

\[
Sz = Tz. \quad \text{(3.3)}
\]

Now, from (iv),

\[
M(Az, \text{BTx}_{2n-1}, c, qt) \geq \Phi \left\{ M(\text{Sz}, \text{TTx}_{2n-1}, c, t) \cdot M(Az, Sz, c, t) \cdot M(BTx_{2n-1}, \text{TTx}_{2n-1}, c, t) \cdot M(Az, \text{TTx}_{2n-1}, c, t) \right\}
\]

which implies that taking limit as \( n \rightarrow \infty \), and using (3.3), (3.2),

\[
M(Az, Tz, c, qt) \geq \Phi \left\{ M(Sz, Sz, c, t) \cdot M(Az, Sz, c, t) \cdot M(Bz, Tz, c, t) \cdot M(Az, Tz, c, t) \right\}
\]

and hence

\[
Az = Tz. \quad \text{(3.4)}
\]

From (iv), (3.3) and (3.4),

\[
M(Az, Bz, c, qt) \geq \Phi \left\{ M(Sz, Tz, c, t) \cdot M(Az, Sz, c, t) \cdot M(Bz, Tz, c, t) \cdot M(Az, Tz, c, t) \right\}
\]

and so

\[
Az = Bz. \quad \text{(3.5)}
\]

From (3.3), (3.4) and (3.5),

\[
Az = Bz = Tz = Sz. \quad \text{(3.6)}
\]

Now, we show that \( Bz = z \). From (iv),

\[
M(Ax_{2n}, Bz, c, qt) \geq \Phi \left\{ M(Sx_{2n}, Tz, c, t) \cdot M(Ax_{2n}, Sx_{2n}, c, t) \cdot M(Bz, Tz, c, t) \cdot M(Ax_{2n}, Tz, c, t) \right\}
\]

which implies that taking limit as \( n \rightarrow \infty \) and using (3.9),

\[
M(z, Bz, c, qt) \geq \Phi \left\{ M(z, Tz, c, t) \cdot M(z, z, c, t) \cdot M(Bz, Tz, c, t) \cdot M(z, Tz, c, t) \right\}
\]

and hence \( Bz = z \). Thus from (3.6), \( z \) is a common fixed point of \( A, B, S \) and \( T \).
For uniqueness, let \( w \) be another common fixed point of \( A, B, S \) and \( T \). Then

\[
M(z, w, c, qt) = M(Az, Bw, c, qt) \\
\geq \Phi\{M(Sz, Tw, c, t) \cdot M(Az, Sz, c, t) \cdot M(Bw, Tw, c, t) \cdot M(Az, Tw, c, t)\} \\
\geq \Phi\{M(z, w, c, t)\} \\
> M(z, w, c, t).
\]

From Lemma 2.2, \( z = w \). This completes the proof of theorem.

**Theorem 3.1:** Let \((X, M, \ast)\) be a complete fuzzy 2- metric space and let \( A, B, S \) and \( T \) be mappings from \( X \) into itself such that the following conditions are satisfied:

(i) \( A^aX = T^uX \), \( B^bX = S^sX \), where \( a, b, s, u \in \mathbb{N} \),

(ii) \( S \) and \( T \) are continuous,

(iii) \( AS = SA \) and \( TB = BT \),

(iv) there exists \( q \in (0, 1) \) such that for every \( x, y \in X \) and \( t > 0 \),

\[
M(A^ax, B^by, c, qt) \geq \Phi\{M(S^sx, T^uy, c, t) \cdot M(A^ax, S^sx, c, t) \cdot M(B^by, T^uy, c, t) \cdot M(A^ax, T^uy, c, t)\}. 
\]

Then \( A, B, S \) and \( T \) have a unique common fixed point in \( X \).

**Proof.** From (iii), \( A^aS^s = S^sA^a \) and \( T^uB^b = B^bT^u \). We know that commutativity implies compatibility, and from Proposition 3.1, there exists a unique \( z \in X \) such that

\[
z=A^az=B^bz=S^sz=T^uz. \tag{3.1.1}
\]

Then we have

\[
Az = A(A^az) = A^a(Az), \quad Az = A(S^sz) = S^s(Az), \\
Bz = B(B^bz) = B^b(Bz), \quad Bz = B(T^uz) = T^u(Bz). \tag{3.1.2}
\]

Similarly,

\[
Sz = A^a(Sz), Tz = B^b(Tz), \\
Sz = S^s(Sz) \quad \text{and} \quad Tz = T^u(Tz). \tag{3.1.3}
\]

From (iv), (3.1.1) and (3.1.2), we get

\[
M(A^az, B^bz, c, qt) \geq \Phi\{M(S^sz, T^uz, c, t) \cdot M(A^az, S^sz, c, t) \cdot M(B^bz, T^uz, c, t) \cdot M(A^az, T^uz, c, t)\},
\]

and from (3.1.2) and (3.1.3),

\[
M(Az, Bz, qt) \geq \Phi\{M(Az, Bz, c, t) \cdot M(Az, Az, c, t) \cdot M(Bz, Bz, c, t) \cdot M(Az, Bz, c, t)\} \\
\geq \Phi\{(Az, Bz, c, t)\} \\
> (Az, Bz, c, t),
\]

and hence

\[
Az = Bz. \tag{3.1.4}
\]

Similarly,

\[
Sz = Tz \quad \text{and} \quad Az = Tz. \tag{3.1.5}
\]

From (3.1.4) and (3.1.5), we have

\[
Az = Bz = Sz = Tz. \tag{3.1.6}
\]

From (iv), (3.1.1) and (3.1.2), we have

\[
M(z, Bz, c, qt) = M(A^az, B^bz, c, t) \\
\geq \Phi\{M(S^sz, T^uz, c, t) \cdot M(A^az, S^sz, c, t) \cdot M(B^bz, T^uz, c, t) \cdot M(A^az, T^uz, c, t)\} \\
= \Phi\{M(z, Bz, c, t) \cdot M(z, z, c, t) \cdot M(Bz, Bz, c, t) \cdot M(Bz, Bz, c, t)\} \\
> M(z, Bz, c, t).
\]

203
From (3.1.6) and (3.1.7) 
\[ Z = Az = Bz = Sz = Tz. \]  
(3.1.8)

**Corollary 3.1 [11]:** Let \((X, M, *)\) be a complete fuzzy metric space and let \(A, B, S\) and \(T\) be mappings from \(X\) into itself satisfying (i) – (iii) of Theorem 3.1 and there exists \(q \in (0, 1)\) such that for every \(x, y \in X\), \(t > 0\) and \(c \in X\), 
\[ M(A^x, B^y, c, qt) \geq \Phi \{ M(S^x, T^y, c, t) \} \cdot M(A^x, S^x, c, t) \cdot M(B^y, T^y, c, t) \cdot M(B^y, S^x, c, 2t) \cdot M(A^x, T^y, c, t) \}. \]

Then \(A, B, S\) and \(T\) have a unique fixed point in \(X\).

**Proof.** Choose \(q \in (0, 1)\) such that for every \(x, y \in X\) and \(t > 0\), 
\[ M(A^x, B^y, c, qt) \geq \Phi \{ M(S^x, T^y, c, t) \} \cdot M(A^x, S^x, c, t) \cdot M(B^y, T^y, c, t) \cdot M(B^y, S^x, c, 2t) \cdot M(A^x, T^y, c, t) \}. \]

and hence, from Theorem 3.1, \(A, B, S\) and \(T\) have a unique fixed point in \(X\).

**Corollary 3.2.** Let \((X, M, *)\) be a complete fuzzy metric space and let \(A, B, S\) and \(T\) be mappings from \(X\) into itself satisfying (i) – (iii) of Theorem 3.1 and \(q \in (0, 1)\) such that for every \(x, y \in X\) and \(t > 0\), 
\[ M(A^x, B^y, c, qt) \geq \Phi \{ M(S^x, T^y, c, t) \} \cdot M(A^x, S^x, c, t) \cdot M(B^y, T^y, c, t) \cdot M(B^y, S^x, c, 2t) \cdot M(A^x, T^y, c, t) \}. \]

Then \(A, B, S\) and \(T\) have a unique fixed point in \(X\).

**Proof.** Choose \(q \in (0, 1)\) such that for every \(x, y \in X\) and \(t > 0\), 
\[ M(A^x, B^y, c, qt) \geq \Phi \{ M(S^x, T^y, c, t) \} \cdot M(A^x, S^x, c, t) \cdot M(B^y, T^y, c, t) \cdot M(B^y, S^x, c, 2t) \cdot M(A^x, T^y, c, t) \}. \]

and hence, from Corollary 3.1, \(A, B, S\) and \(T\) have a unique fixed point in \(X\).

**Corollary 3.3.** Let \((X, M, *)\) be a complete fuzzy metric space and let \(A, B, S\) and \(T\) be mappings from \(X\) into itself satisfying (i) – (iii) of Theorem 3.1 and \(q \in (0, 1)\) such that for every \(x, y \in X\) and \(t > 0\), 
\[ M(A^x, B^y, c, qt) \geq \Phi \{ M(S^x, T^y, c, t) \} \cdot M(A^x, S^x, c, t) \cdot M(B^y, T^y, c, t) \cdot M(B^y, S^x, c, 2t) \cdot M(A^x, T^y, c, t) \}. \]

Then \(A, B, S\) and \(T\) have a unique fixed point in \(X\).

**Proof.** Choose \(q \in (0, 1)\) such that for every \(x, y, c \in X\) and \(t > 0\), 
\[ M(A^x, B^y, c, qt) \geq \Phi \{ M(S^x, T^y, c, t) \} \cdot M(A^x, S^x, c, t) \cdot M(B^y, T^y, c, t) \cdot M(B^y, S^x, c, 2t) \cdot M(A^x, T^y, c, t) \}. \]

and hence, from Corollary 3.1, \(A, B, S\) and \(T\) have a unique fixed point in \(X\).
(iii) there exists $q \in (0, 1)$ such that for every $x, y, c \in X$, $t > 0$
\[ M(A^a x, A^a y, c, qt) \geq \Phi\{M(S^s x, T^u y, c, t) \cdot M(A^a x, S^s x, c, t) \cdot M(A^a y, T^u y, c, t) \cdot M(A^a x, T^u y, c, t)\} \]

In fact $A$, $S$ and $T$ have a unique common fixed point in $X$.

**Proof.** First, we show that the necessity of the conditions (i)-(iii). Suppose that $S z = z = T z$ for some $z \in X$.

Let $A x = z$ for all $x \in X$. Then we have $A^a X \cap T^u X \cap S^s X$ for $a, s, u \in N$ and the condition (i) is satisfied. For any $x \in X$, $A S x = z = S z = S A x$ and $A T x = z = T z = T A x$ and so $A S = S A$, $A T = T A$ and hence the condition (ii) is satisfied.

For some $q \in (0, 1)$, we have
\[ M(A^a x, A^a y, c, qt) = 1 \geq \Phi\{M(S^s x, T^u y, c, t) \cdot M(A^a x, S^s x, c, t) \cdot M(A^a y, T^u y, c, t) \cdot M(A^a x, T^u y, c, t)\} \]

for every $x, y \in X$ and $t > 0$. Thus the condition (iii) is satisfied.

Now, for the sufficiency of the conditions, let $A^a = B^b$ in Theorem 3.1. Then $A$, $S$ and $T$ have a unique common fixed point in $X$.

**Corollary 3.4.** Let $(X, M, *)$ be a complete fuzzy metric space. Then continuous mappings $S, T : X \to X$ have a common fixed point in $X$ if and only if there exists a mapping $A : X \to X$ satisfying (i)-(ii) of Theorem 3.2 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0,$
\[ M(A^a x, A^a y, c, qt) \geq \Phi\{M(S^s x, T^u y, c, t) \cdot M(S^s x, A^a x, c, t) \cdot M(A^a x, T^u y, c, t) \cdot M(A^a x, T^u y, c, t)\} \]

In fact $A, S$ and $T$ have a unique common fixed point in $X$.

**Corollary 3.5.** Let $(X, M, *)$ be a complete fuzzy metric space. Then continuous mappings $S, T : X \to X$ have a common fixed point in $X$ if and only if there exists a mapping $A : X \to X$ satisfying (i)-(ii) of Theorem 3.2 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0,$
\[ M(A^a x, A^a y, c, qt) \geq \Phi\{M(S^s x, T^u y, c, t) \cdot M(S^s x, A^a x, c, t) \cdot M(A^a x, T^u y, c, t) \cdot M(A^a x, T^u y, c, t)\} \]

In fact $A, S$ and $T$ have a unique common fixed point in $X$.

**References:**
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