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# Common Fixed Point Theorems for Four Mappings in Fuzzy 2-Metric Spaces

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**Abstract**
In this paper some common fixed point theorem have been proved as a generalization of result of Seong Hoon Cho [1] the conditions for continuous self mappings S,T of complete fuzzy 2-metric space (X, M,$*$) have been characterised to have a unique common fixed point in X.
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## 1. Introduction

The concept of fuzzy sets was developed extensively by many authors[2, 3, 6] and used in various fields. Recently sushil Sharma [9], Urmila Mishra and et. al.[8 ] proved fixed point theorems in fuzzy metric space and fuzzy 2-metric space.

Bijendra Singh and M. S. Chauhan [10] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sence of George and Veeramani [4 ]. Recently Seong Hoon Cho[1]
generalized this results and characterized the conditions for two continuous self mappings of complete fuzzy metric space.

In this paper, we have a generalization of the result obtained in [1] in fuzzy 2-metric space including a continuous function $\Phi:[0,1] \to [0,1]$.

## 2. Preliminaries

In this section, we give some definitions and lemmas. A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm on [0,1] if ([0, 1], $*$) is an abelian topological monoid with 1 such that $a * b \le c * d$, whenever $a \le c, b \le d$ for all a, b, c, d [0, 1]. Examples of t-norm are $a*b = ab$ and $a*b = \min\{a, b\}$.

**Definition 2.1**: The 3-tuple (X, M, $*$) is called a fuzzy 2-metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions:
(1) $M(x, y, z, t) > 0$,
(2) $M(x, y, z, t) = 1$ if and only if at least two out of three points are equal,
(3) $M(x, y z, t) = M(x, z, y, t) = M(y, z, x, t)$
(4) $M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3) \le M(x, y, z, t_1+t_2+t_3)$,
(5) $M(x, y, z \cdot) : (0, \infty) \to [0, 1]$ is continuous, for all x, y, z X and t, s > 0.
.
A sequence $\{x_n\}$ in a fuzzy 2-metric space (X, M, $*$) is said to be convergent to a point x X, if for each > 0 and each t>0, there exists $n_0 \ge N$ such that $M(x_n, x, c, t) > 1-\varepsilon$ for all c X for all $n \ge n_0$. Equivalently, a sequence $\{x_n\}$ in a fuzzy 2-metric space (X, M, $*$) converges to a point x X if
$\lim_{n\to\infty} M(x_n, x, c, t) = 1$, for all c X and t>0.

A sequence $\{x_n\}$ in a fuzzy metric space (X, M, $*$) is called Cauchy sequence if for each > 0 and each t > 0, there exists $n_0$ N such that $M(x_n, x_m, c, t) > 1-\varepsilon$ for all n, $m \ge n_0$ and for all c X. A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Self mappings A and B of a fuzzy 2-metric space (X, M, $*$) is said to be compatible if

$$\lim_{n\to\infty} M(ABx_n, BAx_n, c, t) = 1 \text{ , for all c X ,for all } t > 0,$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z, \text{ for some } z \quad X.$$

Fuzzy 2-metric space version of the following results will be used to prove our main results.

Lemma 2.1 [5]. Let (X, M, $*$) be a fuzzy metric space. Then for all x, y X, M(x, y, ·) is non-decreasing.

Lemma 2.2[1]. Let (X, M, $*$) be a fuzzy metric space with $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y X and $r * r \geq r$ for all r [0, 1]. If there exists $0 < q < 1$ such that for all x, y X and $t > 0$, $M(x, y, qt) \geq M(x, y, t)$, then x = y.

Lemma 2.3 [1]. Let (X, M, $*$) be a fuzzy metric space with $\lim_{t\to\infty} M(x, y ,t) = 1$
for all x, y X and $r * r \geq r$ for all r [0, 1] and let A and S be continuous self mappings of X and the pair [A,S] be compatible. Let $\{x_n\}$ be a sequence in X such that $Ax_n \to z$ and $Sx_n \to z$. Then $ASx_n \to Sz$.

Lemma 2.4[7]. The only t-norm $*$ satisfying $r * r \geq r$ for all r [0, 1] is the minimum t-norm, that is, $a * b = \min\{a, b\}$ for all a, b [0, 1].

From now on, let (X, M, $*$) be a fuzzy 2- metric space such that $\lim_{t\to\infty} M(x, y, z,t) = 1$
for all x, y X, $r * r \geq r$ for all r [0, 1] and $\Phi:[0,1] \to [0,1]$. A continuous function
Such that $\Phi(t) > t$, $o < t < 1$

**3. Main Results:**

In this section, we prove some common fixed point theorems. To prove our main result we will use the next Proposition which is a generalization of the result of [1].

**Proposition 3.1:** Let A, B, S and T be self maps on a complete fuzzy 2-metric space (X,M, $*$) where $*$ is a continuous t-norm defined by $a*b = \min\{a, b\}$ such that the following conditions are satisfied:

(i) AX TX, BX SX,
(ii) S and T are continuous,
(iii) the pairs [A,S] and [B, T] are compatible,
(iv) there exists q (0, 1) such that for every x, y X and $t > 0$,

$$M(Ax, By ,c,qt) \geq \Phi\{M(Sx,Ty,c,t) * M(Ax, Sx,c,t) * M(By,Ty,c,t) * M(Ax,Ty,c,t)\}.$$

Then A,B, S and T have a unique common fixed point in X.

**Proof.** Let $x_0$ X. From (i), there exists $x_1$ X such that $Ax_0 = Tx_1$ and for this $x_1$ X, from (i), there exists $x_2$ X such that $Bx_1 = Sx_2$. Inductively, we can find a sequence $\{y_n\}$ in X as follows:

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \text{ and } y_{2n} = Sx_{2n} = Bx_{2n-1} \text{ for } n = 1, 2 \cdots.$$

From (iv), we have

$$\begin{aligned} M(y_{2n+1}, y_{2n+2},c, qt) &= M(Ax_{2n}, Bx_{2n+1},c, qt) \\ &\geq \Phi\{M(Sx_{2n},Tx_{2n+1},c,t) * M(Ax_{2n},Sx_{2n},c,t) * \\ &\qquad M(Bx_{2n+1},Tx_{2n+1},c,t) * M(Ax_{2n},Tx_{2n+1},c,t)\} \\ &= \Phi\{M(y_{2n},y_{2n+1},c,t) * M(y_{2n+1},y_{2n},c,t) * \\ &\qquad M(y_{2n+2},y_{2n+1},c,t) * M(y_{2n+1},y_{2n+1},c,t)\} \\ &\geq \Phi\{M(y_{2n},y_{2n+1},c,t) * M(y_{2n+1},y_{2n+2},c,t)\}. \end{aligned}$$

From Lemma 2.1 and 2.4, we have

$$M(y_{2n+1}, y_{2n+2},c, qt) \geq \Phi\{M(y_{2n}, y_{2n+1},c, t)\} > M(y_{2n}, y_{2n+1},c, t).$$

Similarly, we have also $M(y_{2n+2}, y_{2n+3},c, qt) > M(y_{2n+1}, y_{2n+2},c, t)$.

Thus we have $M(y_{n+1}, y_{n+2},c, qt) > M(y_n, y_{n+1},c, t)$ for $n = 1, 2 \cdots$, and so

$$\begin{aligned} M(y_n, y_{n+1},c, t) &> M(y_n, y_{n-1},c, t/q) \\ &> M(y_{n-2}, y_{n-1},c, t/q^2) \\ &> \cdots > M(y_1,y_2,c,t/q^n) \to 1, \text{as } n \to \infty, \end{aligned}$$

and hence $M(y_n, y_{n+1},c, t) \to 1$ as $n \to \infty$ for any $t > 0$ and c

For each $> 0$ and each $t > 0$, we can choose $n_0$ N such that

$$M(y_n, y_{n+1},c, t) > 1-\varepsilon \text{ for all } n > n_0.$$

For m, n N, we suppose $m \geq n$. Then we have that

$$\begin{aligned} M(y_n, y_m, c, t) &\geq \Phi\{M(y_n,y_{n+1},c,t/m-n) * M(y_{n+1},y_{n+2},c,t/m-n) * \cdots \\ &* M(y_{m-1},y_m,c,t/m-n)\} \\ &> (1-\varepsilon) * (1-\varepsilon) * \cdots * (1-\varepsilon) \geq 1-\varepsilon \end{aligned}$$

And hence $\{y_n\}$ is a Cauchy sequence in X.

Since (X, M, $*$) is complete, $\{y_n\}$ converges to some point z X, and so $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z. From Lemma 2.3 and (iii),

$$ASx_{2n} \to Sz \tag{3.1}$$

and

$$BTx_{2n-1} \to Tz. \tag{3.2}$$

From (iv),

$$M(ASx_{2n}, BTx_{2n-1},c, qt) \geq \Phi\{M(SSx_{2n},TTx_{2n-1},c,t) * M(ASx_{2n},SSx_{2n},c,t) * M(BTx_{2n-1},TTx_{2n-1},c,t) * M(ASx_{2n},TTx_{2n-1},c,t)\}$$

Taking limit as $n \to \infty$, and using (3.1) and (3.2),

$$\begin{aligned} M(Sz, Tz,c, qt) &\geq \Phi\{M(Sz,Tz,c,t)*M(Sz,Sz,c,t)* M(Tz,Tz,c,t)*M(Sz,Tz,c,t)\} \\ &\geq \Phi\{M(Sz,Tz,c,t) * M(Sz,Tz,c,t)\} \\ &\geq \Phi\{M(Sz,Tz,c,t)\} \\ &> M(Sz,Tz,c,t), \end{aligned}$$

and hence,

$$Sz=Tz. \tag{3.3}$$

Now, from (iv),

$$M(Az,BTx_{2n-1},c, qt) \geq \Phi\{M(Sz,TTx_{2n-1},c,t) * M(Az,Sz,c,t) * M(BTx_{2n-1},TTx_{2n-1},c,t)*M(Az,TTx_{2n-1},c,t)\}$$

which implies that taking limit as $n \to \infty$, and using (3.2), (3.3),

$$\begin{aligned} M(Az,Tz,c, qt) &\geq \Phi\{M(Sz,Sz,c,t) * M(Az,Tz,c,t) * M(Tz,Tz,c,t) * M(Az,Tz,c,t)\} \\ &\geq \Phi\{M(Az,Tz,c,t)\} \\ &> M(Az,Tz,c,t), \end{aligned}$$

and hence

$$Az = Tz. \tag{3.4}$$

From (iv), (3.3) and (3.4),

$$\begin{aligned} M(Az, Bz,c, qt) &\geq \Phi\{M(Sz,Tz,c,t) * M(Az,Sz,c,t) * M(Bz,Tz,c,t) * M(Az,Tz,c,t)\} \\ &= \Phi\{M(Az,Az,c,t)*M(Az,Az,c,t) * M(Bz,Az,c,t) * M(Az,Az,c,t)\} \\ &\geq \Phi\{M(Az,Bz,c,t)\} \\ &> M(Az,Bz,c,t), \end{aligned}$$

and so

$$Az = Bz. \tag{3.5}$$

From (3.3), (3.4) and (3.5),

$$Az = Bz = Tz = Sz. \tag{3.6}$$

Now, we show that Bz = z. From (iv),

$$M(Ax_{2n}, Bz,c,qt) \geq \Phi\{M(Sx_{2n},Tz,c,t) * M(Ax_{2n},Sx_{2n},c,t) * M(Bz,Tz,c,t) * M(Ax_{2n},Tz,c,t)\}$$

which implies that taking limit as $n \to \infty$ and using (3.9),

$$\begin{aligned} M(z, Bz,c,qt) &\geq \Phi\{M(z,Tz,c,t) * M(z,z,c,t) * M(Bz,Tz,c,t) * M(z,Tz,c,t)\} \\ &\geq \Phi\{M(z,Bz,c,t) * M(Bz,Bz,c,t) * M(z,Bz,c,t)\} \\ &\geq \Phi\{M(z,Bz,c,t)\} \\ &> M(z,Bz,c,t), \end{aligned}$$

And hence Bz = z. Thus from (3.6), z is a common fixed point of A,B, S and T.

For uniqueness, let w be another common fixed point of A,B, S and T. Then

$$\begin{aligned} M(z, w, c, qt) &= M(Az,Bw,c, qt) \\ &\geq \Phi\{M(Sz,Tw,c,t)*M(Az,Sz,c,t)* \\ &\quad M(Bw,Tw,c,t * M(Az,Tw,c,t)\} \\ &\geq \Phi\{M(z,w,c,t)\} \\ &> M(z,w,c,t). \end{aligned}$$

From Lemma 2.2, z = w. This complete the proof of theorem.

**Theorem 3.1:** Let (X, M, $*$) be a complete fuzzy 2- metric space and let A,B, S and T be mappings from X into itself such that the following conditions are satisfied:

(i) $A^aX \quad T^uX$, $B^bX \quad S^sX$, where a, b, s, u  N,

(ii) S and T are continuous,

(iii) AS = SA and TB = BT,

(iv) there exists q  (0, 1) such that for every x, y  X and t > 0,

$$M(A^a x, B^b y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t) * M(A^a x, S^s x,c, t) * M(B^b y, T^u y,c, t) * M(A^a x, T^u y,c, t)\}.$$

Then A, B, S and T have a unique common fixed point in X.

**Proof.** From (iii), $A^aS^s = S^sA^a$ and $T^uB^b = B^bT^u$. We know that commutatively implies compatibility, and from Proposition 3.1, there exists a unique z  X such that

$$z=A^az=B^bz=S^sz=T^uz. \quad (3.1.1)$$

Then we have

$$\begin{aligned} &Az = A(A^az) = A^a(Az),\ Az = A(S^sz) = S^s(Az), \\ &Bz = B(B^bz) = B^b(Bz) \text{ and } Bz = B(T^uz) = T^u(Bz). \end{aligned} \quad (3.1.2)$$

Similarly,

$$\begin{aligned} &Sz=A^a(Sz),Tz=B^b(Tz), \\ &Sz=S^s(Sz) \text{ and } Tz=T^u(Tz). \end{aligned} \quad (3.1.3)$$

From (iv), (3.1.1) and (3.1.2), we get

$$M(A^aAz,B^bBz,c, qt) \geq \Phi\{M(S^sAz,T^uBz,c,t) * M(A^aAz,S^sAz,c,t) * M(B^bBz,T^uBz,c,t) * (A^aAz,T^uBz,c,t)\},$$

and from (3.1.2) and (3.1.3),

$$\begin{aligned} M(Az,Bz, qt) &\geq \Phi\{M(Az,Bz,c,t) * M(Az,Az,c,t) * \\ &\quad M(Bz,Bz,c,t) * M(Az,Bz,c,t)\} \\ &\geq \Phi\{(Az,Bz,c, t)\} \\ &> (Az,Bz,c, t), \end{aligned}$$

and hence

$$Az = Bz. \quad (3.1.4)$$

Similarly,

$$Sz=Tz \text{ and } Az=Tz. \quad (3.1.5)$$

From (3.1.4) and (3.1.5), we have

$$Az = Bz = Sz = Tz \quad (3.1.6)$$

From (iv), (3.1.1) and (3.1.2), we have

$$\begin{aligned} M(z, Bz, c, qt) &= M(A^az, B^bBz,c, t) \\ &\geq \Phi\{M(S^sz,T^uBz,c,t) * M(A^az,S^sz,c,t) * \\ &\quad M(B^bBz,T^uBz,c,t) * M(A^az,T^uBz,c,t)\} \\ &= \Phi\{M(z,Bz,c,t)*M(z,z,c,t)* \\ &\quad M(Bz,Bz,c,t)*M(Bz,Bz,c,t)\} \\ &\geq \Phi\{M(z,Bz,c ,t)\} \end{aligned}$$

$> M(z,Bz,c,t),$

and hence

$z = Bz.$ (3.1.7)

From (3.1.6) and (3.1.7)

$$Z = Az = Bz = Sz = Tz. \quad (3.1.8)$$

**Corollary 3.1 [11]:** Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S and T be mappings from X into itself satisfying (i) − (iii) of Theorem 3.1 and there exists q (0, 1) such that for every x, y X , t > 0 and c X

$$M(A^a x, B^b y, c, qt) \geq \Phi\{M(S^s x, T^u y, c, t) * M(A^a x, S^s x ,c, t) * M(B^b y, T^u y, c, t) * M(B^b y, S^s x, c, 2t) * M(A^a x, T^u y, c, t)\}.$$

Then A, B, S and T have a unique fixed point in X.

**Proof.** We have

$$M(A^a x, B^b y, qt) \geq \Phi\{M(S^s x, T^u y, t) * M(A^a x, S^s x, t) * M(B^b y, T^u y, t) * M(B^b y,S^s x,2t) * M(A^a x,T^u y,t) \}$$

$$\geq \Phi\{M(S^s x,T^u y,t) * M(A^a x,S^s x,t) * M(B^b y,T^u y,t) * M(S^s x,T^u y,t) * M(T^u y,B^b y,t) * M(A^a x,T^u y,t) \}$$

$$\geq \Phi\{M(S^s x,T^u y,t) * M(A^a x,S^s x,t) * M(B^b y,T^u y,t) * M(A^a x,T^u y,t) \}$$

and hence, from Theorem 3.1, A,B, S and T have a unique fixed point in X.

**Corollary 3.2** . Let (X, M, ∗) be a complete fuzzy metric space and let A,B, S and T be mappings from X into itself satisfying (i) − (iii) of Theorem 3.1 and there exists q (0, 1) such that for every x, y X and t > 0, $M(A^a x, B^b y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t)\}$. Then A, B, S and T have a unique common fixed point in X.

**Proof**. Choose q (0, 1) such that for every x, y X and t > 0,

$$M(A^a x, B^b y, c, qt) \geq M(S^s x, T^u y, t).$$

Then we have

$$\begin{aligned} M(A^a x, B^b y,c, qt) &\geq \Phi\{M(S^s x, T^u y,c, t)\} \\ &= \Phi\{M(S^s x, T^u y,c, t) * 1\} \\ &\geq \Phi\{M(S^s x,T^u y,c,t) * M(A^a x,S^s x,c,t) * M(S^s x,B^b y,c,2t) * M(B^b y,T^u y,c,t) * (T^u y,A^a x,c,t)\} \end{aligned}$$

and hence, from Corollary 3.1, A,B, S and T have a unique fixed point in X. □

**Corollary 3.3**. Let (X, M, ∗) be a complete fuzzy metric space and let A,B, S and T be mappings from X into itself satisfying (i) − (iii) of Theorem 3.1 and there exists q (0, 1) such that for every x, y X and t > 0, $M(A^a x, B^b y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t) * M(S^s x, A^a x,c, t) * M(A^a x, T^u y,c, t)\}$. Then A, B, S and T have a unique common fixed point in X.

**Proof**. Choose q (0, 1) such that for every x, y, c X and t > 0, $M(A^a x, B^b y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t) * M(S^s x, A^a x,c, t) * M(A^a x, T^u y,c, t)\}$.

Then we have

$$\begin{aligned} &\Phi\{M(S^s x, T^u y,c, t) * M(S^s x, A^a x,c, t) * M(A^a x, T^u y,c, t) \\ &= \Phi\{M(S^s x,T^u y,c,t) * M(S^s x,A^a x,c,t) * M(A^a x,T^u y,c,t) * 1\} \\ &\geq \Phi\{M(S^s x,T^u y,c,t) * M(S^s x,A^a x,c,t) * M(A^a x,T^u y,c,t) * M(S^s x,B^b y,c,2t) * M(B^b y,T^u y,c,t) * M(T^u y,S^s x,c,t) \} \\ &\geq \Phi\{M(S^s x,T^u y,c,t) * M(S^s x,A^a x,c,t) * M(A^a x,T^u y,c,t) * M(S^s x,B^b y,c,2t) * M(B^b y,T^u y,c,t)\} \end{aligned}$$

and hence, from Corollary 3.1, A,B, S and T have a unique fixed point in X. □

**Theorem 3.2**. Let (X, M, ∗) be a complete fuzzy metric space. Then continuous mappings S, T : X → X have a common fixed point in X if and only if there exists a mapping A : X → X such that the following conditions are satisfied:

(i) $A^a X \quad T^u X \cap S^s X$, where a, s,u N,

(ii) AS = SA and TA = AT,

(iii) there exists q   (0, 1) such that for every x, y, c   X, t > 0

$$M(A^a x, A^a y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t) * M(A^a x, S^s x,c, t) * M(A^a y,T^u y,c,t) * M(A^a x,T^u y,c,t)\}.$$

In fact A, S and T have a unique common fixed point in X.

**Proof**. First, we show that the necessity of the conditions (i)-(iii). Suppose that Sz =z =Tz for some z   X. Let Ax = z for all x   X. Then we have $A^a X$   $T^u X \cap S^s X$ for a, s,u   N and the condition (i) is satisfied. For any x   X, ASx = z = Sz = SAx and ATx = z = Tz = TAx and so AS = SA, AT = TA and hence the condition (ii) is satisfied.

For some q   (0, 1), we have

$$M(A^a x, A^a y,c, qt) = 1 \geq \Phi\{M(S^s x, T^u y,c, t) * M(A^a x, S^s x,c, t) * M(A^a y,T^u y,c,t) * M(A^a x,T^u y,c,t) \}$$

for every x, y   X and t > 0. Thus the condition (iii) is satisfied.

Now, for the sufficiency of the conditions, let $A^a = B^b$ in Theorem 3.1. Then A, S and T have a unique common fixed point in X.

**Corollary 3.4**. Let (X, M, $*$) be a complete fuzzy metric space. Then continuous mappings S, T : X → X have a common fixed point in X if and only if there exists a mapping A : X → X satisfying (i)−(ii) of Theorem 3.2 and there exists q   (0,1) such that for every x,y   X and t>0,

$$M (A^a x, A^a y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t)\}.$$

In fact A,S and T have a unique common fixed point in X.

**Corollary 3.5.** Let (X, M, $*$) be a complete fuzzy metric space. Then continuous mappings S, T : X → X have a common fixed point in X if and only if there exists a mapping A : X → X satisfying (i)−(ii) of Theorem 3.2 and there exists q   (0, 1) such that for every x, y   X and t > 0,

$$M(A^a x, A^a y,c, qt) \geq \Phi\{M(S^s x, T^u y,c, t) * M(S^s x, A^a x,c, t) * M(A^a x, T^u y,c, t)\}.$$

In fact A, S and T have a unique common fixed point in X.

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