Mathematical Theory and Modeling www.iiste.org
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.3, No.6, 2013-Selected from International Conference on Recent Trends in Applied Sciences with Engineering Applications

A Common Fixed Point Theorem for Six Mappings in G-Banach Space with Weak-Compatibility

Ranjeeta Jain
Infinity Management and Engineering college,Pathariya jat Road Sagar (M.P.)470003

ABSTRACT
The aim of this paper is to introduce the concept of G-Banach Space and prove a common fixed point theorem for six mappings in G-Banach spaces with weak–compatibility.
Keywords: Fixed point , common fixed point , G-Banach Space , Continuous mappings , weak compatible mappings.

Introduction : This is well know that the fundamental contraction principle for proving fixed points results is the Banach Contraction principle. There have been a number of generalization of metric space and Banach space. One such generalization is G-Banach space.The concept of G-Banach space is introduce by Shrivastava R,Animesh,Yadav R.N.[4] which is a probable modification of the ordinary Banach Space.

Recently in 2012,R.K. Bharadwaj [2] introduced fixed point theorems in G-Banach Space through weak compatibility and gave the following fixed point theorems for four mappings-

Theorems[A]: Let X be a G-Banach Space , such that ∇ Satisfy property with α - α ≤ 1. If A , B , S and T be mapping from X into itself satisfying the following condition:

I. $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$ and T(X) or S(X) is a closed subset of X.
II. The Pair (A,S) and (B,T) are weakly compatible,
III. For all $x,y \in X$

$$\|Ax - By\|_g \le \le k_1\left(\frac{\|Sx - Ax\|_g \|Sx - By\|_g}{\|Sx - Ty\|_g} \nabla \frac{\|Tx - Ax\|_g \|Ty - By\|_g}{\|Sx - Ty\|_g}\right)$$
$$+ k_2 \max\left(\frac{\|Sx - Ax\|_g \|Ty - By\|_g}{\|Sx - Ty\|_g} \nabla \frac{\|Sx - By\|_g \|Ty - Ax\|_g}{\|Sx - Ty\|_g}\right)$$
$$+ k_3\left(\|Sx - Ax\|_g \nabla \|Ty - By\|_g \nabla \|Sx - By\|_g \nabla \|Ty - Ax\|_g \nabla \|Sx - Ty\|_g\right)$$

Where $k_1, k_2, k_3 > 0$ and $0 < k_1 + k_2 + k_3 < 1$. Then A , B , S and T have a unique common fixed point in X.

In 1980, Singh and Singh[5] gave the following theorem on metric space for self maps which is use to our main result-

Theorems[B]: Let P, Q and T be self maps of a metric space (X, d) such that
(i) PT = TP and QT = TQ, (ii) $P(X) \cup Q(X) \subseteq T(X)$, (iii) T is continuous,
(iv) $d(Px , Qy) \le c\,\lambda(x, y)$,

where $\lambda(x, y) = \max\{d(Tx, Ty), d(Px, Tx), d(Qy, Ty), \frac{1}{2}[d(Px, Ty)+d(Qy, Ty)]\}$

for all $x, y \in X$ and $0 \le < 1$. Further if
(v) X is complete then P, Q, T have a unique common fixed point in X.

Just we recall the some definition of G-Banach space for the sake of completeness which as follows-

N be the set of natural numbers and R^+ be the set of all positive numbers let binary operation $\nabla : R^+ \times R^+ \to R^+$ satisfies the following conditions:

i. ∇ is associative and commutative,
ii. ∇ is continuous.

Five typical example are as follows:

i. $a \nabla b = \max(a,b)$
ii. $a \nabla b = a+b$
iii. $a \nabla b = a.b$
iv. $a \nabla b = a.b+a+b$

v. $a \nabla b = \dfrac{ab}{\max(a,b,1)}$

Definition 1: The binary operation ∇ is said to satisfy α-property if there exists a positive real number α , such that $a \nabla b \le \alpha \max(a,b)$ for every a,b Є R^+

Example:If we define $a \nabla b = a + b$ for each a,b Є R^+ then for $\alpha \ge 2$, we have

$a \nabla b \le \alpha \max(a,b)$

if we define $a \nabla b = \dfrac{ab}{\max(a,b,1)}$ for each a,b Є R^+ then for $\alpha \ge 1$, we have

$a \nabla b \le \alpha \max(a,b)$

Definition 2: Let x be a nonempty set ,A Generalized Normed Space on X is a function $\| \|_g : x \times x \to R^+$ that satisfies the following conditions for each x,y,z, Є X

1. $\|x-y\|_g > 0$
2. $\|x-y\|_g = 0$ if and only if x = y
3. $\|x-y\|_g = \|y-x\|_g$
4. $\|\alpha x\|_g = |\alpha| \|x\|_g$ for any scalar α
5. $\|x-y\|_g \le \|x-z\|_g \nabla \|z-x\|_g$

The pair (x, $\| \|_g$) is called generalized Normed Space or simply G-Normed Space.

Definition 3: A Sequence in X is said converges to x if $\|x_n-x\|_g \to 0$, as $n \to \infty$. That is for each $\epsilon > 0$ there exists n_0 Є N such that for every $n \ge n_0$ implies that $\|x_n-x\|_g < \epsilon$

Definition 4: A sequence $\{x_n\}$ is said to be Cauchy sequence if for every $\epsilon > 0$ there exists $n_0 \in N$ such that $\|x_m-x_n\|_g < \epsilon$ for each $m,n \ge n_0$. G-Normed Space is said to be G-Banach Space if for every Cauchy sequence is converges in it.

Definition 5: Let (x, $\| \|_g$) be a G-Normed Space for r>0 we define

$B_g(x,r) = \{y \in X : \|x-y\|_g < r\}$

Let X be a G-normed Space and A be a subset of X, then for every x ϵ A, there exists r > 0 such that $B_g(x,r) \subseteq A$, then the subset A is called open subset of X. A subset A of X is said to be closed if the complement of A is open in X.

Definition 6: Let A and S be mappings from a G-Banach space X into itselt. Then the mappings are said to be weakly compatible if they are commute at there coincidence point that is Ax= Sx implies that ASx = SAx

Here we generalized and extend the results of R.K.Bhardwaj[2] (theorem A) for six mappings opposed to four mappings in G-Banach space using the concept of weak-compatibility.

Main Result :

THEOREM(1) : Let X be a G-Banach Space , such that ∇ Satisfy property with $\alpha - \alpha \le 1$. If P , Q , A , B , S and T be mapping from X into itself satisfying the following condition:

I. $A(X) \subseteq Q(X) \cup T(X)$, $B(X) \subseteq P(X) \cup S(X)$ and T(X) or S(X) is a closed subset of X.

II. The Pair (A,S) and (P,S) , (B,T) and (Q ,T) are weakly compatible,

III. For all $x,y \in X$

$$\|Ax - By\|_g \le$$

$$\le \alpha \max \begin{pmatrix} \|Sx-By\|_g \|Ty-Ax\|_g \nabla \dfrac{\|Px-Ax\|_g \|Qy-By\|_g}{\|Sx-Py\|_g} \nabla \dfrac{\|Sx-Ty\|_g \|Qy-By\|_g}{\|Qy-Ax\|_g + \|Px-Ax\|_g} \nabla \\ \left\{ \dfrac{\|Px-Qx\|_g + \|Bx-Ty\|_g}{\|Px-By\|_g} \right\} \|Sx-By\|_g \end{pmatrix}$$

$$+ \beta \max \begin{pmatrix} \|Sx-Ty\|_g \nabla \|Px-Sx\|_g \nabla \|Qy-By\|_g \nabla \|Px-Ty\|_g \nabla \|Qy-Sx\|_g \nabla \\ \dfrac{\|Px-Ax\|_g \|Qy-By\|_g}{\|Px-Qy\|_g} \nabla \dfrac{\|Sx-Ty\|_g \|Qy-Sx\|_g}{\|Px-Qy\|_g} \end{pmatrix}$$

Where α , β > 0 and $0 < \alpha + \beta < 1$. Then P , Q , A , B , S and T have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary point in X then by (i) we choose a point x_1 in X such that $y_0 = Ax_0 = Tx_1 = Qx_1$ and $y_1 = Bx_1 = Sx_2 = Px_2$.

In general there exists a sequence $\{y_n\}$ such that
$y_{2n} = Ax_{2n} = Tx_{2n+1} = Qx_{2n+1}$ and
$y_{2n+1} = Bx_{2n+1} = Sx_{2n+2} = Px_{2n+2}$, for $n = 1, 2, 3,$ --------------
we claim that the sequence $\{y_n\}$ is a Cauchy sequence.
By (iii) we have

$$\|y_{2n} - y_{2n+1}\|_g = \|Ax_{2n} - Bx_{2n+1}\|_g$$

$$\leq \alpha \max\left(\begin{array}{l} \|Sx_{2n} - Bx_{2n+1}\|_g \|Tx_{2n+1} - Ax_{2n}\|_g \nabla \dfrac{\|Px_{2n} - Ax_{2n}\|_g \|Qx_{2n+1} - Bx_{2n+1}\|_g}{\|Sx_{2n} - Px_{2n+1}\|_g} \nabla \\ \dfrac{\|Sx_{2n} - Tx_{2n+1}\|_g \|Qx_{2n+1} - Bx_{2n+1}\|_g}{\|Qx_{2n+1} - Ax_{2n}\|_g + \|Px_{2n} - Ax_{2n}\|_g} \nabla \left\{ \dfrac{\|Px_{2n} - Qx_{2n}\|_g + \|Bx_{2n} - Tx_{2n+1}\|_g}{\|Px_{2n} - Bx_{2n+1}\|_g} \right\} \|Sx_{2n} - Bx_{2n+1}\|_g \end{array} \right)$$

$$+ \beta \max\left(\begin{array}{l} \|Sx_{2n} - Tx_{2n+1}\|_g \nabla \|Px_{2n} - Sx_{2n}\|_g \nabla \|Qx_{2n+1} - Bx_{2n+1}\|_g \nabla \|Px_{2n} - Tx_{2n+1}\|_g \nabla \|Qx_{2n+1} - Sx_{2n}\|_g \nabla \\ \dfrac{\|Px_{2n} - Ax_{2n}\|_g \|Qx_{2n+1} - Bx_{2n+1}\|_g}{\|Px_{2n} - Qx_{2n+1}\|_g} \nabla \dfrac{\|Sx_{2n} - Tx_{2n+1}\|_g \|Qx_{2n+1} - Sx_{2n}\|_g}{\|Px_{2n} - Qx_{2n+1}\|_g} \end{array} \right)$$

$$\leq \alpha \max\left(\begin{array}{l} \|y_{2n-1} - y_{2n-1}\|_g \|y_{2n} - y_{2n}\|_g \nabla \dfrac{\|y_{2n-1} - y_{2n}\|_g \|y_{2n} - y_{2n+1}\|_g}{\|y_{2n-1} - y_{2n}\|_g} \nabla \\ \dfrac{\|y_{2n-1} - y_{2n}\|_g \|y_{2n} - y_{2n+1}\|_g}{\|y_{2n} - y_{2n}\|_g + \|y_{2n-1} - y_{2n}\|_g} \nabla \left\{ \dfrac{\|y_{2n-1} - y_{2n-1}\|_g + \|y_{2n} - y_{2n+1}\|_g}{\|y_{2n-1} - y_{2n}\|_g} \right\} \|y_{2n-1} - y_{2n}\|_g \end{array} \right)$$

$$+ \beta \max\left(\begin{array}{l} \|y_{2n-1} - y_{2n}\|_g \nabla \|y_{2n-1} - y_{2n-1}\|_g \nabla \|y_{2n} - y_{2n+1}\|_g \nabla \|y_{2n-1} - y_{2n}\|_g \nabla \|y_{2n} - y_{2n-1}\|_g \nabla \\ \dfrac{\|y_{2n-1} - y_{2n}\|_g \|y_{2n} - y_{2n+1}\|_g}{\|y_{2n-1} - y_{2n}\|_g} \nabla \dfrac{\|y_{2n-1} - y_{2n}\|_g \|y_{2n} - y_{2n-1}\|_g}{\|y_{2n-1} - y_{2n}\|_g} \end{array} \right)$$

$$\|y_{2n} - y_{2n+1}\|_g \leq (\alpha + \beta) \|y_{2n-1} - y_{2n}\|_g$$

That is by induction we can show that

$$\|y_{2n} - y_{2n+1}\|_g \leq (\alpha + \beta)^n \|y_0 - y_1\|_g \|y_{2n} - y_{2n+1}\|_g$$

As $n \to \infty$ $\|y_{2n} - y_{2n+1}\|_g \to 0$, for any integer $m \geq n$
It follows that the sequence $\{y_n\}$ is a Cauchy sequence which converges to $y \in X$.
This implies that $\lim n \to \infty\ y_n = \lim n \to \infty\ Ax_{2n} = \lim n \to \infty\ Tx_{2n+1} = \lim n \to \infty\ Qx_{2n+1} = \lim n \to \infty$ $Bx_{2n+1} = \lim n \to \infty\ Sx_{2n+2} = \lim n \to \infty\ Px_{2n+2} = y$
Now let us assume that T(X) is closed subset of X, then there exists $v \in X$ such that
$Tv = Qv = y$
We now prove that $Bv = y$,
By (iii) we get

$$\|Ax_{2n} - Bv\|_g \leq \alpha \max\left(\begin{array}{l} \|Sx_{2n} - Bv\|_g \|Tv - Ax_{2n}\|_g \nabla \dfrac{\|Px_{2n} - Ax_{2n}\|_g \|Qv - Bv\|_g}{\|Sx_{2n} - Pv\|_g} \nabla \\ \dfrac{\|Sx_{2n} - Tv\|_g \|Qv - Bv\|_g}{\|Qv - Ax_{2n}\|_g + \|Px_{2n} - Ax_{2n}\|_g} \nabla \left\{ \dfrac{\|Px_{2n} - Qx_{2n}\|_g + \|Bx_{2n} - Tv\|_g}{\|Px_{2n} - Bv\|_g} \right\} \|Sx_{2n} - Bv\|_g \end{array} \right)$$

$$+ \beta \max\left(\begin{array}{l} \|Sx_{2n} - Tv\|_g \nabla \|Px_{2n} - Sx_{2n}\|_g \nabla \|Qv - Bv\|_g \nabla \|Px_{2n} - Tv\|_g \nabla \|Qv - Sx_{2n}\|_g \nabla \\ \dfrac{\|Px_{2n} - Ax_{2n}\|_g \|Qv - Bv\|_g}{\|Px_{2n} - Qv\|_g} \nabla \dfrac{\|Sx_{2n} - Tv\|_g \|Qv - Sx_{2n}\|_g}{\|Px_{2n} - Qv\|_g} \end{array} \right)$$

$$\|y - Bv\|_g \leq (\alpha + \beta) \|y - Bv\|_g$$

Which is contradiction , it follows that Bv = y = Tv = Qv . Since (B,T) and (Q,T) are weakly compatible mappings , then we have

BTv = TBv and QTv = TQv

By = Ty and Qy = Ty

Which implies By = Qy.

Now we prove that By = y for this by using (iii) we get

$$\|Ax_{2n} - By\|_g \le \alpha \max\left(\begin{array}{l} \|Sx_{2n} - By\|_g \|Ty - Ax_{2n}\|_g \,\nabla\, \dfrac{\|Px_{2n} - Ax_{2n}\|_g \|Qy - By\|_g}{\|Sx_{2n} - Py\|_g} \,\nabla \\ \dfrac{\|Sx_{2n} - Ty\|_g \|Qy - By\|_g}{\|Qy - Ax_{2n}\|_g + \|Px_{2n} - Ax_{2n}\|_g} \,\nabla \left\{\dfrac{\|Px_{2n} - Qx_{2n}\|_g + \|Bx_{2n} - Ty\|_g}{\|Px_{2n} - By\|_g}\right\} \|Sx_{2n} - By\|_g \end{array}\right)$$

$$+ \beta \max\left(\begin{array}{l} \|Sx_{2n} - Ty\|_g \,\nabla\, \|Px_{2n} - Sx_{2n}\|_g \,\nabla\, \|Qy - By\|_g \,\nabla\, \|Px_{2n} - Ty\|_g \,\nabla\, \|Qy - Sx_{2n}\|_g \,\nabla \\ \dfrac{\|Px_{2n} - Ax_{2n}\|_g \|Qy - By\|_g}{\|Px_{2n} - Qy\|_g} \,\nabla\, \dfrac{\|Sx_{2n} - Ty\|_g \|Qy - Sx_{2n}\|_g}{\|Px_{2n} - Qy\|_g} \end{array}\right)$$

$$\|Ax_{2n} - By\|_g \le (\alpha + \beta)\ \|y - By\|_g$$

Which is contradiction.

Thus By = y = Ty = Qy --------------------(A)

Since

$B(X) \subseteq P(X) \cup S(X)$, there exists $w \in X$. such that sw = y = Pw . we show that Aw = y

From (iii) we have

$$\|Aw - By\|_g \le \alpha \max\left(\begin{array}{l} \|Sw - By\|_g \|Ty - Aw\|_g \,\nabla\, \dfrac{\|Pw - Aw\|_g \|Qy - By\|_g}{\|Sw - Py\|_g} \,\nabla \\ \dfrac{\|Sw - Ty\|_g \|Qy - By\|_g}{\|Qy - Aw\|_g + \|Pw - Aw\|_g} \,\nabla \left\{\dfrac{\|Pw - Qw\|_g + \|Bw - Ty\|_g}{\|Pw - By\|_g}\right\} \|Sw - By\|_g \end{array}\right)$$

$$+ \beta \max\left(\begin{array}{l} \|Sw - Ty\|_g \,\nabla\, \|Pw - Sw\|_g \,\nabla\, \|Qy - By\|_g \,\nabla\, \|Pw - Ty\|_g \,\nabla\, \|Qy - Sw\|_g \,\nabla \\ \dfrac{\|Pw - Aw\|_g \|Qy - By\|_g}{\|Pw - Qy\|_g} \,\nabla\, \dfrac{\|Sw - Ty\|_g \|Qy - Sw\|_g}{\|Pw - Qy\|_g} \end{array}\right)$$

$$\|Aw - y\|_g \le (\alpha + \beta)\ \|Aw - y\|_g$$

Which is contradiction , so that Aw = y = Sw = Pw .

Since (A , S) and (P , S) are weakly compatible , then

ASw = Saw and PSw = SPw

Ay = Sy Py = Sy

Therefore Ay = Py

Now we Show that Ay = y,

From (iii) we have

$$\|Ay - By\|_g \le \alpha \max\left(\begin{array}{l} \|Sy - By\|_g \|Ty - Ay\|_g \,\nabla\, \dfrac{\|Py - Ay\|_g \|Qy - By\|_g}{\|Sy - Py\|_g} \,\nabla \\ \dfrac{\|Sy - Ty\|_g \|Qy - By\|_g}{\|Qy - Ay\|_g + \|Py - Ay\|_g} \,\nabla \left\{\dfrac{\|Py - Qy\|_g + \|By - Ty\|_g}{\|Py - By\|_g}\right\} \|Sy - By\|_g \end{array}\right)$$

$$+\beta \max\left(\begin{array}{l} \|Sy - Ty\|_g \nabla \|Py - Sy\|_g \nabla \|Qy - By\|_g \nabla \|Py - Ty\|_g \nabla \|Qy - Sy\|_g \nabla \\ \dfrac{\|Py - Ay\|_g \|Qy - By\|_g}{\|Py - Qy\|_g} \nabla \dfrac{\|Sy - Ty\|_g \|Qy - Sy\|_g}{\|Py - Qy\|_g} \end{array} \right)$$

$\|Ay - y\|_g \leq (\alpha + \beta)\, \|Ay - y\|_g$

Which is contradiction thus $Ay = y$ and therefore

$Ay = Sy = Py = y$ ---(B)

Now from equation (A) and (B) we get

$Ay = By = Py = Qy = Sy = Ty = y$.

Hence y is a common fixed point of A ,B , S , T , P and Q .

Uniqueness:

Let us assume that x is another fixed point of A ,B , S , T , P and Q different from y in X.

Then from (iii) we have

$$\|Ax - By\|_g \leq$$

$$\leq \alpha \max\left(\begin{array}{l} \|Sx - By\|_g \|Ty - Ax\|_g \nabla \dfrac{\|Px - Ax\|_g \|Qy - By\|_g}{\|Sx - Py\|_g} \nabla \dfrac{\|Sx - Ty\|_g \|Qy - By\|_g}{\|Qy - Ax\|_g + \|Px - Ax\|_g} \nabla \\ \left\{ \dfrac{\|Px - Qx\|_g + \|Bx - Ty\|_g}{\|Px - By\|_g} \right\} \|Sx - By\|_g \end{array} \right)$$

$$+\beta \max\left(\begin{array}{l} \|Sx - Ty\|_g \nabla \|Px - Sx\|_g \nabla \|Qy - By\|_g \nabla \|Px - Ty\|_g \nabla \|Qy - Sx\|_g \nabla \\ \dfrac{\|Px - Ax\|_g \|Qy - By\|_g}{\|Px - Qy\|_g} \nabla \dfrac{\|Sx - Ty\|_g \|Qy - Sx\|_g}{\|Px - Qy\|_g} \end{array} \right)$$

$$\|x - y\|_g \leq (\alpha + \beta)\, \|x - y\|_g$$

Which is contradiction . Thus $x = y$. This completes the proof of the theorem.

COROLLARY:

Let X be a G-Banach Space such that ∇ Satisfy α- property with $\alpha \leq 1$. If T be a mapping from X into itself, satisfying the following condition:

$$\|T^r x - T^s y\|_g$$

$$\leq \alpha \max\left(\begin{array}{l} \|x - T^s y\|_g \|y - T^r x\|_g \nabla \dfrac{\|x - T^r x\|_g \|y - T^s y\|_g}{\|x - y\|_g} \nabla \dfrac{\|x - y\|_g \|y - T^s y\|_g}{\|y - T^r x\|_g + \|x - T^r x\|_g} \nabla \\ \left\{ \dfrac{\|x - T^r x\|_g + \|y - T^s y\|_g}{\|x - T^s y\|_g} \right\} \|x - T^r x\|_g \end{array} \right)$$

$$+\beta \max\left(\begin{array}{l} \|x - y\|_g \nabla \|x - T^r x\|_g \nabla \|y - T^s y\|_g \nabla \|x - T^s y\|_g \nabla \|y - T^r x\|_g \nabla \\ \dfrac{\|x - T^r x\|_g \|y - T^s y\|_g}{\|x - y\|_g} \nabla \dfrac{\|x - T^r x\|_g \|y - T^r x\|_g}{\|x - y\|_g} \end{array} \right)$$

For non-negative α , β suvh that $0 < \alpha + \beta < 1$ and r , s $\in$ N(set of natural number). Then T has a unique common fixed point in X.

REFERENCES :

1. Ahmad A. and Shakil M. "Some fixed point theorems in Banach spaces ", Nonlinear func. Anal. Appl.11(2006) 343-349.
2. Bhardwaj R.K. " Some contraction on G-Banach Space",Global journal of Science frontier research Mathematics and Decision Sciences vol.12 issue 1 version 1.o, jaanuary (2o12).pp.81-89
3. Ciric Lj.B and J.S Ume "some fixed point theorems for weakly compatible mapping s"J.Math.Anal.Appl.314(2) (2006) 488-499.
4. Shrivastava R ,Animesh,yadav R.N. "some Mapping on G-Banach Space" international journal of Mathematical Science and Engineering Application, Vol.5, No. VI , (2011),pp.245-260.
5. Singh,S.L. and Singh,s.p. "A fixed point theorm" Indian journals pure and Applied. Math.,vol no.11(1980), 1584-1586.
6. Saini R.K. and Chauhan S.S "Fixed point theorem for eight mappings on metric space"journal of mathematics and system science,vol.1 no.2 (2005).
7. V.Popa," A general fixed point theorems for weakly compatible mapping satisfying an Implict relation",Filomat,19(2005) 45-51.

This academic article was published by The International Institute for Science, Technology and Education (IISTE). The IISTE is a pioneer in the Open Access Publishing service based in the U.S. and Europe. The aim of the institute is Accelerating Global Knowledge Sharing.

More information about the publisher can be found in the IISTE's homepage: http://www.iiste.org

CALL FOR PAPERS

The IISTE is currently hosting more than 30 peer-reviewed academic journals and collaborating with academic institutions around the world. There's no deadline for submission. **Prospective authors of IISTE journals can find the submission instruction on the following page:** http://www.iiste.org/Journals/

The IISTE editorial team promises to the review and publish all the qualified submissions in a **fast** manner. All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Printed version of the journals is also available upon request of readers and authors.

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library , NewJour, Google Scholar