Generalized Fixed Point Theorems for Compatible Mapping in Fuzzy 2-Metric Space for Integral Type Mapping

Rasik M.Patel, Ramakant Bhardwaj*
The Research Scholar of CMJ University, Shillong (Meghalaya)
*Truba Institute of Engineering & Information Technology, Bhopal (M.P)
Email: rasik_maths@yahoo.com, drrkbhardwaj100@gmail.com

Abstract
In this paper, we give some new definitions of compatible mappings of types (I) and (II) in fuzzy-2 metric space and prove some common fixed point theorems for four mappings under the condition of compatible mappings of types (I) and (II) in complete fuzzy-2 metric space. Our results extend, generalize and improve the corresponding results given by many authors.

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1. Introduction and Preliminaries

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [37] in 1922. In the general setting of complete metric space, this theorem runs as the follows, **Theorem 1.1**(Banach's contraction principle) Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y∈X, d (fx, fy) ≤ c d(x, y) Then f has a unique fixed point a∈X, such that for each x∈ X,$\lim_{n\to\infty} f^n x = a$. After the classical result, R.Kannan [46] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In 2002, A. Branciari [1] analyzed the existence of fixed point for mapping f defined on a complete metric space (X,d) satisfying a general contractive condition of integral type.

Theorem 1.2(Branciari) Let (X, d) be a complete metric space, c∈ (0, 1) and let f: X→X be a mapping such that for each x, y ∈ X, $\int_0^{d(fx,fy)} \varphi(t)dt \le c\int_0^{d(x,y)} \varphi(t)dt$. Where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each ε >o, $\int_0^{\varepsilon} \varphi(t)dt,$ then f has a unique fixed point a∈X such that for each x∈X, $\lim_{n\to\infty} f^n x = a$ After the paper of Branciari, a lot of a research works have been carried out on generalizing contractive conditions of integral type for a different contractive mapping satisfying various known properties. A fine work has been done by Rhoades [2] extending the result of Brianciari by replacing the condition [1.2] by the following

$\int_0^{d(fx,fy)} \varphi(t)dt \le \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt$(1.3) The aim of this paper is to generalize some mixed type of contractive conditions to the mapping and then a pair of mappings, satisfying a general contractive mappings such as R. Kannan type [46], S.K. Chatrterjee type [38], T. Zamfirescu type [48], etc.The concept of fuzzy sets was introduced initially by Zadeh [50] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. For example, Deng [12], Ereeg [13], Fang [18], George [19], Kaleva and Seikkala [28], Kramosil and Michalek [29] have introduced the concept of fuzzy metric spaces in different ways. In fuzzy metric spaces given by Kramosil and Michalek [29], Grabiec [20] obtained the fuzzy version of Banach's contraction principle, which has been improved, generalized and extended by some authors. Sessa [35] defined a generalization of commutativity introduced by Jungck [24], which is called the weak commutativity. Further, Jungck [25] introduced more generalized commutativity, so called compatibility. Mishra et al. [30] obtained some common fixed point theorems for compatible mappings in fuzzy metric spaces. Recently, Jungck et al. [26] introduced the concept of compatible mappings of type (A) in metric spaces, which is equivalent to the concept of compatible mappings under some conditions, and proved common fixed point theorems in metric spaces. Cho [09] introduced the concept of compatible mappings of type (a) in fuzzy metric spaces. Again, Pathak et al. [32] introduced the concept of compatible mappings of type (B) in metric spaces and Cho et al. [10] introduced the concept of compatible mappings of type (b) in fuzzy metric spaces. Pathak et al. [33] introduced the concept of compatible mappings of type (I) and (II) in metric spaces. On the other hand, George and Veeramani [19], Kramosil and

Michalek[29] introduced the concept of fuzzy topological spaces induced by fuzzy metrics which have very important applications in quantum particle physics particularly in connections with both string and $\epsilon^{(\infty)}$ theory which were given and studied by El Naschie [14–17,49]. Many authors [21,31,39,40,41] have also proved some fixed point theorems in fuzzy (probabilistic) metric spaces (see [2–7,18,20,22,23,36]).

Definition 1.1: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:
(1) * is associative and commutative,
(2) * is continuous,
(3) a * 1 = a for all a ∈ [0, 1],
(4) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],
Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 1.2: A 3-tuple (X, M,*) is called a fuzzy metric space if X is an arbitrary (Non-empty) set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions: for all x, y, z ∈ X and t, s > 0,
(1) M(x, y, t) > 0.
(2) M(x, y, t) $= 1$ if and only if x = y,
(3) M(x, y, t) $= \mathrm{M(y, x, t)}$.
(4) M(x, y, t) $* \mathrm{M(y, z, s)} \leq \mathrm{M(x, z, t+s)}$.
(5) M(x, y, .) $: (0,\infty) \to [0,1]$ is continuous.

Let M(x, y, t) be a fuzzy metric space. For any t > 0, the open ball B(x, r, t) with center x∈ X and radius 0 < r < 1 is defined by B(x, r, t) = {y ∈ X: M(x, y, t) $> 1 - r\}$. Let (X, M,*) be a fuzzy metric space. Let s be the set of all $A \subset S$ with x ∈ A if and only if there exist t > 0 and 0 < r < 1 such that B(x, r, t) ⊂ A. Then s is a topology on X (induced by the fuzzy metric M). This topology is Hausdorff and first countable. A sequence $\{x_n\}$ in X converges to x if and only if M $(x_n, x, \mathrm{t}) \to 1$ as n → ∞ for all t > 0. It is called a Cauchy sequence if, for any 0 < e < 1 and t > 0, there exits
$n_0 \in$ N such that M $(x_n, x_m,$ t) $> 1 - \varepsilon$ for any n, m $\geq n_0$ The fuzzy metric space (X, M,*) is said to be complete if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exists t > 0 and 0 < r < 1 such that M(x, y, t) $> 1 - r$ for all x, y ∈ A.

Example 1.3 [11]: Let X = R and denote a * b = ab for all a, b ∈ [0, 1]. For any t ∈ (0,∞), define
M(x, y, t) $= \frac{t}{t+|x-y|}$ for all x, y ∈ X. Then M is a fuzzy metric in X.

Lemma 1.4 [11]: Let (X, M,*) be a fuzzy metric space. Then M(x, y, t) is non-decreasing with respect to t for all x, y in X.

Definition 1.5: Let (X, M,*) be a fuzzy metric space. M is said to be continuous on $X^2 \times (0,\infty)$ if

$$\lim_{n\to\infty} \mathrm{M}\,(x_n, x_n, \mathrm{t_n}) = \mathrm{M(x, y, t)}$$

Whenever a sequence $\{(x_n, x_n, \mathrm{t_n})\}$ in $X^2 \times (0,\infty)$ converges to a point (x, y, t) ∈ $X^2 \times (0,\infty)$,

$$\text{i.e.} \lim_{n\to\infty} \mathrm{M}\,(x_n, x, \mathrm{t}) = \lim_{n\to\infty} \mathrm{M}\,(y_n, y, \mathrm{t}) = 1 \quad \text{and}$$

$$\lim_{n\to\infty} \mathrm{M}\,(x, y, \mathrm{t_n}) = \mathrm{M(x, y, t)}$$

Lemma 1.6: Let (X, M,*) be a fuzzy metric space. Then M is continuous function on $X^2 \times (0,\infty)$.

Proof: See Proposition 1 of [36].

Lemma 1.7 [11]: Let (X, M,*) be a fuzzy metric space. If we define $E_{\lambda,M}: X^2 \to R^+ U\{0\}$

$E_{\lambda,M} = \inf\{\, t > 0$: M(x, y, t) $> 1 - \lambda\,\}$ for all $\lambda \in (0,1)$ and x, y ∈ X. Then we have

(1) For all $\mu \in (0,1)$, there exists $\lambda \in (0,1)$ such that

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)\,, for\ all\ x_1, x_2, x_3 \ldots . x_n \in X.$$

(2) The sequence $\{x_n\}_{n\in N}$ is convergent in fuzzy metric space (X, M,*) if and only if $E_{\lambda,M}(x_n, x) \to 0$. Also, the

Sequence $\{x_n\}_{n\in N}$ is a Cauchy sequence if and only if it is a Cauchy sequence with $E_{\lambda,M}$.

Lemma 1.8 [11]: Let (X, M,*) be a fuzzy metric space. If a sequence $\{x_n\}$ in X is such that, for any n ∈ N, M

$(x_n, x_{n+1}, t) \geq$ M $(x_0, x_1, k^n t)$ for all k > 1, then the sequence $\{x_n\}$ is a Cauchy sequence.

2. Some definitions of compatible mappings

In this section, we give some definitions of compatible mappings with types, some properties and examples of compatible mappings in fuzzy metric spaces.

Definition 2.1: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is, Ax = Sx implies that ASx = SAx.

Definition 2.2 [30]: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings are said to be compatible if, for all t > 0,

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AS}x_n, \mathrm{SA}x_n, \mathrm{t}) = 1$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{S}x_n = x \in X.$$

Proposition 2.3 [40]: If the self-mappings A and S of a fuzzy metric space (X, M,*) are compatible, then they are weak compatible.

The converse is not true as seen in following example.

Example 2.4 [11]: Let (X, M,*) be a fuzzy metric space, where X = [0, 2] with t-norm defined a * b = min {a, b}, for all a, b ∈ [0,1] and $\mathrm{M}(x, y, t) = \frac{t}{t+|x-y|}$, for all x, y ∈ X. Define the self-mappings A and S on X as follows:

$$\mathrm{A}x = \begin{cases} 2 & \text{if } 0 \le x \le 1, \\ \frac{x}{2} & \text{if } 1 \le x \le 2, \end{cases} \qquad \mathrm{S}x = \begin{cases} 2 & \text{if } x = 1, \\ \frac{x+3}{5} & \text{if otherwise} \end{cases}$$

Then we have S1 = A1 = 2 and S2 = A2 = 1. Also SA1 = AS1 = 1 and SA2 = AS2 = 2. Thus (A, S) is weak compatible. Again,

$\mathrm{A}x_n = 1 - \frac{1}{4n}$, $\mathrm{S}x_n = 1 - \frac{1}{10n}$,

Thus we have $\mathrm{A}x_n \to 1$, $\mathrm{S}x_n \to 1$. Further, it follows that $\mathrm{SA}x_n = \frac{4}{5} - \frac{1}{20n}$, $\mathrm{AS}x_n = 2$.

Therefore, we have

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AS}x_n, \mathrm{SA}x_n, \mathrm{t}) = \lim_{n\to\infty} \mathrm{M}\,(2, \frac{4}{5} - \frac{1}{20n}, \mathrm{t}) = \frac{\mathrm{t}}{\mathrm{t} + \frac{6}{5}} < 1 \quad \text{for all t} > 0.$$

Hence (A, S) is not compatible.

Definition 2.5 [9]: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. The mappings A and S are said to be compatible of type (α) if, for all t > 0,

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AS}x_n, \mathrm{SS}x_n, \mathrm{t}) = 1, \qquad \lim_{n\to\infty} \mathrm{M}\,(\mathrm{SA}x_n, \mathrm{AA}x_n, \mathrm{t}) = 1$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{S}x_n = x \in X.$$

Definition 2.6 [10]: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. The mappings A and S are said to be compatible of type (β) if, for all t > 0,

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AA}x_n, \mathrm{SS}x_n, \mathrm{t}) = 1$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{S}x_n = x \in X.$$

Proposition 2.7 [09]: Let (X, M,*) be a fuzzy metric space with t * t = t for all t ∈ [0,1] and A, S be continuous mappings from X into itself. Then A and S are compatible if and only if they are compatible of type (α).

Proposition 2.8 [10]: Let (X, M,*) be a fuzzy metric space with t * t = t for all t ∈ [0,1] and A, S be continuous mappings from X into itself. Then A and S are compatible if and only if they are compatible of type (β).

Proposition 2.9 [10]: Let (X, M,*) be a fuzzy metric space with t * t = t for all t ∈ [0,1] and A, S be continuous mappings from X into itself. Then A and S are compatible of type (α) if and only if they are compatible of type (β).

Now, Turkoglu et al. [45] introduced the following definitions:

Definition 2.10: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the pair (A, S) is called A-compatible if, for all t > 0,

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AS}x_n, \mathrm{SS}x_n, \mathrm{t}) = 1$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X.$$

Definition 2.11: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the pair (A, S) is called S-compatible if and only if (S, A) is S-compatible.

Definition 2.12: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the pair (A, S) is said to be compatible of type (I) if, for all t > 0,

$$\lim_{n\to\infty} M\,(ASx_n, x, t) \le M\,(Sx, x, t)$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X.$$

Remark 2.13: in [47], the above Definition was introduced as follows:

$$\lim_{n\to\infty} M\,(ASx_n, x, \lambda t) \le M\,(Sx, x, t)$$

Whenever $\lambda \in (0,1]$ and $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X.$$

If $\lambda = 1$, then, by Lemma 1.4, it is easily verified that, for all $\lambda \in (0,1]$ by using above Definition, we have Definition of [48].

Definition 2.14: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the pair (A, S) is said to be compatible of type (II) if and only if (S, A) is compatible of type (I).

Remark 2.15: In [25, 26, 33, 34], we can find the equivalent formulations of above Definitions and their examples in metric spaces (see [25, 26, 33, 34]). Such mappings are independent of each other and more general than commuting and weakly commuting mappings (see [25, 35]).

Proposition 2.16: Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Suppose that the pair (A, S) is compatible of type (I) (respectively, (II)) and A z = S z for some z ∈ X.Then, for all t > 0, $M\,(Az, SSz, t) \ge M\,(Az, ASz, t)$(Respectively, $M\,(Sz, AAz, t) \ge M\,(Sz, SAz, t)$).

Proof: See Proposition 7 of [47].

Definition 2.17: (S.GÄahler [43]) Let X be a nonempty set. A real valued function *d* on X×X×X is said to be a 2-metric on X if

(1) Given distinct elements x; y of X, there exists an element z ∈ X such that d(x, y, z) ≠0,
(2) d(x, y, z) = 0 when at least two of x, y, z ∈ X are equal,
(3) d(x, y, z) = d(x, z, y) = d(y, z, x) for all x, y, z ∈ X,
(4) d(x, y, z) ≤ d(x, y, w)+d(x, w, z)+d(w, y, z) for all x, y, z, w ∈ X.

The pair (X, d) is called a 2-metric space.

Example 2.18 [27]: Let X = R^3 and let d(x, y, z):= the area of the triangle spanned by x, y and z which may be given explicitly by the formula, d(x, y, z) =$|x_1(y_2z_3 - y_3z_2) - x_2(y_1z_3 - y_3z_1) + x_3(y_1z_2 - y_2z_1)|$,
Where x = (x_1, x_2, x_3) y =(y_1, y_2, y_3), z = (z_1, z_2, z_3). Then (X, d) is a 2-metric space.

Definition 2.19: (S. Sharma [44]) the 3-tuple (X, M,*) is called a fuzzy 2-metric space if X is an arbitrary set, * is a continuous t-norm of H-type and M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following conditions: for all x, y, z, u ∈ X and $t_1, t_2, t_3 > 0$,

(1) M(x, y, z, 0) = 0,
(2) M(x, y, z, t) = 1 for all t > 0 if and only if at least two of the three points are equal,
(3) M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t) for all t > 0,
(Symmetry about first three variables)
(4) M(x, y, z,$t_1 + t_2 + t_3$) ≥∗ {(M(x, y, u, t_1), M(x, u, z,t_2), M (u, y, z,t_3))},
(This corresponds to tetrahedron inequality in 2-metric space. The function value M(x, y, z, t) may be interpreted as the probability that the area of triangle is less than t.)
(5) M(x, y, z,.): [0, 1) × [0, 1] is left continuous.

Example 2.20 [27]: Let (X, d) be a 2-metric space and denote *(a, b) = ab for all a, b ∈ [0, 1]. For each h, m, n ∈ R^+ and t > 0, define M(x, y, z, t) = $\frac{ht^n}{ht^n+md(x,y,z)}$. Then (X, M,*) is a fuzzy -2 metric space.

Definition 2.21 [27]: Let (X, M,*) be a fuzzy- 2 metric space.

(1) A sequence $\{x_n\}$ in fuzzy-2 metric space X is said to be convergent to a point x ∈ X (denoted by

$$\lim_{n\to\infty} x_n = x \quad or \quad x_n \to x$$

if for any $\lambda \in (0,1)$ and t > 0, there exists n_0 ∈ N such that for all n ≥ n_0 and a ∈ X, $M\,(x_n, x, a, t) > 1 - \lambda$.
That is

$$\lim_{n\to\infty} M\,(x_n, x, a, t) = 1 \text{ for all a} \in X \text{ and } t > 0.$$

(2) A sequence $\{x_n\}$ in fuzzy- 2 metric space X is called a Cauchy sequence, if for any $\lambda \in (0,1)$ and t > 0, there exists
n_0 ∈ N such that for all m, n ≥ n_0 and a ∈ X, M (x_n, x_m, a, t) $> 1 - \lambda$.
(3) A fuzzy- 2 metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.22 [27]: Self mappings A and B of a fuzzy- 2 metric space (X, M,*) is said to be compatible, if

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AB}x_n, \mathrm{BA}x_n, \mathrm{a}, \mathrm{t}) = 1 \; for\ all\ a \in X\ and\ t > 0,$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{B}x_n = z \quad for\ some\ \ z \in X.\ Then \quad \lim_{n\to\infty} \mathrm{AB}x_n = Bz.$$

Lemma2.23 [27]: Let (X, M,*) be a fuzzy- 2 metric space. If there exists q ∈ (0, 1) such that M(x, y, z, qt + 0) ≥ M(x, y, z, t) for all x, y, z ∈ X with z ≠ x, z ≠ y and t > 0, then x = y,

3. The main results

In this section, we prove a fixed point theorem for four mappings under the condition of compatible mappings of types (I) and (II).

Let φ be the set of all continuous and increasing functions $\emptyset: [0,1]^7 \to [0,1]$ in any coordinate and $\emptyset(t,t,t,t,t,t,t) > t$ for all $t \in [0,1)$.

Theorem 3.1: Let (X, M,*) be a complete fuzzy-2 metric space with t * t = t for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself such that
(i) A(X) ⊆ T(X), B(X) ⊆ S(X),
(ii) There exists a constant k ∈ $(0, \frac{1}{2})$ such that

$$\int_0^{M(Ax, By, a, kt)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx,\ Ty,\ a,\ t), M(Ax, Sy, a, t), M\ (By, Ty, a, t),\ M(Ax,\ Tx, a, \alpha t), \\ M(By,\ Sx,\ a, (2\text{-}\alpha\,)t),\ M(Ax,\ Tx,\ a, t),\ M(Ax,\ Sx, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

for all x, y, a ∈ X, $\alpha \in (0,2), t > 0\ and\ \emptyset \in \varphi$.
If the mappings A, B, S and T satisfy any one of the following conditions:
(iii) The pairs (A, S) and (B, T) are compatible of type (II) and A or B is continuous,
(iv)The pairs (A, S) and (B, T) are compatible of type (I) and S or T is continuous, then A, B, S and T have a unique common fixed point in X.

Proof: Let x_0 ∈ X be an arbitrary point. Since A(X) ⊆ T(X), B(X) ⊆ S(X), there exist x_1, x_2 ∈ X such that Ax_0 = Tx_1, Bx_1 = Sx_2. Inductively, construct the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}, \qquad y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$$

for n = 0, 1, 2. . . Then by $\alpha = 1 - q$ and $q \in (\frac{1}{2}, 1)$, if we set $d_m(t) = M(y_m, y_{m+1}, a, kt)\ for\ t > 0$, then we prove that $\{d_m(t)\}$ is increasing with respect to m. setting m = 2n, then we have

$$\int_0^{d_{2n}(t)} \zeta(t)dt = \int_0^{M(y_{2n}, y_{2n+1}, a, t)} \zeta(t)dt$$

$$= \int_0^{M(Ax_{2n}, Bx_{2n+1}, a, kt)} \zeta(t)dt$$

$$\geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx_{2n}, Tx_{2n+1}, a, t), M(Ax_{2n}, Sx_{2n+1}, a, t) \\ M(Bx_{2n+1}, Tx_{2n+1}, a, t), M(Ax_{2n}, Tx_{2n}, a, (1-q)t) \\ M(Bx_{2n+1}, Sx_{2n}, a, (1+q)t), M(Ax_{2n}, Tx_{2n}, a, t) \\ M(Ax_{2n}, Sx_{2n}, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(y_{2n-1}, y_{2n}, a, t), M(y_{2n}, y_{2n-1}, a, t), M(y_{2n+1}, y_{2n}, a, t) \\ M(y_{2n}, y_{2n}, a, (1-q)t), M(y_{2n+1}, y_{2n-1}, a, (1+q)t), \\ M(y_{2n}, y_{2n-1}, a, t), M(y_{2n}, y_{2n-1}, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

$$= \int_0^{\phi\left\{\left\{\min\{d_{2n-1}(t), d_{2n-1}(t), d_{2n}(t), 1, d_{2n-1}(t), d_{2n-1}(t), d_{2n-1}(t)\right\}\right\}} \zeta(t)dt.$$

The above inequality is true Ø is an increasing function.
We claim that for all n ∈ N,

$$\int_0^{d_{2n}(t)} \zeta(t)dt \geq \int_0^{d_{2n-1}(t)} \zeta(t)dt.$$

In fact, if $\int_0^{d_{2n}(t)} \zeta(t)dt < \int_0^{d_{2n-1}(t)} \zeta(t)dt.$

We have

$$\int_0^{d_{2n}(kt)} \zeta(t)dt \geq \int_0^{\phi\{(d_{2n}(qt), d_{2n}(qt), d_{2n}(qt), d_{2n}(qt), d_{2n}(qt), d_{2n}(qt), d_{2n}(qt)) > d_{2n}(qt)\}} \zeta(t)dt,$$

That is $\int_0^{d_{2n}(kt)} \zeta(t)dt > \int_0^{d_{2n}(qt)} \zeta(t)dt$ which is a contradiction.

Hence $\int_0^{d_{2n}(t)} \zeta(t)dt \geq \int_0^{d_{2n-1}(t)} \zeta(t)dt$ for all $n \in$ N and $t > 0$.

Similarly, for m = 2n+1, we have $\int_0^{d_{2n+1}(t)} \zeta(t)dt \geq \int_0^{d_{2n}(t)} \zeta(t)dt$ and so $\{d_m(t)\}$ is an increasing in [0, 1].

Similarly, for m = 2n+1, we have $\int_0^{d_{2n+1}(kt)} \zeta(t)dt \geq \int_0^{d_{2n}(qt)} \zeta(t)dt$ and so

$\int_0^{d_n(kt)} \zeta(t)dt \geq \int_0^{d_{n+1}(qt)} \zeta(t)dt$ for all $n \in$ N, that is

$$\int_0^{M(y_n, y_{n+1}, a, t)} \zeta(t)dt \geq \int_0^{M(y_{n-1}, y_n, a, \frac{q}{k}t)} \zeta(t)dt \geq \ldots\ldots \geq \int_0^{M\left(y_0, y_1, a, \left(\frac{q}{k}\right)^n t\right)} \zeta(t)dt.$$

Hence $\{y_n\}$ is a Cauchy sequence and, by the completeness of X, $\{y_n\}$ Converges to a point z in X. Let

$\lim_{n\to\infty} y_n = z.$ *Hence we have*

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = z.$$

Now, suppose that T is continuous and the pairs (A, S), (B, T) are compatible of type (I). Hence we have

$$\lim_{n\to\infty} TTx_{2n+1} = Tz, \quad \lim_{n\to\infty} BTx_{2n+1} = Tz, \quad \lim_{n\to\infty} STx_{2n+1} = Tz$$

Now, for $\alpha = 1$, setting $x = x_{2n}$ and $y = Tx_{2n+1}$ in the inequality (3.1), we have

$$\int_0^{M(Ax_{2n},\, BTx_{2n+1},\, a,\, kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx_{2n}, TTx_{2n+1}, a, t), M(Ax_{2n}, STx_{2n+1}, a, t) \\ M(BTx_{2n+1}, TTx_{2n+1}, a, t), M(Ax_{2n}, TTx_{2n}, a, t) \\ M(BTx_{2n+1}, Sx_{2n}, a, t), M(Ax_{2n}, TTx_{2n}, a, t) \\ M(Ax_{2n}, Sx_{2n}, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt.$$

Letting n→ ∞, we have

$$\int_0^{M(z,\lim_{n\to\infty} BTx_{2n+1},a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(z,Tz,a,t),\ M(z,Tz,a,t),\ M(\lim_{n\to\infty} BTx_{2n+1},Tz,a,t)\} \\ M(\lim_{n\to\infty} BTx_{2n+1},Tz,a,t), M(z,Tz,a,t),\ M(z,Tz,a,t) \\ M(z,z,a,t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

Now using (2.22) and (2.23), we have

$$\int_0^{M(z,Tz,a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(z,Tz,a,t),\ M(z,Tz,a,t),\ M(Tz,Tz,a,t) \\ M(Tz,Tz,a,t), M(z,Tz,a,t),\ M(z,Tz,a,t) \\ M(z,z,a,t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

Therefore,

$$\int_0^{M(z,Tz,a,kt)} \zeta(t)dt \geq \int_0^{M(z,Tz,a,t)} \zeta(t)dt$$

So it follow that T z = z.

Again, replacing x *by* x_{2n} and y *by* z in (1), for all $\alpha = 1$, we have

$$\int_0^{M(Ax_{2n},\, Bz,\, a,\, kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx_{2n}, Tz, a, t), M(Ax_{2n}, Sz, a, t), M(Bz, Tz, a, t), \\ M(Ax_{2n}, Tx_{2n}, a, at), M(Bz, Sx_{2n}, a, t), \\ M(Ax_{2n}, Tx_{2n}, a, t), M(Ax_{2n}, Sx_{2n}, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

And so, letting n→ ∞, we have

$$\int_0^{M(Bz,z,a,kt)} \zeta(t)dt > \int_0^{M(Bz,z,a,t)} \zeta(t)dt.$$

Which implies that B z = z. Since B(X) $\subseteq$ S(X), there exist $u \in$ X such that S u = z = B z. So, for $\alpha = 1$, we have

$$\int_0^{M(Au,\, Bz,\, a,\, kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Su, Tz, a, t), M(Au, Sz, a, t), M(Bz, Tz, a, t), \\ M(Au, Tu, a, at), M(Bz, Su, a, t), \\ M(Au, Tu, a, t), M(Au, Su, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

And so,

$$\int_0^{M(Au,z,a,kt)} \zeta(t)dt > \int_0^{M(z,Au,a,t)} \zeta(t)dt.$$

Which implies that Au = z. Since the pair (A, S) is compatible of type (I) and Au = Su = z, by proposition 2.16, we have

$$\int_0^{M(Au,SSu,a,kt)} \zeta(t)dt > \int_0^{M(Au,ASu,a,t)} \zeta(t)dt.$$

And so,

$$\int_0^{M(z,Sz,a,kt)} \zeta(t)dt > \int_0^{M(z,Az,a,t)} \zeta(t)dt.$$

Again, for $\alpha = 1$, we have

$$\int_0^{M(Az,Bz,a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(Sz,Tz,a,t), M(Az,Sz,a,t), M(Bz,Tz,a,t), \\ M(Az,Tz,a,at), M(Bz,Sz,a,t), \\ M(Az,Tz,a,t), M(Az,Sz,a,t) \end{array}\right\}\right\}} \zeta(t)dt$$

Hence we have

$$\int_0^{M(Az,z,a,kt)} \zeta(t)dt > \int_0^{M(z,Az,a,t)} \zeta(t)dt.$$

And so A z = z. Therefore, A z = B z = S z = T z = z and z is a common fixed point of the self-mappings A, B, S and T. The uniqueness of a common fixed point of the mappings A, B, S and T be easily verified by using (3.1) In fact, if z' be another fixed point for A, B, S and T, then, for $\alpha = 1$, we have

$$\int_0^{M(z,z',a,kt)} \zeta(t)dt = \int_0^{M(Az,Bz',a,kt)} \zeta(t)dt$$

$$\geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(Sz,Tz',a,t), M(Az,Sz',a,t), M(Bz',Tz',a,t), \\ M(Az,Tz,a,at), M(Bz',Sz,a,t), \\ M(Az,Tz,a,t), M(Az,Sz',a,t) \end{array}\right\}\right\}} \zeta(t)dt$$

$$> \int_0^{M(z,z',a,kt)} \zeta(t)dt$$

so z = z'.

Example 3.2: Let (X, d) be a fuzzy-2 metric space and denote *(a, b) = ab for all a, b ∈ [0, 1]. For each h, m, n ∈ R^+ and t > 0 and x, y, a ∈ X define M(x, y,a, t) = $\frac{ht^n}{ht^n+md(x,y,a)}$. Define the self-mappings A, B ,S ,T on X by
Ax= Bx =1,
S x = $\frac{3-x}{2}$, T x =$\frac{2x+3}{5}$
If we define a sequence $\{x_n\}$ in X by $1 - {}^1/_n$, then we have

$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{S}x_n = 1$, $\lim_{n\to\infty} \mathrm{M}\,(\mathrm{SA}x_n, 1, \mathrm{a}, \mathrm{t}) \leq \mathrm{M}\,(\mathrm{A1}, 1, \mathrm{a}, \mathrm{t}) = 1.$

$\lim_{n\to\infty} \mathrm{B}x_n = \lim_{n\to\infty} \mathrm{T}x_n = 1$, $\lim_{n\to\infty} \mathrm{M}\,(\mathrm{TB}x_n, 1, \mathrm{a}, \mathrm{t}) \leq \mathrm{M}\,(\mathrm{B1}, 1, \mathrm{a}, \mathrm{t}) = 1.$

This is the pairs (A, S), (B, T) are compatible of type (II) and A, B are continuous. Consider a function

$\emptyset: [0,1]^7 \to [0,1]$ defined by $\emptyset(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (\min\{x_i\})^h$ for some 0 < h <1. Then we have

M (Ax, By,a, t) ≥ $\emptyset(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$.

Therefore, all the conditions of theorem 3.1 are satisfied and so A, B, S and T have a unique common fixed point in X.

Corollary 3.3: Let (X, M,*) be a complete fuzzy-2 metric space with t * t = t for all $t \in [0,1]$. Let A, B, S and T be
mappings from X into itself such that

(i) A(X) $\subseteq$ T(X), B(X) $\subseteq$ S(X),
(ii) There exists a constant k $\in (0, \frac{1}{2})$ such that

$$\int_0^{M(Ax,By,a,kt)} \zeta(t)dt \geq \int_0^{\left\{\min\left\{\begin{array}{l} a_1(t)M(Sx,Ty,a,t)+a_2(t)M(Ax,Sy,a,t)+a_3(t)M(By,Ty,a,t)+ \\ a_4(t)M(Ax,Tx,a,\alpha t)+a_5(t)M(By,Sx,a,(2-\alpha)t)+a_6(t)M(Ax,Tx,a,t) \\ +a_7(t)M(Ax,Sx,a,t) \end{array}\right\}\right\}^{\frac{1}{2}}} \zeta(t)dt$$

for all x, y, $a \in$ X, $\alpha \in (0,2), t > 0$ *and* $a_i: R^+ \to (0,1]$ *such that* $\sum_{i=1}^{7} a_i(t) = 1$.
If the mappings A, B, S and T satisfy any one of the following conditions:
(iii) The pairs (A, S) and (B, T) are compatible of type (II) and A or B is continuous,
(iv) The pairs (A, S) and (B, T) are compatible of type (I) and S or T is continuous, then A, B, S and T have a unique common fixed point in X.
Proof: By Theorem 3.1, if we define

$$\emptyset(x_1,x_2,x_3,x_4,x_5,x_6,x_7) = \{(a_1(t)x_1 + a_2(t)x_2 + a_3(t)x_3 + a_4(t)x_4 + a_5(t)x_5 + a_6(t)x_6 + a_7(t)x_7)\}^{\frac{1}{2}},$$

Then we have conclusion.

Corollary 3.4: Let (X, M,*) be a complete fuzzy-2 metric space with t * t = t for all $t \in [0,1]$. Let A, B, R, S, H and T be mappings from X into itself such that
(i) A(X) $\subseteq$ TH(X), B(X) $\subseteq$ SR(X),
(ii) There exists a constant k $\in (0, \frac{1}{2})$ such that

$$\int_0^{M(Ax,By,a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(SRx,Ty,a,t), M(Ax,SRy,a,t), M(By,THy,a,t), M(Ax,THx,a,\alpha t), \\ M(By,SRx,a,(2-\alpha)t), M(Ax,THx,a,t), M(Ax,SRx,a,t) \end{array}\right\}\right\}} \zeta(t)dt$$

for all x, y, $a \in$ X, $\alpha \in (0,2), t > 0$ *and* $\emptyset \in \varphi$.
If the mappings A, B, SR and TH satisfy any one of the following conditions:
(iii) The pairs (A, SR) and (B, TH) are compatible of type (II) and A or B is continuous,
(iv)The pairs (A, SR) and (B, TH) are compatible of type (I) and SR or TH is continuous,
(v) TH= HT, AR = RA, BH = HB and SR = RS, then A, B, R, S, H and T have a unique common fixed point in X.
Proof: By theorem 3.1 A, B, TH and SR have a unique common fixed point in X. That is, there exists z $\in$ X Such that A z = B z =THz = SR z = z. Now, we prove that R z = z. In fact, by the condition (ii), it follows that

$$\int_0^{M(ARz,Bz,a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(SRRz,THz,a,t), M(ARz,SRRz,a,t), M(Bz,THz,a,t), M(ARz,THz,a,\alpha t), \\ M(Bz,SRRz,a,(2-\alpha)t), M(ARz,THz,a,t), M(ARz,SRRz,a,t) \end{array}\right\}\right\}} \zeta(t)dt$$

For $\alpha = 1$, we have

$$\int_0^{M(Rz,z,a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(Rz,z,a,t), M(Rz,Rz,a,t), M(z,z,a,t), M(Rz,z,a,t), \\ M(z,Rz,a,t), M(Rz,z,a,t), M(Rz,Rz,a,t) \end{array}\right\}\right\}} \zeta(t)dt$$

$$> \int_0^{M(Rz,z,a,t)} \zeta(t)dt$$

Which is a contradiction. Therefore, it follows that R z = z. Hence S z = SR z = z. Similarly, we get T z = H z = z.

Corollary 3.5: Let (X, M,*) be a complete fuzzy-2 metric space with t * t = t for all $t \in [0,1]$. Let S and T be mappings from X into itself such that

(i) There exists a constant k ∈ $(0, \frac{1}{2})$ such that

$$\int_0^{M(x,y,a,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx, Ty, a, t), M(x, Sy, a, t), M(y, Ty, a, t), M(x, Tx, a, \alpha t), \\ M(y, Sx, a, (2-\alpha)t), M(x, Tx, a, t), M(x, Sy, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

for all x, y, $a \in$ X, $\alpha \in (0,2), t > 0$ and $\emptyset \in \varphi$. Then A, B, S and T have a unique common fixed point in X.

Proof: If we get A = B = I (: the identity mapping) in theorem 3.1, then it is easy to check that the pairs (I, S) and (I, T)
are compatible of type (II) and the identity mapping I is continuous. Hence, by theorem 3.1, T and S have a unique common fixed point in X.

Corollary 3.6: Let (X, M,*) be a complete fuzzy-2 metric space with t * t = t for all $t \in [0,1]$. Let A and S be mappings from X into itself such that

(i)$A^n(X) \subseteq S^m(X)$,

(ii) There exists a constant k ∈ $(0, \frac{1}{2})$ such that

$$\int_0^{M(A^n x, A^n y, a, kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(S^m x, S^m y, a, t), M(A^n x, S^m y, a, t), M(A^n y, S^m y, a, t), M(A^n x, S^m x, a, \alpha t), \\ M(A^n y, S^m x, a, (2-\alpha)t), M(A^n x, S^m x, a, t), M(A^n x, S^m y, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

for all x, y, $a \in$ X, $\alpha \in (0,2), t > 0$ and $\emptyset \in \varphi$ and for $some$ $n, m \in N$.

If the mappings A^n and S^m satisfy any one of the following conditions:

(iii) The pairs (A^n,S^m) is compatible of type (II) $A^n$and is continuous,

(iv)The pairs (A^n,S^m) is compatible of type (I) and S^m is continuous,

(v) $A^nS = SA^n$ and $AS^m = S^mA$, then A and S have a unique common fixed point in X.

Proof: If we set A = B = A^n and S = T = S^m in theorem 3.1, then A^n and S^m have a unique fixed point in X. That is,

There exists $z \in$ X such that $A^nz = S^mz = z$. since $A^nAz = A\,A^nz = Az$ and $S^mAz = AS^mz = Az$, it follows that A z is a fixed point of A^n and S^m and hence A z = z. Similarly, we have S z = z.

Theorem 3.7: Let (X, M,*) be a complete fuzzy-2 metric space with t * t = t for all $t \in [0,1]$. Let S, T and two sequences$\{A_i\}, \{B_j\}$ for all i, j ∈ N be mappings from X into itself such that

(i) There exists $i_0, j_0 \in N$ such that A_{i_0}(X) ⊆T(X), B_{j_0}(X) ⊆S(X),

(ii) There exists a constant k ∈ $(0, \frac{1}{2})$ such that

$$\int_0^{M(A_i x, B_j y, a, kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx, Ty, a, t), M(A_i x, Sy, a, t), M(B_j y, Ty, a, t), M(A_i x, Tx, a, \alpha t), \\ M(B_j y, Sx, a, (2-\alpha)t), M(A_i x, Tx, a, t), M(A_i x, Sy, a, t) \end{matrix}\right\}\right\}} \zeta(t)dt$$

for all x, y, $a \in$ X, $\alpha \in (0,2), t > 0$ and $\emptyset \in \varphi$.

If the mappings A_{i_0}, B_{j_0} $, S$ and T satisfy any one of the following conditions:

(iii) The pairs (A_{i_0}, S) and (B_{j_0}, T) are compatible of type (II) and A_{i_0} or B_{j_0} is continuous,

(iv)The pairs (A_{i_0}, S) and (B_{j_0}, T) are compatible of type (I) and S or T is continuous, thenA_i,B_j ,S and T have a unique common fixed point in X for i, j = 1 ,2 ,3…

Proof: By theorem 3.1, the mappings S, T and A_{i_0} and B_{j_0} for some $i_0, j_0 \in N$ have a unique common fixed point in X.

That is, there exists a unique point z∈ X such that S z = T z =A_{i_0}z = B_{j_0} z = z. Suppose that there exists i ∈ N such that

$I \neq I_0$. Then we have

$$\int_0^{M(A_i z, z, a, kt)} \zeta(t)dt = \int_0^{M(A_i z, B_{j0} z, a, kt)} \zeta(t)dt$$
$$\geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(Sz, Tz, a, t), M(A_i z, Sz, a, t), M(B_{j0} z, Ty, a, t), M(A_i z, Tz, a, \alpha t), \\ M(B_{j0} z, Sz, a, (2-\alpha)t), M(A_i z, Tz, a, t), M(A_i z, Sz, a, t) \end{array}\right\}\right\}} \zeta(t)dt$$

For $\alpha = 1$, we have

$$\int_0^{M(A_i z, z, a, kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{array}{l} M(z, z, a, t), M(A_i z, z, a, t), M(z, z, a, t), M(A_i z, z, a, t), \\ M(z, z, a, t), M(A_i z, z, a, t), M(A_i z, z, a, t) \end{array}\right\}\right\}} \zeta(t)dt$$
$$> \int_0^{M(A_i z, z, a, t)} \zeta(t)dt,$$

Which is a contradiction. Hence, for all i∈ N, it follows that $A_i z = z$. similarly, for j∈ N, we have $B_j z = z$. Therefore, for all i, j∈ N, we have $A_i z = B_j z = Sz = Tz = z$.

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References:

[1] A.Branciari. A fixed point theorem for mappings satisfying a general contractive condition of integral type. Int.J.Math.Sci. 29(2002), no.9, 531 - 536.
[2] B.E. Rhoades, Two fixed point theorems for mappings satisfying a general contractive condition of integral type. International Journal of Mathematics and Mathematical Sciences, 63, (2003), 4007 - 4013.
[3] Badard R.Fixed point theorems for fuzzy numbers. Fuzzy Set Syst 1984; 13:291–302.
[4] Bose BK, Sabani D. Fuzzy mappings and fixed point theorems. Fuzzy Set Syst 1987; 21:53–58.
[5] Butnarin D. Fixed point for fuzzy mappings. Fuzzy Set Syst 1982; 7:191–207.
[6] Chang SS. Fixed point theorems for fuzzy mappings. Fuzzy Set Syst 1985; 17:181–187.
[7] Chang SS, Cho YJ, Lee BS, Jung JS, Kang SM. Coincidence point and minimization theorems in fuzzy metric spaces. Fuzzy Set Syst 1997; 88:119–128.
[8] Chang SS, Cho YJ, Lee BE, Lee GM. Fixed degree and fixed point theorems for fuzzy mappings. Fuzzy Set Syst 1997; 87:325–334.
[9] Cho YJ. Fixed points in fuzzy metric spaces. J Fuzzy Math 1997; 5:949–962.
[10] Cho YJ, Pathak HK, Kang SM, Jung JS. Common fixed points of compatible maps of type (b) on fuzzy metric spaces. Fuzzy Set Syst 1998; 93:99–111.
[11] Cho YJ, Shaban Sedghi, Nabi Shobe. Generalized fixed point theorems for compatible mappings in fuzzy metric spaces.Choas, Solitons and Fractals 39 (2009) 2233-2244.
[12] Deng ZK. Fuzzy pseudo-metric spaces. J Math Anal Appl 1982; 86:74–95.
[13] Ereeg MA. Metric spaces in fuzzy set theory. J Math Anal Appl 1979; 69:338–353.
[14] El Naschie MS. On the uncertainty of Cantorian geometry and two-slit experiment. Chaos, Solitons & Fractals 1998; 9:517–529.
[15] El Naschie MS. A review of E-infinity theory and the mass spectrum of high energy particle physics. Chaos, Solitons & Fractals 2004; 19:209–236.
[16] El Naschie MS. On a fuzzy Kahler-like Manifold which is consistent with two-slit experiment. Int J Nonlin Sci Numer Simul 2005; 6:95–98.
[17] El Naschie MS. The idealized quantum two-slit gedanken experiment revisited-criticism and reinterpretation. Chaos, Solitons & Fractals 2006; 27:9–13.
[18] Fang JX. On fixed point theorems in fuzzy metric spaces. Fuzzy Set Syst 1992; 46:107–113.
[19] George A, Veeramani P. On some result in fuzzy metric space. Fuzzy Set Syst 1994; 64:395–399.
[20] Grabiec M. Fixed points in fuzzy metric spaces. Fuzzy Set Syst 1988; 27:385–389.
[21] Gregori V, Sapena A. On fixed-point theorem in fuzzy metric spaces. Fuzzy Set Syst 2002; 125:245–252.
[22] Hadzic O. Fixed point theorems for multi-valued mappings in some classes of fuzzy metric spaces. Fuzzy Set Syst 1989; 29:115–125.

[23] Jung JS, Cho YJ, Chang SS, Kang SM. Coincidence theorems for set-valued mappings and Ekland's variational principle in fuzzy metric spaces. Fuzzy Set Syst 1996; 79:239–250.
[24] Jungck G. Commuting maps and fixed points. Amer Math Monthly 1976; 83:261–263.
[25] Jungck G. Compatible mappings and common fixed points. Int J Math Sci 1986; 9:771–779.
[26] Jungck G, Murthy PP, Cho YJ. Compatible mappings of type (A) and common fixed points. Math Jpn 1993; 38:381–390.
[27] Jinkyu Han. Comman fixed point theorem on fuzzy-2 metric spaces. Int J Chungcheong Math Soci.2010:302-729.
[28] Kaleva O, Seikkala S. On fuzzy metric spaces. Fuzzy Set Syst 1984; 12:215–229.
[29] Kramosil I, Michalek J. Fuzzy metric and statistical metric spaces. Kybernetica 1975; 11:326–334.
[30] Mishra SN, Sharma SN, Singh SL. Common fixed points of maps in fuzzy metric spaces. Int J Math Sci 1994; 17:253–258.
[31] Mihet D. A Banach contraction theorem in fuzzy metric spaces. Fuzzy Set Syst 2004; 144:431–439.
[32] Pathak HK, Cho YJ, Kang SM, Lee BS. Fixed point theorems for compatible mappings of type (P) and applications to dynamic programming. Le Matematische 1995;L:15–33.
[33] Pathak HK, Cho YJ, Chang SS, Kang SM. Compatible mappings of type (P) and fixed point theorems in metric spaces and probabilistic metric spaces. Novi Sad J Math 1996;26:87–109.
[34] Pathak HK, Mishra N, Kalinde AK. Common fixed point theorems with applications to nonlinear integral equation. Demonstratio Math 1999; 32:517–564.
[35] Sessa S. On some weak commutativity condition of mappings in fixed point considerations. Publ Inst Math (Beograd) 1982; 32:149–153.
[36] Sharma S, Deshpande B. Common fixed points of compatible maps of type (b) on fuzzy metric spaces. Demonsratio Math 2002; 35:165–174.
[37]. S. Banach, Sur les oprations dans les ensembles abstraits et leur application aux quations intgrales, Fund. Math.3,(1922)133-181 (French).
[38]. S.K.Chatterjea, Fixed point theorems, C.R.Acad.Bulgare Sci. 25(1972), 727-730.
[39] Schweizer B, Sherwood H, Tardiff RM. Contractions on PM-space examples and counterexamples. Stochastica 1988; 1:5–17.
[40] Singh B, Jain S. A fixed point theorem in Menger space through weak compatibility. J Math Anal Appl 2005; 301:439–448.
[41] Saadati R, Sedghi S. A common fixed point theorem for R-weakly commutiting maps in fuzzy metric spaces. 6th Iranian conference on fuzzy systems 2006:387–391.
[42] Sedghi S, Turkoglu D, Shobe N. Generalization common fixed point theorem in complete fuzzy metric spaces. J. Comput. Anal. Applictat 2007; 9(3):337–348.
[43] S. GÄahler, 2-metrische Raume and ihre topologische structure, Math. Nachr. 26 (1983), 115-148.
[44] S. Sharma. On fuzzy metric spaces, Southeast Asian Bull. of Math. 26 (2002), no. 1, 133-145.
[45] Rodrı´guez Lo´pez J, Ramaguera S. The Hausdorff fuzzy metric on compact sets. Fuzzy Set Syst 2004; 147:273–283.
[46] R. Kannan, Some results on fixed points, Bull. Calcutta Math. Soc., 60(1968), 71-76.
[47] Turkoglu D, Altun I, Cho YJ. Compatible maps and compatible maps of types (a) and (b) in intuitionistic fuzzy metric spaces, in press.
[48] T.Zamfrescu, Fixed point theorems in metric spaces, Arch. Math. (Basel) 23(1972), 292-298.
[49] Tanaka Y, Mizno Y, Kado T. Chaotic dynamics in Friedmann equation. Chaos, Solitons & Fractals 2005; 24:407–422.
[50] Zadeh LA. Fuzzy sets. Inform Control 1965; 8:338–353.