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Some Common Fixed Point Theorems For Compatible Mapping In Fuzzy Metric Spaces For Integral Type Mapping

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Abstract
In this paper, we prove some common fixed point theorems for six mappings in fuzzy metric space for integral type mapping. Our main results extend generalize and fuzzify some known results in fuzzy metric spaces.
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1. Introduction

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [28] in 1922. In the general setting of complete metric space, this theorem runs as the follows, **Theorem 1.1**(Banach's contraction principle) Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y∈X, d (fx, fy) ≤ c d(x, y) Then f has a unique fixed point a∈X, such that for each x∈ X,$\lim_{n\to\infty} f^n x = a$. After the classical result, R.Kannan [23] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In 2002, A. Branciari [1] analyzed the existence of fixed point for mapping f defined on a complete metric space (X,d) satisfying a general contractive condition of integral type.

Theorem 1.2(Branciari) Let (X, d) be a complete metric space, c∈ (0, 1) and let f: X→X be a mapping such that for each x, y ∈ X, $\int_0^{d(fx,fy)} \varphi(t)dt \le c\int_0^{d(x,y)} \varphi(t)dt$. Where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each ε >o, $\int_0^{\varepsilon} \varphi(t)dt,$ then f has a unique fixed point a∈X such that for each x∈X, $\lim_{n\to\infty} f^n x = a$ After the paper of Branciari, a lot of a research works have been carried out on generalizing contractive conditions of integral type for a different contractive mapping satisfying various known properties. A fine work has been done by B.E. Rhoades [2, 4] extending the result of Brianciari by replacing the condition [1.2] by the following
$\int_0^{d(fx,fy)} \varphi(t)dt \le \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt$ (1.3) The aim of this paper is to generalize some mixed type of contractive conditions to the mapping and then a pair of mappings, satisfying a general contractive mappings such as R. Kannan type [23], S.K. Chatrterjee type [34], T. Zamfirescu type [36], etc. The concept of fuzzy sets was introduced initially by Zadeh [17] in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [39], Erceg [19], Kaleva and Seikkala [22], Kramosil and Michalek [13] have introduced the concept of fuzzy metric space in different ways. Grabiec [20] followed Kramosil and Michalek [13] and obtained the fuzzy version of Banach contraction principle. Moreover, it appears that the study of Kramosil and Michalek [13] of fuzzy metric spaces pave the way for developing smoothing machinery in the field of fixed point theorems, in particular for the study of contractive type maps. Fang [14] proved some fixed point theorems in fuzzy metric spaces, which improve, generalize, unify and extend some main results of Banach [28], Edelstein [18], Istratescu [12], Sehgal and Bharucha-Reid [37]. Sessa [30] defined a generalization of commutativity, which is called weak commutativity. Further Jungck [10] introduced more generalized commutativity, so called compatibility. Mishra et al. [29] obtained common fixed point theorems for compatible maps on fuzzy metric spaces. Recently, Jungck et al. [11] introduced the concept of compatible mappings of type (α) in metric spaces, which is equivalent to the concept of compatible mappings under some conditions and proved common fixed point theorems in metric spaces. Cho [38] introduced the concept of compatible mappings of type (α) in fuzzy metric spaces. Many authors have studied the fixed point theory in fuzzy metric spaces. The most interesting references in this direction are [14,15,16,20,24,26,32] and fuzzy mappings [6,3,7,25,27]. Recently, George and Veeramani [13] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [14] and defined the Hausdorff topology on the fuzzy metric spaces. They showed also that every metric induces a fuzzy metric. In this paper, we prove common fixed point theorems for six mappings satisfying some conditions in fuzzy metric

spaces in the sense of Kramosil and Michalek [14]. Our main theorems extend, generalize and fuzzify some known results in fuzzy metric spaces, probabilistic metric spaces and uniform spaces
[9,11,21,31,33,38]. We also give an example to illustrate our main theorem.

2. Preliminaries

Definition 2.1 [5]: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:
(1) * is associative and commutative,
(2) * is continuous,
(3) a * 1 = a for all a ∈ [0, 1],
(4) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],
Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 2.2 [14]: A 3-tuple (X, M,*) is called a fuzzy metric space if X is an arbitrary (Non-empty) set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions: for all x, y, z ∈ X and t, s > 0,
(1) M(x, y, t) > 0.
(2) M(x, y, t) $= 1$ if and only if x = y,
(3) M(x, y, t) $= \mathrm{M}(y,x,t)$.
(4) M(x, y, t) $* \mathrm{M}(y,z,s) \leq \mathrm{M}(x,z,t+s)$.
(5) M(x, y, .) $: (0,\infty) \to [0,1]$ is continuous.

Let M(x, y, t) be a fuzzy metric space. For any t > 0, the open ball B(x, r, t) with center x∈ X and radius 0 < r < 1 is defined by B(x, r, t) = {y ∈ X: M(x, y, t) $> 1-r$}. Let (X, M,*) be a fuzzy metric space. Let s be the set of all $A \subset S$ with x ∈ A if and only if there exist t > 0 and 0 < r < 1 such that B(x, r, t) ⊂ A. Then s is a topology on X (induced by the fuzzy metric M). This topology is Hausdorff and first countable. A sequence $\{x_n\}$ in X converges to x if and only if M $(x_n, x$, t) → 1 as n → ∞ for all t > 0. It is called a Cauchy sequence if, for any 0 < e < 1 and t > 0, there exits
n_0 ∈ N such that M $(x_n, x_m,$ t) $> 1-\varepsilon$ for any n, m ≥ n_0 The fuzzy metric space (X, M,*) is said to be complete if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exists t > 0 and 0 < r < 1 such that M(x, y, t) $> 1-r$ for all x, y ∈ A.

Example 2.1 [8]: Let X = R and denote a * b = ab for all a, b ∈ [0, 1]. For any t ∈ (0,∞), define
M(x, y, t) $= \frac{t}{t+|x-y|}$ for all x, y ∈ X. Then M is a fuzzy metric in X.

Lemma 2.1 [20]: Let (X, M,*) be a fuzzy metric space. Then M(x, y, t) is non-decreasing with respect to t for all x, y in X.

Definition 2.3 [20]: Let (X, M,*) be a fuzzy metric space. M is said to be continuous on $X^2 \times (0,\infty)$ if

$$\lim_{n\to\infty} \mathrm{M}\,(x_n, x_n, \mathrm{t_n}) = \mathrm{M}(\mathrm{x},\mathrm{y},\mathrm{t})$$

Whenever a sequence $\{(x_n, x_n, \mathrm{t_n})\}$ in $X^2 \times (0,\infty)$ converges to a point (x, y, t) ∈ $X^2 \times (0,\infty)$,

$$\text{i.e. } \lim_{n\to\infty} \mathrm{M}\,(x_n, x, \mathrm{t}) = \lim_{n\to\infty} \mathrm{M}\,(y_n, y, \mathrm{t}) = 1 \text{ and}$$

$$\lim_{n\to\infty} \mathrm{M}\,(x, y, \mathrm{t_n}) = \mathrm{M}(\mathrm{x},\mathrm{y},\mathrm{t})$$

Lemma 2.2 [29]: Let $\{y_n\}$ be a sequence in fuzzy metric space (X, M,*) with the condition (FM-6). If there exists a number $k \in (0,1)$ such that $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$ for all $t > 0$ and n =1, 2, 3….then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.3 [29]: Let (X, M,*) be a fuzzy metric space. If there exists k ∈ (0, 1) such that
M(x, y, kt) ≥ M(x, y, t) for all x, y ∈ X and t > 0, then x = y,

3. Some definitions of compatible mappings

In this section, we give some definitions of compatible mappings with type (α) in fuzzy metric spaces.

Definition 3.1 [29]: Let A and B be mappings from a fuzzy metric space (X, M,*) into itself. The mappings A and B are said to be compatible of if, for all t > 0,

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AB}x_n, \mathrm{BA}x_n, \mathrm{t}) = 1$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{B}x_n = z \in X.$$

Definition 3.2 [38]: Let A and B be mappings from a fuzzy metric space (X, M,*) into itself. The mappings A and B are said to be compatible of type (α) if, for all t > 0,

$$\lim_{n\to\infty} \mathrm{M}\,(\mathrm{AB}x_n, \mathrm{BB}x_n, \mathrm{t}) = 1\,, \qquad \lim_{n\to\infty} \mathrm{M}\,(\mathrm{BA}x_n, \mathrm{AA}x_n, \mathrm{t}) = 1$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{B}x_n = z \in X.$$

Proposition 3.1 [38]: Let (X, M,*) be a fuzzy metric space with t * t = t for all t ∈ [0,1] and A, B be continuous mappings from X into itself. Then A and B are compatible of type (α) and Az = Bz for some $z \in X$, then $ABz = BBz = BAz = AAz$.

Proposition 3.2 [38]: Let (X, M,*) be a fuzzy metric space with t * t = t for all t ∈ [0,1] and A, B be continuous mappings from X into itself. Then A and B are compatible of type (α) and $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} \mathrm{A}x_n = \lim_{n\to\infty} \mathrm{B}x_n = z \in X.$$

$\lim_{n\to\infty} \mathrm{BA}x_n = Az \; if \; A \; is \; continuous \; at \; z \;\; and \; ABz = BAz, \qquad Az = Bz, if \; A \; and \; B \; are \; continuous \; at \; z.$

4. Main results

In this section, we prove a fixed point theorem for six mappings satisfying some conditions.

Theorem 4.1: Let (X, M,*) be a complete fuzzy metric space with t * t = t for all $t \in [0,1]$ and the condition (FM-6) Let A, B, S, T, P and Q be mappings from X into itself such that

(1) P(X) ⊆ AB(X), Q(X) ⊆ ST(X),

(2) AB = BA, ST = TS, PB = BP, QS = SQ, QT = TQ,

(3) A, B, S and T are continuous,

(4) the pair (P , AB) and (Q , ST) are compatible of type (α),

(5) there exists a number $k \in (0\,,1)$ such that

$$\int_0^{M(Px,\,Qy,kt)} \zeta(t)dt \ge \int_0^{\left\{\begin{matrix} M\,(ABx,\,Px,t) * M\,(STy,Qy,\,t) * M\,(STy,\,Px,\,\beta t) * M\,(ABx,Qy,(2-\beta)t) * \\ M\,(ABx,\,STy,\,t) * M\,(Px,\,Qx,\,t)\end{matrix}\right\}} \zeta(t)dt$$

for all x, y ∈ X, $\beta \in (0{,}2), t > 0$.

Then A, B, S, T , P and Q have a unique common fixed point in X.

Proof: By (1), Since P(X) ⊆ AB(X), Q(X) ⊆ ST(X), there exist x_1, x_2 ∈ X such that

Px_0 = ABx_1, Qx_1 = STx_2. Inductively, construct the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n} = Px_{2n} = ABx_{2n+1}\,, \qquad y_{2n+1} = Qx_{2n+1} = STx_{2n+2}$$

for n = 0, 1, 2. . . By (5) Then by $\beta = 1 - q$ and $q \in (0\,,1)$, we have

$$\int_0^{M(y_{2n+1},\,y_{2n+2},\,kt)} \zeta(t)dt = \int_0^{M(Px_{2n+1},\,Qx_{2n+2},\,kt)} \zeta(t)dt$$

$$\ge \int_0^{\left\{\begin{matrix} M\,(ABx_{2n+1},\,Px_{2n+1},t) * M\,(STx_{2n+2},\,Qx_{2n+2},t) * \\ M\,(STx_{2n+2},\,Px_{2n+1},\,\beta t) * M\,(ABx_{2n+1},Qx_{2n+2},(2-\beta)t) \\ * M\,(ABx_{2n+1},\,STx_{2n+2},t) * M\,(Px_{2n+1},\,Qx_{2n+1},t)\end{matrix}\right\}} \zeta(t)dt$$

$$= \int_0^{\left\{\begin{matrix} M\,(y_{2n},\,y_{2n+1},\,t) * M\,(y_{2n+1},y_{2n+2},t) * M\,(y_{2n+1},y_{2n+1},(1-q)t) \\ * M\,(y_{2n},y_{2n+2},(1+q)t) * M\,(y_{2n},y_{2n+1},t) * M\,(y_{2n+1},y_{2n+1},\,t)\end{matrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(y_{2n+1},\,y_{2n+2},\,kt)} \zeta(t)dt \ge \int_0^{\left\{\begin{matrix} M\,(y_{2n},\,y_{2n+1},\,t) * M\,(y_{2n+1},y_{2n+2},t) * 1 \\ * M\,(y_{2n},y_{2n+1},t) * M\,(y_{2n+1},y_{2n+2},qt) * \\ M\,(y_{2n},y_{2n+1},\,t) * 1\end{matrix}\right\}} \zeta(t)dt$$

$$\ge \int_0^{\left\{\begin{matrix} M\,(y_{2n},\,y_{2n+1},\,t) * M\,(y_{2n+1},y_{2n+2},t) * 1 \\ M\,(y_{2n+1},y_{2n+2},qt)\end{matrix}\right\}} \zeta(t)dt$$

Since the t-norm $*$ is continuous and M (x, y, .) is left continuous, letting $q \to 1$ in above, we have

$$\int_0^{M(y_{2n+1}, y_{2n+2}, kt)} \zeta(t)dt \geq \int_0^{\{M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t)\}} \zeta(t)dt$$

Similarly we have also

$$\int_0^{M(y_{2n+2}, y_{2n+3}, kt)} \zeta(t)dt \geq \int_0^{\{M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+2}, y_{2n+3}, t)\}} \zeta(t)dt$$

Thus

$$\int_0^{M(y_{n+1}, y_{2n+2}, kt)} \zeta(t)dt \geq \int_0^{\{M(y_n, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, t)\}} \zeta(t)dt$$

For n = 1, 2, 3………and so, for positive integers n, p

$$\int_0^{M(y_{n+1}, y_{n+2}, kt)} \zeta(t)dt \geq \int_0^{\left\{M(y_n, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, \frac{t}{k^p})\right\}} \zeta(t)dt$$

Thus, since $M(y_{n+1}, y_{n+2}, \frac{t}{k^p}) \to 1$ as $p \to \infty$, we have

$$\int_0^{M(y_{n+1}, y_{n+2}, kt)} \zeta(t)dt \geq \int_0^{\{M(y_n, y_{n+1}, t)\}} \zeta(t)dt$$

By lemma (2.2), $\{y_n\}$ is a Cauchy sequence and, by the completeness of X, $\{y_n\}$ Converges to a point z in X. Let

$\lim_{n\to\infty} y_n = z.$ *Hence we have*

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Px_{2n} = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} ABx_{2n+1} = \lim_{n\to\infty} STx_{2n+2} = z.$$

Now, suppose that A, B, S and T are continuous and the pairs (P, AB) and (Q, ST) are compatible of type (α). Hence we have

$$\lim_{n\to\infty} P(AB)x_{2n+1} = ABz, \quad \lim_{n\to\infty} (AB)^2 x_{2n+1} = ABz, \quad \lim_{n\to\infty} Q(ST)x_{2n+2} = STz \quad \lim_{n\to\infty} (ST)^2 x_{2n+2} = STz$$

Now, for $\beta = 1$, setting $x = (AB)x_{2n+1}$ and $y = x_{2n+2}$ in the inequality (5), we have

$$\int_0^{M(P(AB)x_{2n+1}, Qx_{2n+2}, kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{array}{l} M((AB)^2 x_{2n+1}, P(AB)x_{2n+1}, t) * M(STx_{2n+2}, Qx_{2n+2}, t) * \\ M(STx_{2n+2}, P(AB)x_{2n+1}, t) * M((AB)^2 x_{2n+1}, Qx_{2n+2}, t) \\ * M((AB)^2 x_{2n+1}, STx_{2n+2}, t) * M(P(AB)x_{2n+1}, Q(AB)x_{2n+1}, t) \end{array}\right\}} \zeta(t)dt$$

Which implies that as $n \to \infty$

$$\int_0^{M(ABz,z,kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(ABz,ABz,t) * M(z,z,t) * M(z,ABz,t) \\ * M(ABz,z,t) * M(ABz,z,t) * M(ABz,ABz,t) \end{Bmatrix}} \zeta(t)dt$$

$$\geq \int_0^{\{1 * 1 * M(z,ABz,t) * M(ABz,z,t) * M(ABz,z,t) * 1\}} \zeta(t)dt$$

$$\geq \int_0^{\{M(ABz,z,t)\}} \zeta(t)dt$$

Therefore, by lemma (2.3), we have $ABz = z$.
By putting $\beta = 1$, setting $x = Px_{2n}$ and $y = x_{2n+1}$ in the inequality (5), we have

$$\int_0^{M(P(Px_{2n}),Qx_{2n+1},kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(AB(Px_{2n}),P(Px_{2n}),t) * M(STx_{2n+2},Qx_{2n+1},t) * \\ M(STx_{2n+1},P(Px_{2n}),t) * M(AB(Px_{2n}),Qx_{2n+1},t) \\ * M(AB(Px_{2n}),STx_{2n+1},t) * M(P(Px_{2n}),Q(Px_{2n}),t) \end{Bmatrix}} \zeta(t)dt$$

Taking the limit $n \to \infty$, we have

$$\int_0^{M(Pz,z,kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(z,Pz,t) * M(z,z,t) * M(z,Pz,t) \\ * M(z,z,t) * M(z,z,t) * M(Pz,z,t) \end{Bmatrix}} \zeta(t)dt$$

$$= \int_0^{\{M(z,Pz,t) * 1 * M(z,Pz,t) * 1 * 1 * M(Pz,z,t)\}} \zeta(t)dt$$

$$\geq \int_0^{\{M(Pz,z,t)\}} \zeta(t)dt$$

By lemma (2.3), we have $Pz = z$. Therefore $ABz = Pz = z$.

Now, we show that $Bz = z$. by putting $\beta = 1$, setting $x = Bz$ and $y = x_{2n+1}$ in the inequality (5), and using (2) we have

$$\int_0^{M(P(Bz),Qx_{2n+1},kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(AB(Bz),P(Bz),t) * M(STx_{2n+1},Qx_{2n+1},t) * \\ M(STx_{2n+1},P(Bz),t) * M(AB(Bz),Qx_{2n+1},t) \\ * M(AB(Bz),STx_{2n+1},t) * M(P(Bz),Q(Bz),t) \end{Bmatrix}} \zeta(t)dt$$

$$\int_0^{M(Bz,z,kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(Bz,Bz,t) * M(z,z,t) * M(z,Bz,t) \\ * M(Bz,z,t) * M(Bz,z,t) * M(Bz,z,t) \end{Bmatrix}} \zeta(t)dt$$

$$= \int_0^{\begin{Bmatrix} 1 * 1 * M(z,Bz,t) * M(Bz,z,t) \\ * M(Bz,z,t) * M(Bz,z,t) \end{Bmatrix}} \zeta(t)dt$$

$$\geq \int_0^{\{M(Bz,z,t)\}} \zeta(t)dt$$

Therefore, by lemma (2.3), we have $Bz = z$. since $ABz = z$, therefore $Az = z$.
By putting $\beta = 1$, setting $x = z$ and $y = STx_{2n+2}$ in the inequality (5), we

$$\int_0^{M(Pz,Q(ST)x_{2n+2},kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{array}{l} M(ABz,Pz,t) * M((ST)^2 x_{2n+2}, Q(ST)x_{2n+2},t) * \\ M((ST)^2 x_{2n+2}, Pz, t) * M(ABz, Q(ST)x_{2n+2}, t) \\ * M(ABz, (ST)^2 x_{2n+2}, t) * M(Pz, z, t) \end{array}\right\}} \zeta(t)dt$$

Taking the limit $n \to \infty$, we have

$$\int_0^{M(z,STz,kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{array}{l} M(z,z,t) * M(STz,STz,t) * M(STz,z,t) \\ * M(z,STz,t) * M(z,STz,t) * M(z,z,t) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\left\{\begin{array}{l} 1 * 1 * M(STz,z,t) * M(z,STz,t) \\ * M(z,STz,t) * 1 \end{array}\right\}} \zeta(t)dt$$

$$\geq \int_0^{\{M(z,STz,t)\}} \zeta(t)dt$$

Therefore, by lemma (2.3), we have $STz = z$.
By putting $\beta = 1$, setting $x = z$ and $y = Qx_{2n+1}$ in the inequality (5), we have

$$\int_0^{M(Pz,Q(Qx_{2n+1}),kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{array}{l} M(ABz,Pz,t) * M((ST(Qx_{2n+1}), Q(Qx_{2n+1}),t) * \\ M(ST(Qx_{2n+1}), Pz, t) * M(ABz, Q(Qx_{2n+1}), t) \\ * M(ABz, ST(Qx_{2n+1}), t) * M(Pz, z, t) \end{array}\right\}} \zeta(t)dt$$

$$\int_0^{M(z,Qz,kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{array}{l} M(z,z,t) * M(z,Qz,t) * M(z,z,t) \\ * M(z,Qz,t) * M(z,z,t) * M(z,z,t) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\{1 * M(z,Qz,t) * 1 * M(z,Qz,t) * 1 * 1\}} \zeta(t)dt$$

$$\geq \int_0^{\{M(z,Qz,t)\}} \zeta(t)dt$$

Therefore, by lemma (2.3), we have $Qz = STz = z$. Finally , we show that $Tz = z$.
By putting $\beta = 1$, setting $x = z$ and $y = Tz$ in the inequality (5), we have

$$\int_0^{M(Pz,QTz,kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{array}{l} M(ABz,Pz,t) * M(ST(Tz),Q(Tz),t) * M(ST(Tz),Pz,t) * M(ABz,Q(Tz),t) * \\ M(ABz,ST(Tz),t) * M(Pz,Qz,t) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\left\{\begin{array}{l} M(z,z,t) * M(Tz,Tz,t) * M(Tz,z,t) * M(z,Tz,t) * \\ M(z,Tz,t) * M(z,z,t) \end{array}\right\}} \zeta(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{l} 1 * 1 * M(Tz,z,t) * M(z,Tz,t) * \\ M(z,Tz,t) * 1 \end{array}\right\}} \zeta(t)dt$$

$$\int_0^{M(z,Tz,kt)} \zeta(t)dt = \int_0^{M(Pz,QTz,kt)} \zeta(t)dt \geq \int_0^{\{M(z,\,Tz,\,t)\}} \zeta(t)dt$$

Therefore, by lemma (2.3), we have $Tz = z$. Since $STz = z$, therefore $Sz = z$. By combining the above results, we have $Az = Bz = Sz = Tz = Pz = Qz = z$,

I.e. z is a common fixed point of A, B, S, T, P and Q.

For uniqueness, let $w(w \neq z)$ be another common fixed point of A, B, S, T, P and Q and $\beta = 1$, then by (5), we write

$$\int_0^{M(Pz,\,Qw,\,kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(ABz,\,Pw,t) * M(STw,Qw,\,t) * M(STw,\,Pz,\,t) * M(ABz,\,Qw,t) * \\ M(ABz,\,STw,\,t) * M(Pz,\,Qz,\,t) \end{Bmatrix}} \zeta(t)dt$$

$$\geq \int_0^{\begin{Bmatrix} M(z,\,w,t) * M(w,w,\,t) * M(w,\,z,\,t) \\ * M(z,\,w,t) * M(z,\,w,\,t) * M(z,z,\,t) \end{Bmatrix}} \zeta(t)dt$$

$$= \int_0^{\{M(z,\,w,t) * 1 * M(w,\,z,\,t) * M(z,\,w,t) * M(z,\,w,\,t) * 1\}} \zeta(t)dt$$

It follows that

$$\int_0^{M(z,\,w,\,kt)} \zeta(t)dt \geq \int_0^{\{M(z,\,w,t)\}} \zeta(t)dt$$

Therefore, by lemma (2.3), we have z = w. This completes the proof of theorem.

Remark (4.1). In Theorem 4.1, if we put P = Q, our theorem reduces to the result due to Cho [38].

If we put B = T = I (The identity map on X) in theorem 4.1, we have the following:

Corollary 4.2: Let (X, M,*) be a complete fuzzy metric space with t * t = t for all $t \in [0,1]$ and the condition (FM-6) Let A, S, P and Q be mappings from X into itself such that

(1) P(X) $\subseteq$ A(X), Q(X) $\subseteq$ S(X),

(2) A and S are continuous,

(3) the pair (P , A) and (Q , S) are compatible of type (α),

(4) there exists a number $k \in (0\,,1)$ such that

$$\int_0^{M(Px,\,Qy,\,kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(Ax,\,Px,t) * M(Sy,Qy,\,t) * M(Sy,\,Px,\,\beta t) * M(Ax,Qy,(2-\beta)t) * \\ M(Ax,\,Sy,\,t) * M(Px,\,Qx,\,t) \end{Bmatrix}} \zeta(t)dt$$

for all x, y ∈ X, $\beta \in (0,2), t > 0$.

Then A, S, P and Q have a unique common fixed point in X.

If we put A= B = S = T = I (The identity map on X) in theorem 4.1, we have the following:

Corollary 4.3: Let (X, M,*) be a complete fuzzy metric space with t * t = t for all $t \in [0,1]$ and the condition (FM-6) Let P and Q be mappings from X into itself such that

(1) P(X) $\subseteq$ Q(X),

(2) Q is a continuous,

(3) the pair (P , Q) is compatible of type (α),

(4) there exists a number $k \in (0\,,1)$ such that

$$\int_0^{M(Px,\,Qy,\,kt)} \zeta(t)dt \geq \int_0^{\begin{Bmatrix} M(x,\,Px,t) * M(y,Qy,\,t) * M(y,\,Px,\,\beta t) * M(x,Qy,(2-\beta)t) * \\ M(x,\,y,\,t) * M(Px,\,Qx,\,t) \end{Bmatrix}} \zeta(t)dt$$

for all x, y ∈ X, $\beta \in (0,2), t > 0$.
Then P and Q have a unique common fixed point in X.
If we put P = Q, A = S, B = T = I (The identity map on X) in theorem 4.1, we have the following:
Corollary 4.4: Let (X, M,*) be a complete fuzzy metric space with t * t = t for all $t \in [0,1]$ and the condition (FM-6) Let P and S be mappings from X into itself such that
(1) P(X) $\subseteq$ S(X),
(2) S is a continuous,
(3) the pair (P , S) is compatible of type (α),
(4) there exists a number $k \in (0\,,1)$ such that

$$\int_0^{M(Px,Py,kt)} \zeta(t)dt \geq \int_0^{\left\{\begin{matrix} M(Sx,\,Px,t) * M(Sy,\,Py,\,t) * M(Sy,\,Px,\,\beta t) * M(Sx,Py,(2-\beta)t) * \\ M(Sx,\,Sy,\,t) * M(Px,\,Px,\,t) \end{matrix}\right\}} \zeta(t)dt$$

for all x, y ∈ X, $\beta \in (0,2), t > 0$.
Then P and S have a unique common fixed point in X.

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