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Some Fixed Point Theorems of Integral Type Satisfies Contraction

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Abstract
In this paper we proof some fixed point theorem for a pair of self maps of integral type which satisfies the contraction mapping.
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INTRODUCTION :-
The first well known result on fixed point for contraction mapping was Banach fixed point theorem , Published in 1922 ,[12],In general setting of Complete metric space ,Smart [2] & A. Meir and E.Keeler [6] ,Theorems on contraction mappings presented the following results.
Theorem 1.1: Let (X, d) be a complete metic space , $\alpha \in [0.1)$ and let $T: X \to X$ be a map such that for each $x, y \in X$, $\quad d(Tx, Ty) \leq \alpha\, d(x, y)$
Then, T has a unique fixed point $z \in X$ such that for each $x \in X$, $\lim_{n\to\infty} T^n x = z$.
After this result , more theorems with contraction mapping satisfies different types of contractive inequalities have been established see in [18],[11], [1].
Theorem 1.2 : Let (X, d) be a complete metic space , $\alpha \in [0.1)$ and let $T: X \to X$ be a map such that for each $c \in X$, $\quad \int_0^{d(Tx,Ty)} \xi(t)dt \leq \int_0^{d(x,y)} \xi(t)dt$
Where $\xi: [0, +\infty] \to [0, +\infty]$ is a lebesgue integrable mapping which is summable on each compact subset of $[0, +\infty]$, non negative, and such that, $\forall\ \varepsilon > 0$, $\int_0^{\varepsilon} \xi(t)dt > 0$ Then, T has [1]
Unique fixed point $z \in X$, such that for each $x \in X$, $T^n x = z$ as $n \to \infty$.
It can be proved in [17], that theorem 1.2 could be extended to more general contractive conditions, e.g. ,in [15], Rhoades established that Theorem 1.2 holds.
If we replace $d(x, y)$ by $\max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x,Tx)+d(y,Ty)}{2}\right\}$ other work in this particular case of the famous Meir-Keeler fixed point.
Theorem [6], More precisely, he proved that under hypotheses of Theorem 1.2, , that is for every $\delta > 0$ such that, Then T has a unique fixed point.
In this paper, we obtain an extension of Theorem 1.2 through rational expression.

MAIN RESULT:-
Theorem1: Let (X,d) be a complete metric space and $T: X \to X$ be a given mapping , then for each $x, y \in X$,

$$\int_0^{d(Tx,Ty)} \psi(t)dt \leq \alpha \int_0^{d(x,Tx)+d(y,Ty)} \psi(t)dt \qquad (1)$$

Where $\alpha > 0, 0 < \alpha < 1$ and $\psi : (0,1) \to (0,1)$ is a Lebesgue integrable mapping which Is summable on each compact subset of $(0,\infty)$, non negative , such that

$$\int_0^{\epsilon} \psi(t)dt > 0\ ,\forall \epsilon > 0 \qquad (2)$$

Then T has unique fixed point $x \in X$ such that for each $x \in X$, $T^n x = z$ as $n \to \infty$.
Proof: Let for any point $x_0 \in X$, $\exists\, x_1 \in X$ such that

$$x_1 = Tx_0$$

Similarly for any point $x_1 \in X$, $\exists\, x_2 \in X$ such that

$$x_2 = Tx_1$$

Proceeding the same way we construct a sequence $\{x_n\}$ of element x in X, as

$$x_{n+1} = Tx_n \ \ \forall \text{ integer } n \geq 1$$

Case I – Firstly we have to prove that the sequence $\{x_n\}$ is a cauchy sequence

Now, from (1)

$$\int_0^{d(Tx_n,Tx_{n+1})}\psi(t)dt \le \alpha\int_0^{d(x_n,Tx_n)+d(x_{n+1},Tx_{n+1})}\psi(t)dt$$

$$\int_0^{d(x_{n+1},x_{n+2})}\psi(t)dt \le \alpha\int_0^{d(x_n,Tx_n)+d(x_{n+1},Tx_{n+1})}\psi(t)dt$$

Similarly,

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \alpha\int_0^{d(x_{n-1},Tx_{n-1})+d(x_n,Tx_n)}\psi(t)dt$$

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \alpha\int_0^{d(x_{n-1},x_n)+d(x_n,x_{n+1})}\psi(t)dt$$

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \alpha\int_0^{d(x_{n-1},x_n)}\psi(t)dt + \alpha\int_0^{d(x_n,x_{n+1})}\psi(t)dt$$

$$(1-\alpha)\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \alpha\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \frac{\alpha}{(1-\alpha)}\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$

$$\le q\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$

Where $q = \frac{\alpha}{(1-\alpha)}$, $\alpha \in (0,1)$

Proceeding the same we can write,

$$\int_0^{d(x_1,x_2)}\psi(t)dt \le q\int_0^{d(x_0,x_1)}\psi(t)dt$$

:

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$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le q^n\int_0^{d(x_0,x_1)}\psi(t)dt$$

since ψ is a lebeasgue measurable function and continous so, we can write

$$d(x_n,x_{n+1}) \le q^n d(x_0,x_1)$$
$$\lim_{n\to\infty} T\,x_n = T\lim_{n\to\infty} x_n = Tx$$

for $m,n \ge M$

$$d(x_n,x_m) \le d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + \_\_\_ + d(x_{m-1},x_m)$$
$$d(x_n,x_m) \le q^n d(x_0,x_1) + q^{n+1}d(x_0,x_1) + \_\_\_ + q^{m-1}d(x_0,x_1)$$

$$\le q^n(1+q+q^2+\_\_\_+q^{m-1-n})\,d(x_0,x_1)$$
$$d(x_n,x_m) \le \frac{q^n}{1-q}d(x_0,x_1)$$

as $n\to\infty$, we have $\lim_{n\to\infty} d(x_n,x_m) \to 0$

$\lim_{n\to\infty} x_n = x$

Now, for fixed point Let $z \in X$ such that $Tx_n \to z$ as $n\to\infty$ we prove that $Tz = z$

Then we have to substitute $x = z$, $y = z_n$ in (1)

$$\int_0^{d(Tz,Tz_n)}\psi(t)dt \le \alpha\int_0^{d(z,Tz)+d(z_n,Tz_n)}\psi(t)dt$$

$$\int_0^{d(Tz,Tz_n)}\psi(t)dt \le \alpha\int_0^{d(z,Tz)}\psi(t)dt + \alpha\int_0^{d(z_n,Tz_n)}\psi(t)dt$$

$$\lim_{n\to\infty}\int_0^{d(Tz,Tz_n)}\psi(t)dt \le \alpha\int_0^{d(z,Tz)}\psi(t)dt$$

$$\lim_{n\to\infty} d(Tz,Tz_n) \le \alpha\, d(z,Tz)$$

as $n\to\infty$, $d(z,Tz)\to 0$

$Tz = z$

Which deduce that z is a fixed point of T.

Uniqueness: - Suppose that there is another fixed point of T say w, distinct from z in X then from (1) we have ,

$$\int_0^{d(Tz,Tw)} \psi(t)dt \le \int_0^{d(z,Tz)+d(w,Tw)} \psi(t)dt$$
$$\le \int_0^{d(z,Tz)+d(w,Tw)} \psi(t)dt$$
$$= \int_0^{d(Tz_n,Tz_n)+d(Tz_n,Tz_n)} \psi(t)dt$$
$$\int_0^{d(Tz,Tw)} \psi(t)dt < 0$$

Which is a contraction. So, T has a unique fixed point in X.

Theorem2: Let (M, d) be a complete metric space and Let $T : M \to M$ be a mapping , we assume that for each $x, y \in M$,

$$\int_0^{d(Tx,Ty)} \psi(t)dt \le a\int_0^{d(x,y)} \psi(t)dt + b\int_0^{\frac{d^2(x,Tx)+d(x,Ty)d(y,Tx)+d^2(y,Ty)}{1+d(x,Tx)+d(y,Ty)}} \psi(t)dt \qquad (1)$$

[4]

for all $x, y \in M, a > 0, b > 0, 0 < a + 2b < 1$ and $\psi : R_+ \to R_+$ is a Lebesgue integrable mapping which is summable on each compact subset of $[0, +\infty)$, non negative and such that

$$\int_0^{\epsilon} \psi(t)dt > 0 \ , \forall \epsilon > 0 \qquad (2)$$

Then T has a unique fixed point $z_0 \in M$ such that for each $x \in M$

$\lim_{n\to\infty} T^n x = z_0$

Proof: Let$\psi : R_+ \to R_+$ be a condition as we define $\psi_0(t) = \int_0^t \psi(t)dt$, $t \in R_+$. It is clear that $\psi_0(0) = 0$, ψ_0 is a monotonically non decreasing and by condition ψ_0 is absolutly continuous. Then for any for any point $x_0 \in X, \exists\, x_1 \in X$ such that

$$x_1 = Tx_0$$

Similarly for any point $x_1 \in X$, $\exists\, x_2 \in X$ such that

$$x_2 = Tx_1$$

Proceeding the same way we construct a sequence $\{x_n\}$ of element x in X, as

$$x_{n+1} = Tx_n \ \ \forall \text{ integer } n \ge 1$$

Case I – Firstly we have to prove that the sequence $\{x_n\}$ is a cauchy sequence

Now, from (1)

$$\int_0^{d(Tx_n,Tx_{n+1})} \psi(t)dt \le a\int_0^{d(x_n,x_{n+1})} \psi(t)dt + b\int_0^{\frac{d^2(x_n,Tx_n)+d(x_n,Tx_{n+1})d(x_{n+1},Tx_n)+d^2(x_{n+1},Tx_{n+1})}{1+d(x_n,Tx_n)+d(x_{n+1},Tx_{n+1})}} \psi(t)dt$$

$$\int_0^{d(x_{n+1},x_{n+2})} \psi(t)dt \le a\int_0^{d(x_n,x_{n+1})} \psi(t)dt + b\int_0^{\frac{d^2(x_n,Tx_n)+d(x_n,Tx_{n+1})d(x_{n+1},Tx_n)+d^2(x_{n+1},Tx_{n+1})}{1+d(x_n,Tx_n)+d(x_{n+1},Tx_{n+1})}} \psi(t)dt$$

Similarly $\int_0^{d(x_n,x_{n+1})} \psi(t)dt \le a\int_0^{d(x_{n-1},x_n)} \psi(t)dt + b\int_0^{\frac{d^2(x_{n-1},Tx_{n-1})+d(x_{n-1},Tx_n)d(x_n,Tx_{n-1})+d^2(x_n,Tx_n)}{1+d(x_{n-1},Tx_{n-1})+d(x_n,Tx_n)}} \psi(t)dt$

$$\le a\int_0^{d(x_{n-1},x_n)} \psi(t)dt + b\int_0^{\frac{d^2(x_{n-1},Tx_{n-1})+d(x_{n-1},Tx_n)d(x_n,Tx_{n-1})+d^2(x_n,Tx_n)}{1+d(x_{n-1},Tx_{n-1})+d(x_n,Tx_n)}} \psi(t)dt$$

$$\le a\int_0^{d(x_{n-1},x_n)} \psi(t)dt + b\int_0^{\frac{d^2(x_{n-1},x_n)+d(x_{n-1},x_{n+1})d(x_n,x_n)+d^2(x_n,x_{n+1})}{1+d(x_{n-1},x_n)+d(x_n,x_{n+1})}} \psi(t)dt$$

$$\le a\int_0^{d(x_{n-1},x_n)} \psi(t)dt + b\int_0^{\frac{d^2(x_{n-1},x_n)+d^2(x_n,x_{n+1})}{1+d(x_{n-1},x_n)+d(x_n,x_{n+1})}} \psi(t)dt$$

$$\le a\int_0^{d(x_{n-1},x_n)} \psi(t)dt + b\int_0^{d(x_{n-1},x_n)} \psi(t)dt + b\int_0^{d(x_n,x_{n+1})} \psi(t)dt$$

$$(1-b)\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le (a+b)\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$
$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \frac{(a+b)}{(1-b)}\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$
$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le q\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$

Proceeding as we get, $\int_0^{d(x_1,x_2)}\psi(t)dt \le q\int_0^{d(x_0,x_1)}\psi(t)dt$

$$\vdots \qquad \vdots$$
$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le q^n\int_0^{d(x_0,x_1)}\psi(t)dt$$

since ψ is a lebeasgue measurable function and continous So we can write

$$d(x_n,x_{n+1}) \le q^n d(x_0,x_1)$$
$$\lim_{n\to\infty} T x_n = T\lim_{n\to\infty} x_n = Tx$$

for $m,n \ge M$

$$d(x_n,x_m) \le d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + \_\_\_ + d(x_{m-1},x_m)$$
$$d(x_n,x_m) \le q^n d(x_0,x_1) + q^{n+1}d(x_0,x_1) + \_\_\_ + q^{m-1}d(x_0,x_1)$$
$$\le q^n(1+q+q^2+\_\_\_+q^{m-1-n})\, d(x_0,x_1)$$
$$d(x_n,x_m) \le \frac{q^n}{1-q}d(x_0,x_1)$$

as $n\to\infty$, we have $\lim_{n\to\infty} d(x_n,x_m)\to 0$

$\lim_{n\to\infty} x_n = x$

Which is a contradiction, we proved that $\{Tx_n\}$ is Cauchy.

Now for fixed point: Let $z \in X$ such that $Tx_n \to z$ as $n\to\infty$ we prove that $Tz = z$

We have to substitute $x = z$ and $y = z_n$ in (1)

$$\int_0^{d(Tz,Tz_n)}\psi(t)dt \le a\int_0^{d(z,z_n)}\psi(t)dt + b\int_0^{\frac{d^2(z,Tz)+d(z,Tz_n)d(z_n,Tz)+d^2(z_n,Tz_n)}{1+d(z,Tz)+d(z,Tz)}}\psi(t)dt$$
$$\text{as } \lim_{n\to\infty}\int_0^{d(Tz,Tz_n)}\psi(t)dt \le a\int_0^{d(z,z)}\psi(t)dt + b\int_0^{\frac{d^2(z,Tz)+d(z,Tz)d(z,Tz)+d^2(z,Tz)}{1+d(z,Tz)+d(z,Tz)}}\psi(t)dt$$

[6]

$$\lim_{n\to\infty}\int_0^{d(Tz,Tz_n)}\psi(t)dt \le b\int_0^{d(z,Tz)}\psi(t)dt$$
$$\text{as } \lim_{n\to\infty} d(Tz,Tz_n) \le b\, d(z,Tz)$$

As $n\to\infty$, we have $d(z,Tz)\to 0$

$Tz = z$, which shows that z is a fixed point of T.

Uniqueness :– Assume that there is aanother fixed point say w of T which is distinct from z in X, then from (1) we have,

$$\int_0^{d(Tw,Tz)}\psi(t)dt \le a\int_0^{d(w,z)}\psi(t)dt + b\int_0^{\frac{d^2(w,Tw)+d(w,Tz)d(z,Tw)+d^2(z,Tz)}{1+d(w,Tw)+d(z,Tz)}}\psi(t)dt$$
$$\int_0^{d(Tw,Tz)}\psi(t)dt \le a\int_0^{d(w,z)}\psi(t)dt$$

which is a contradiction so T has a unique fixed point in X.

Theorem3: Let(X, d) be a complete metric space and $S,T: X\to X$ be a given mapping , then for each $x,y \in X$,

$$\int_0^{d(Sx,Ty)}\psi(t)dt \le \alpha\int_0^{d(x,Sx)+d(y,Ty)}\psi(t)dt \; (1)$$

Where $\alpha > 0, 0 < \alpha < 1$ and $\psi : (0,1) \to (0,1)$is a Lebesgue integrable mapping which Is summable on each compact subset of $(0,\infty)$, nonnegative, such that

$$\int_0^{\epsilon}\psi(t)dt > 0 \;, \forall \epsilon > 0 \qquad (2)$$

Then S and T hasa unique fixed point $x \in X$ such that for each $x \in X, T^n x = z$ as $n \to \infty$.

Proof: LetX be a non empty set $S, T: X \to X$ then for any point $x_0 \in X, \exists\, x_1 \in X$ such that

$$x_1 = Tx_0$$

Similarly for any point $x_1 \in X, \exists\, x_2 \in X$ such that

$$x_2 = Tx_1$$

Proceeding the same way we construct a sequence $\{x_n\}$ of element x in X, as

$$x_{n+1} = Sx_n \text{ and } x_{n+2} = Tx_{n+1} \forall \text{ integer } n \geq 1$$

Case I – Firstly we have to prove that the sequence $\{x_n\}$ is a cauchy sequence

Now, from (1)

$$\int_0^{d(Sx_n,Tx_{n+1})} \psi(t)dt \leq \alpha \int_0^{d(x_n,Sx_n)+d(x_{n+1},Tx_{n+1})} \psi(t)dt$$

$$\int_0^{d(x_{n+1},x_{n+2})} \psi(t)dt \leq \alpha \int_0^{d(x_n,Sx_n)+d(x_{n+1},Tx_{n+1})} \psi(t)dt$$

Similarly,

$$\int_0^{d(x_n,x_{n+1})} \psi(t)dt \leq \alpha \int_0^{d(x_{n-1},Sx_{n-1})+d(x_n,Tx_n)} \psi(t)dt$$

$$\int_0^{d(x_n,x_{n+1})} \psi(t)dt \leq \alpha \int_0^{d(x_{n-1},x_n)+d(x_n,x_{n+1})} \psi(t)dt$$

$$\int_0^{d(x_n,x_{n+1})} \psi(t)dt \leq \alpha \int_0^{d(x_{n-1},x_n)} \psi(t)dt + \alpha \int_0^{d(x_n,x_{n+1})} \psi(t)dt$$

$$(1-\alpha) \int_0^{d(x_n,x_{n+1})} \psi(t)dt \leq \alpha \int_0^{d(x_{n-1},x_n)} \psi(t)dt$$

$$\int_0^{d(x_n,x_{n+1})} \psi(t)dt \leq \frac{\alpha}{(1-\alpha)} \int_0^{d(x_{n-1},x_n)} \psi(t)dt$$

$$\leq q \int_0^{d(x_{n-1},x_n)} \psi(t)dt$$

Where $q = \frac{\alpha}{(1-\alpha)}$, $\alpha \in (0,1)$

Proceeding the same we can write,

$$\int_0^{d(x_1,x_2)} \psi(t)dt \leq q \int_0^{d(x_0,x_1)} \psi(t)dt$$

:
:

$$\int_0^{d(x_n,x_{n+1})} \psi(t)dt \leq q^n \int_0^{d(x_0,x_1)} \psi(t)dt$$

since ψ is a lebeasgue measurable function and continous so, we can write

$$d(x_n, x_{n+1}) \leq q^n d(x_0, x_1)$$

$$\lim_{n\to\infty} S\, x_n = S \lim_{n\to\infty} x_n = Sx$$

$$\lim_{n\to\infty} T\, x_{n+1} = T \lim_{n\to\infty} x_{n+1} = Tx$$

for $m, n \geq M$

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \_\_\_ + d(x_{m-1}, x_m)$$

$$d(x_n, x_m) \leq q^n d(x_0, x_1) + q^{n+1} d(x_0, x_1) + \_\_\_ + q^{m-1} d(x_0, x_1)$$

$$\leq q^n (1 + q + q^2 + \_\_\_ + q^{m-1-n})\, d(x_0, x_1)$$

$$d(x_n, x_m) \leq \frac{q^n}{1-q} d(x_0, x_1)$$

$$\text{as } n \to \infty \text{, we have} \lim_{n\to\infty} d(x_n, x_m) \to 0$$

$\lim_{n\to\infty} x_n = x$

Therefore sequence $\{x_n\}$ is a cauchy sequence in x.

Now, for fixed point Let $z \in X$ such that $Tx_n \to z$ as $n \to \infty$ we prove that $Tz = z$
Then we have to substitute $x = z, y = z_n$ in (1)

$$\int_0^{d(Sz,Tz_n)} \psi(t)dt \leq \alpha \int_0^{d(z,Sz)+d(z_n,Tz_n)} \psi(t)dt$$

$$\int_0^{d(Sz,Tz_n)} \psi(t)dt \leq \alpha \int_0^{d(z,Sz)} \psi(t)dt + \alpha \int_0^{d(z_n,Tz_n)} \psi(t)dt$$

$$\lim_{n\to\infty} \int_0^{d(Sz,Tz_n)} \psi(t)dt \leq \alpha \int_0^{d(z,Sz)} \psi(t)dt$$

$$\lim_{n\to\infty} d(Sz, Tz_n) \leq \alpha\, d(z, Sz)$$

And $\lim_{n\to\infty} d(Sz, Tz_n) \leq \alpha\, d(z, Tz)$
as $n \to \infty$, $d(z, Sz) \to 0$ And $d(z, Tz) \to 0$

$$Sz = Tz = z$$

Which proves that z is a fixed point of T.
Uniqueness:-Let if possible we assume that w be another fixed point of S and T, then from (1) we have ,

$$\int_0^{d(Sz,Tw)} \psi(t)dt \leq \int_0^{d(z,Sz)+d(w,Tw)} \psi(t)dt$$

$$\leq \int_0^{d(Sz_n,Sz_n)+d(Tz_n,Tz_n)} \psi(t)dt$$

$$\therefore \int_0^{d(Sz,Tw)} \psi(t)dt < 0$$

Which is a contraction so T has a unique fixed point in X.

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