

The Concept of Linear Inequalities in One Variable (A Study in Afadzato South District, Ghana)

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Abstract
The study examined junior high students' awareness of linear disparities in Ghana's Afadzato South District. The study examined male and female students' linear inequalities in learning outcomes in the 'Have' Circuit of Afadzato South District. The study challenge sparked three questions. Using a questionnaire, interview guide, and the Algebra Diagnostic Test, researchers collected quantitative and qualitative data. Data analysis employed descriptive statistics and t-tests. According to the report, 125 (40.3%) students passed the algebra diagnostic test after studying linear inequalities. 146 (47.1%) said linear inequalities helped them understand word problems. p 0.05, mean deviation = 1.516; standard error difference = 1.606. The test enhanced boys' and girls' performance. According to the report, stakeholders should support and motivate students to improve their math study habits. Every arithmetic topic is tied to a higher notion, so kids tend to view math as easy. Headmasters, headmistresses, District and Municipal Chief Executives of Education, and the Ministry of Education should provide teaching and learning materials to stimulate students' interest in math. Math clinics, workshops, and in-service training should be offered often for elementary math teachers to improve their skills. Effective monitoring and encouragement of elementary mathematics teachers can assist pupils to develop an interest in linear inequalities, allowing them to pursue higher studies.
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1.0 INTRODUCTION

Science and math are vital to our existence. Science and math are vital to our existence. Developing nations' education systems prioritize math (CRDD, 2007). Mathematical languages help humans express, communicate, reason, calculate, abstract, generalize, and formalize. Mathematics employs systematic reasoning and argument to generate certainty, generality, and reliability. Teaching math in Ghana seeks to provide students with the skills, insights, attitudes, and values they need to excel in their careers and daily lives. Mathematics is crucial in homes, schools, businesses, markets, and almost every human effort. Math allows for systematic results (Ansari, 2004). Linear inequalities are part of algebra and are taught once pupils have gained mathematical insight, attitudes, and values. Despite their close relationship, linear inequalities are occasionally studied separately from math. With arithmetic, we operate on numbers and get results after each operation, but in linear inequalities, we don't calculate the numbers first. Students must symbolically identify linear inequalities' unknowns, variables, and relations (Martinez, 2002, p. 8). Linear inequalities are tough for students. Many misunderstand linear inequalities. Counting starts in math. Every day, we use math. We learn fundamental math to communicate in these instances (Ansari, 2004). Math skills improve as we grow and learn. Cake baking uses math to measure flour. These situations require common sense, realism, and experience. Daily math doesn't require "brains" (Ansari, 2004). Less educated people can quickly calculate. Many pre-schoolers have numerical skills (Clements & Sarama, 2004; Ginsburg, 2002). Math models scientific ideas derive qualitative consequences, and forecast events (Jaworski, 2006). Space, weather, geology, and finance use math. Math is a key part of human communication and a technique to express patterns, relationships, rationality, and aesthetics (Jaworski, 2006). Ghanaian seniors must pass four core classes and two electives (CRDD, 2007). In today's fast-growing science, technology, production, and industry, math is vital (CRDD, 2007). Math is a daily necessity that spans cultures. Math is a basic topic for all students and a tool and way of thinking for many other courses. Linear inequalities are a common action. (2004)

1.2 Motivation for the study

Math and life often involve inequality. In junior high math books, inequalities are either explicitly used to express numerical links or implicitly incorporated into the domain or behavior of functions. Inequalities dominate advanced math. An analysis of inequalities (Burn, 2005). Optimization uses inequalities. Inequalities define limits and domains in function theory. In Algebra, students learn inequalities. subsets of equations. Many published research papers include inequities (e.g., Kieran, 2004; Dreyfus & Hoch, 2004). Most studies uncover issues with teaching inequality. Students work demonstrates misunderstandings. According to WAEC's BECE 2017 report, students can't solve linear inequality questions (mathematics). According to the poll, kids don't accurately answer linear inequalities. They miss the signs, causing erroneous treatment and answers. This issue prompted the researcher to ask pupils about linear inequalities. The study aims to learn how junior high school students in Have Circuit, Afadzato South District, Volta Region, Ghana, think about one-variable linear inequalities.

2.0 LITERATURE REVIEW

2.1 Theoretical Framework

It is a framework based on an existing theory in a field of inquiry that is related to and/or reflects the hypothesis of a study. It is a blueprint that is often 'borrowed' by the researcher to build his/her own house or research inquiry. It serves as the foundation upon which research is constructed. The theoretical framework guides the researcher so that he or she would not deviate from the confines of the accepted theories to make his or /her final contribution scholarly. The theoretical framework also guides the kind of data to be accrued for a particular study (Lester, 2005). It aids the researcher in finding an appropriate research approach, analytical tools, and procedures for his/her research inquiry. It makes research findings more meaningful and generalizable (Akintoye, 2015).

2.1.1 The perspective of constructivism

The basic and most fundamental assumption of constructivism is that knowledge does not exist independently of the learner, it is constructed. The physical world sets certain boundaries within which multiple perspectives can be negotiated and constructed. For constructivists, learning is meaning-making. Human choices and actions are a result of interpretation of the world. Constructivism has been highly debated in educational research, especially in the context of mathematics, particularly concerning the objectivity of knowledge. Even among those who call themselves constructivists, there is no consensus as to what it implies for learning. There are many versions of constructivism. Richardson (2003) suggests that there might even be up to 18 different versions. To situate the research into the realm of constructivism, the researcher describes

2.1.1.10 Possible characteristics of constructivist pedagogy

Richardson (2003) outlines five characteristics of practice that have been seen in classrooms where teachers claim to use constructivism as a guiding theory. These are :
(1) Attention to the individual and their backgrounds,
(2) Facilitation of group dialogue to develop shared understandings,
(3) Provision of opportunities for students to justify and explain responses and possibly change or add to their existing beliefs and understandings by engaging in the planned tasks,
(4) Development of student's awareness of their understandings and learning processes, and
(5) Planned and unplanned use of direct instruction, reference to the text, or websites.

This last characteristic has been questioned because direct instruction is often seen as representing the transmission view of teaching and learning and is not usually associated with constructivist pedagogy (Richardson, 2003). Constructivist pedagogy can be "thought of as the creation of classroom environments, activities, and methods that are grounded in a constructivist theory of learning, with goals that focus on individual students developing deep understandings in the subject matter of interest and habits of mind that aid in future learning" (Richardson, 2003). and participation metaphors of learning, especially as a starting point in moving towards constructivist pedagogy. The acquisition metaphor focuses on concept development and learning acquisition of the individual. The teacher plays a central role in the transmission of knowledge. Following the participation metaphor, knowing comes about through evolving bonds between individuals and others based on communication and discourse. The teacher plays the role of facilitator and questioner, versus dispenser of knowledge.

2.1.2 Students' misconceptions of linear inequality concepts

Recent studies on errors and misconceptions in school math are hard to discover (Barcellos, 2005), although several prior studies focused on students' errors produced by correctly employing defective algorithms or mistakenly picking elementary arithmetic methods. Fujii (2003) sought to understand students' procedural flaws and defective algorithms. McNeil and Alibali (2005) asserted that "prior learning constrains subsequent learning" (p. 8), that is, students' misconceptions may be induced by previous learning experiences. Due to life experiences or past learning, students approach classes with varied perspectives. Identifying pupils' biases and misconceptions helps them learn arithmetic more effectively. Ignoring pupils' assumptions may hinder new learning and promote the previous misconceptions. This section reviews some misunderstandings and their origins.

2.1.3 Misconceptions about the use of Literal Symbols

Grammar and syntax simplify algebra with linear inequalities. Variables are symbols in this symbolic language (Drijvers et al., 2011, p. 17). Rational number reasoning (learning rational numbers) is difficult for pupils at all levels of teaching (Vosniadou, Vamvakoussi, & Skopeliti, 2008). (Ni & Zhou, 2005). Students find rational number notation difficult (Stafylidou & Vosniadou, 2004). Decimals and fractions can express the same number as different numbers (Vamvakoussi & Vosniadou, 2007). Justification, proving, forecasting, and solving all result in notation errors that are calculated with literal symbols (Kieran, 2007). Multiple-meaning variables confound students. Students "switch from one interpretation to another when solving an issue, making it hard to understand" (2008). Second, we can't add apples and bananas because 3a+2b is the sum. The Letter-as-object fallacy is encouraged by formulas like A = L * B. Calculus students misinterpret the equal sign (Knuth et al., 2008). School and outside experience shape most beliefs (Greer, 2004). When presented with a scientific notion, students may not be abstract. Teachers may simplify controversial topics. Inadequate explanation can hinder student understanding (Liljedahl, 2005). Algebra presentations may require speed and formality. Algebra's cognitive difficulties are ignored by academia. Students lack time to intuitively study algebra or connect it to pre-algebraic notions. Children understand the equal sign as an arithmetic result, not an equivalency (Molina & Ambrose, 2008). This well-documented (MIS)conception may be due to students' elementary school experiences, where equality and its symbolic instantiation were traditionally introduced early, with little explicit instructional time spent on the concept in later grades, and the equal sign was nearly always present in the operations equals answer context (e.g., 2 + 5 = 7). y = 2+3 is a linear function but can be graphed (Bills, Dreyfus, Tsamir, Watson, & Zaslavsky, 2006). So many students spread ideas.

3.0 METHODOLOGY

3.1 Research Design

Research designs are directions for performing research. A study's purpose determines its research strategy (Cohen, Manion, & Morrison, 2004). This study used descriptive survey research since it allows for a high number of respondents (Cohen et al., 2004). A descriptive survey design gives information on the typical group member. By collecting data on a group, a researcher can describe the average member or member's average performance. Descriptive research design is highly appreciated by policymakers in the social sciences, where huge populations are handled by utilizing questionnaires. Educational research uses descriptive surveys to acquire data. Yamane's algorithm helps researchers choose samples from two or more homogenous groups or clusters (Singh & Masuku, 2014). The formula is given by,

$$n = \frac{N}{(1+N* e^2)},$$

where
n = sample size
$N =$ The population size
$e =$ Specified margin of error or significance level

Table 1: Sample Distribution of the Study

Schools	No. of Pupils	Percentage (%)
Have D/A “A” JHS	33	10.6
Have D/A “B” JHS	32	10.3
Agate D/A JHS	34	11.0
Have R/C JHS	32	10.3
Have E.P. JHS	40	12.9
Have Ando No. 1 Presby JHS	35	11.3
Have Alavanyo D/A JHS	34	11.0
Have Ando No. 2 JHS	36	11.6
Hadzidekope D/A JHS	34	11.0
Total	**310**	**100**

3.10 Data Collection Procedures

Before collecting data, the researcher contacted school principals and math teachers. The research's goal was explained to the respondents by the school leaders. The study's data collection took a week. On data collection day, the researcher gave each student a unique index number. Each index number was a letter and number. This was done to assure accurate, anonymous data from study participants and student confidentiality. Three days were used to collect qualitative data through semi-structured interviews and classroom observations, and two days were used to collect quantitative data through a researcher-created and supervisor-validated questionnaire

4.0 RESULTS AND DISCUSSION

Research Question 1:

What conceptions of linear inequalities do pupils in ‘Have Circuit’ of the Afadzato South District have?

The first research question of the study sought to examine the students’ conception of linear inequalities in the

Have Circuit located in the Afadzato South District of the Volta Region of Ghana. The categorical data collected with the questionnaire designed for the pupils were used to respond to this question. The response of the pupils from the questionnaire which answers the first research question has been presented in Table 4.

Table 4: Students' Conceptions of linear Inequalities District

Concepts of linear inequalities	Mode	Percentage (%)
Equality and inequality concepts are the same (125 pupils SD).	125	40.3
I can solve inequality of 2x > 4 (250 pupils SA)	250	80.6
I can solve this inequality 1-2x < 5 (200 pupils SA).	200	64.5
I can solve 1-2x > 2(6-x) (100 pupils SA)	100	32.3
I can work out the solution to this, 2x + 4 ≥ 24 (140 pupils SA).	140	45.2
I can work out the solution of m/3 – 3 ≤ - 6 (86 pupils A).	86	27.7

From Table 4, 125 pupils representing 40.3% strongly disagreed with the assertion that *equality and inequality concepts are the same*. In addition, 250 (80.6%) strongly agreed that they could apply the concepts of inequality to solve the inequality 2x > 4. It could be seen that this percentage was high indicating that about two-thirds of the sample have developed the required to be able to solve 2x > 4. Meanwhile, 200 (64.5%), 100 (32.3%), 140 (45.2%) and 86 (27.7%) acknowledged that they could apply the concepts of inequality to solve 1-2x < 5, 1-2x > 2(6-x), 2x + 4 ≥ 24 and m/3 – 3 ≤ - 6 respectively. There 5, below presents the responses of the mathematics teachers in the various schools sampled for the study in Have Circuit of Afadzato South District on the concepts of linear inequalities.

Table 5: Mathematics Teachers' Conception of linear Inequalities

Concepts of linear inequalities	Mode	Percentage (%)
Examining the students at the end of each topic and making necessary reviews to correct the mistakes of students (23 teachers SA).	23	76.7
Motivate students to learn mathematics on their own (19 teachers SA).	19	63.3
Encourage students to form small groups to practice (23 teachers SA).	23	76.7
Involve students in the teaching and learning process (18 teachers SA).	18	60
My students can solve m /3-3≤ -6 (23 teachers SA)	23	76.7

The table shows what nine math teachers believed about inequality. 23 of 30 teachers who answered the teachers' questionnaire said their students could solve m/3-3-6. 23 teachers (76.7%) said they test their pupils after each topic to ensure they grasp inequality. 19 teachers (63.3%) said they encouraged their students to understand inequality. 23 (76.7%) of the teachers said they encouraged students to create small groups to practice unequal problem-solving, and 18 (60%) said they involved students in the teaching and learning process. Most sampled teachers tried to help their students understand linear inequality ideas.

4.2 Pupils' Responses from the Interview Session

For the first question on the interview guide: *What sorts of images or examples come to mind when you consider the concept of inequality?*

In response to the above question on the interview guide, 300 (96.8%) pupils indicated that the *"symbols of inequality such as ≤, ≥, < and >"* come into mind when they consider the concept of inequality. Also, 150 pupils representing 48.4% indicated the fact that "*the symbol (s) of the inequality is changed or reversed when the inequality is multiplied or divided by a negative value*". The above responses of the pupils indicated that they have a deeper understanding of the integral concepts associated with inequality.

What are how inequalities and equations are the same and/or different?

Furthermore, concerning the second question on the interview guide shown in the italics above, 160 (51.6%) indicated that "*the procedure for solving an inequality problem is similar to the procedure for solving a linear equation of one variable*". Meanwhile, 306 (98.7%) pupils demonstrated that "linear *equations have the equal to sign (=) while inequalities may use any of the symbols, i.e. ≤, ≥, < and >".* More so, 200 (64.5%) pupils indicated that "*inequality sign is changed when it is multiplied with a negative value whereas in the case of linear equation, the sign (equal to, =) does not change when it is multiplied with a negative value*".

For the third interview guide question: *What does a solution to inequality mean?*

In response to that question, 120 (38.7%) of pupils indicated that the solution of inequality means "*a number which when substituted for the variable makes the inequality a true statement*".

In addition, for the fourth interview guide question: *Can you tell me an interesting fact you have learned/discovered lately about inequalities?* 250 (80.6%) indicated that "*adding or subtracting the same number on both sides of the inequality expression does not change the inequality*". Again, 303 (97.7%) demonstrated that "multiplying an inequality with a negative value changes the inequality". For the interview guide questions 5 and 6, the majority of pupils demonstrated mastery of the inequality concept and provided the desired solution to the problem posted.

Research Question 2:
What benefits do people derive from understanding linear inequalities concepts in mathematics?
This second research question of the study sought to examine the benefits of the conception of linear inequalities. The categorical data collected with the questionnaire and the interview guide designed for the pupils were used to answer this question. Table 6 presents descriptively, the responses of pupils on the benefits of the learning of linear inequalities. The modal responses together with their corresponding percentages have been presented in Table 4.5 below.

Table 6: Benefits of Understanding Linear Inequality Concepts

Benefits of understanding linear inequality concepts	Mode	Percentage (%)
Learning of linear inequalities would form the basis for the learning of other lessons (140 pupils A).	140	45.1
Learning of linear inequalities will make word problems associated with it easier (146 pupils SA).	146	47.1
Learning of linear inequalities will arouse pupils' interest (150 pupils SA).	150	48.1
Learning linear inequalities will enable pupils to interpret mathematical statements with confidence (170 pupils SA).	170	54.8
Learning of linear inequalities will enable pupils to logically present solutions relating to them (160 pupils SA).	160	51.6
The learning of the linear inequalities will allay pupils' fears (127 pupils SA).	127	41.0
The learning of linear inequalities will address the misconceptions of it (250 pupils SA)	250	80.6

Table 6 shows that 140 students (45.1%) felt that knowing linear inequalities will help them learn other math and scientific concepts. 146 (47.1%) strongly believed that learning inequalities will help them understand inequality word problems. 150 students, or 48.1%, felt that studying inequality would interest them. 170 (54.8%) students said learning inequality will help them confidently interpret mathematical statements. 160 students (51.6%) said understanding inequality will help them give rational solutions. Again, 127 (41%) said linear inequalities will calm their anxieties. 250 (80.6%) agreed it will assist correct their misperception about learning the idea.

Research Question 3:
What conceptual change in learning outcomes do male and female pupils have in linear inequalities?
This third research topic examined the impact of inequality on boys and girls in the Have Circuit in Ghana's Volta Region. The question was answered using quantitative data from children's linear inequality exam results. Table 7 displays the descriptive statistics of the scores of the sample (pupils) after taking part in the achievement test of the study. The descriptive statistics of the pupils' achievement test scores have been presented in

Table 7: Descriptive Statistics of Pupils' Achievement Test Scores

	Gender	N	Mean	Std. Deviation	Std. Error Mean
Scores	Boys	155	51.06	14.225	1.143
	Girls	155	49.54	14.043	1.128

The boys' mean and standard deviation were (M = 51.06, SD = 14.23) with a standard error mean of (Std. Error Mean = 1.143), while the girls' were (M = 49.54, SD = 14.04) with a standard error mean of (Std. Error Mean = 1.128). Boys did marginally better than girls, even though their standard deviation and standard error mean were practically the same. Boys outperformed girls by 1.52 points. A t-test was used to compare boys' and girls' linear inequality test scores. Tables 7a and 7b show the results of the independent samples t-test for the achievement test scores of the students.

Table 8a: Independent Samples T-Test of Pupils' Achievement Test Scores

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	Df
Scores1	Equal variances assumed	.179	.672	.944	308
	Equal variances not assumed			.944	307.949

Table 7's Levene's Test for Variance Equality (F =.179, p > 0.05) Because p =.672 > 0.05, this statistical test assumed similar achievement test score variances between boys and girls. If the significance threshold was less than 0.05, we would have accepted Equal Variances Not Assumed and used calculated statistics for that row. From Table 7b, a t-test for equality of means revealed a mean difference of 1.516 (Mean Deviation = 1.516), which was statistically significant with a Standard Error Difference of 1.606 (Mean Deviation = 1.516, p 0.05;

Standard Error Difference = 1.606), indicating that the achievement test improved the boys' and girls' performance.

4.4 Discussion

Research Question 1:

What are the students' conceptions of linear inequalities?

For linear disparities, 125 students (40.3%) disagreed that equality and inequality are the same. 250 (80.6%) said inequality could solve 2x > 4. Two-thirds of the sample can answer 2x > 4. These results show students understand linear inequality. According to Greer (2004), ideas come from little experience or a few instances. Both outside and classroom learning are affected. The study's findings support Liljedahl's (2005) conclusion that pupils don't have as much abstract thinking when initially introduced to a scientific subject. To assist students' grasp, teachers may convey only part of a difficult concept. Some students may not understand a simple explanation. Linear inequality has its own symbols, laws, and signs (Drijvers et al., 2011). Linear inequalities have their own grammar and syntax, simplifying algebra. "Variables are just signs that can be changed with rules" (Drijvers et al., 2011, p. 17). For effective learning, a math teacher must help students apply new knowledge to existing concepts (Ausubel, 2000). The researcher desired the instructors' opinions. 23 teachers (76.7%) said their students could solve m/3-3-6. 23 teachers (76.7%) claimed they test students on inequality after each topic. 19 teachers (63.3%) claimed they taught inequality. 23 (76.7%) of the teachers encouraged students to form small groups to practice uneven problem-solving, while 18 (60%) involved students in the teaching and learning process. Most teachers seek to teach linear inequality. When teachers fail to accurately convey a subject's initial interpretation to students, it can obstruct the passage to a higher-order, more general and abstract interpretation. Early knowledge of natural numbers and their properties helps kids understand infinity, but it makes it harder for them to understand the properties and operations of rational numbers (Moskal & Magone, 2000; Yujing & Yong-Di, 2005; Johnstone, 2000; Bills, Dreyfus, Tsamir, Watson, & Zaslavsky, 2006).

Research Question 2:

What are the benefits of understanding linear inequalities concepts in mathematics?

140 students (45.1%) said linear inequalities will help them understand math and science. 146 (47.1%) said understanding inequality will help them understand it. 150 students (48.1%) found inequality interesting. 170 (54.8%) students thought knowing inequality will help them interpret arithmetic. Understanding inequality will help pupils find sensible solutions, 51.6% say. 127 (41%) stated linear inequalities would relax them. 250 individuals (80.6%) agreed it will help them learn the idea. This shows that students' understanding of linear inequality affects higher-order math and informal instruction. Some beliefs can be stable, widely held, resistant to change, and not hinder algebra learning (Vosniadou, Vamvakoussi, & Skopeliti, 2008; Richardson, 2003; Anderson, 2002; Knuth et al., 2008).

Research Question 3:

How did the evidence of students' conceptual change in linear inequalities concepts impact learning outcomes?

According to the t-test, boys outperformed girls on the achievement test. This statistical test assumed equal achievement test score variances between boys and girls since p = 0.672 > 0.05 (p > 0.05). We would have accepted Equal Variances Not Assumed and used calculated statistics. A t-test for equality of means revealed a mean difference of 1.516 (Mean Deviation = 1.516), which was statistically significant with a standard error difference of 1.606 (Mean Deviation = 1.516, p 0.05; Standard Error Difference = 1.606), indicating that the achievement test improved boys' and girls' performance. Prior learning might cause pupils' assumptions and beliefs, according to McNeil and Alibali (2005). If arithmetic concepts are presented clearly, students will do well on tests and exams.

5.1 Summary of the Findings

According to the study, 125 students, or 40.3%, strongly disputed that equality and inequality are the same for linear inequalities. 250 (80.6%) agreed they could use inequality to solve 2x > 4. This high percentage indicates that two-thirds of the sample can solve 2x > 4. 200 (64.5%), 100 (32.3%), 140 (45.2%), and 86 (27.7%) indicated they could solve 1-2x 5, 1-2x > 2(6-x), 2x + 4-24, and m/3-3-3-6 using inequality concepts.23 (76.7%) of 30 teachers said their students could solve m/3-6-6, according to the study. 23 teachers (76.7%) said they test their pupils at the conclusion of each topic to ensure they comprehend inequality. 19 teachers (63.3%) said they encouraged their students to understand inequality. 23 (76.7%) of the teachers said they encouraged students to create small groups to practice unequal problem-solving, and 18 (60%) said they involved students in the teaching and learning process. 140 students (45.1%) felt that knowing linear inequalities would help them learn other math and scientific concepts. 146 (47.1%) strongly believed that studying inequalities would help them grasp inequality word problems. 150 students (48.1%) said learning about inequality would attract pupils. Understanding inequality, according to 170 (54.8%) students, will help them grasp math statements.

Understanding inequality, according to 160 students (51.6%), will help them provide rational solutions. 127 (41%) said linear inequalities would ease their anxieties. Still, 250 (80.6%) participants highly felt that it would assist them learn the concept. A statistically significant mean difference of 1.516 (Mean Deviation = 1.516) and a standard error difference of 1.606 (Mean Deviation = 1.516, p 0.05; Standard Error Difference = 1.606) show that both boys and girls did better on the achievement test.

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