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Mathematical Model and Analysis of Drug Addiction among Adolescent's Populace in Nigeria

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Abstract

Drug addiction has become a menace especially among adolescents which has massively contributed to the social vices perpetrated by youths in Nigeria. We constructed a five-compartment model to explain the transmission dynamics of addiction leading to a non-linear deterministic equation. The next generation approach was employed to obtain the basic reproduction number (R_0). The drug free equilibrium point was obtained and showed to be locally asymptotically stable when the threshold quantity is less than one. A suitable Lyapunov function was constructed for the global stability which was found to be globally asymptotically stable. Sensitivity analysis was conducted to ascertain the behavior of the various parameters on the threshold quantity to deduce a suitable intervention strategy. Numerical simulations are carried out, the analysis is discussed, and the results are presented in graphical form.

Keywords: Addiction, model, drug, stability, basic reproduction number, sensitivity analysis.

1. Introduction

Drug addiction has become an emerging menace globally resulting in public health issues among the adolescent population. Drugs can be used both positively and negatively. A drug is used positively when the user takes it as prescribed by a physician. A drug is used negatively when the user takes it without the prescription of a physician. In this case, we assume that the chemical substances of the drug negatively affect the tissues of the human body. Although these substances may be very useful to the human body when applied appropriately under the directive of a physician, they can become harmful under self-medication.

Public health organizations have used drugs to treat many illnesses. The use of drugs becomes an abuse when they are taken unlawfully, excessively, and illegitimately without a physician's prescription (Matonya et al., 2019). The danger of inappropriate drug usage cannot be overemphasized as it affects not only the victim's health, but also the entire development of the nations (WHO, 2022). Reports from UNODC (2022) indicated that those who suffered from substance abuse and those with similar drug-related disorders account for a quarter of a billion cases of abuse each year. This is responsible for about 183,000 deaths within the globe in 2012 (UNODC, 2014). The same UNODC report estimated between 162 million and 324 million people between the ages of 15 and 64 had used drugs illicitly (UNODC, 2014).

It has been observed that drug usage by individuals has increased to 22% between the years 2010 – 2019 globally. UNODC projects an 11% increase in the number of drug users globally by 2030 with an increase of 40% in Africa, since Africa has the potential for rapid growth in the adolescent population. In particular, the prediction shows that in 2030, Nigeria would have about 20 million drug users (UNODC, 2022). According to the World Health Organization (WHO, 2022), mental and narcolepsy disorder among teenagers has continuously increased because of the misuse of hard drugs. The usual drugs used by youths in Nigeria includes alcohol, amphetamines, nicotine, cocaine, synthetic opioids, and others. The Nigerian Government established a ministry known as the National Drug Law Enforcement Agency to combat drug abuse in the nation. Yet there has been a continuous increase in cases of violent activities recorded among young people (15 – 30 years of age) aided by drug usage. Mamman et al. (2014) had shown that 37.47% of Nigeria's drug users are in the northwest zone, 17.32% in the southwest zone, 13.5% in the southeast zone, 11.71% in the north-central zone, and 8.54% in the north-east zone.

At present, Nigeria and the globe are suffering from the negative impacts of drug misuse. Several works have been conducted on drug abuse by other researchers. Matonya and Kuznetsov (2019) formulated a mathematical model

to describe drug abuse in Tanzania. They used a next generation method to obtain the threshold quantity which was applied to compute the stabilities of their model. They showed that their analytical result was mathematically meaningful. They explicitly stated the conditions for both drug-free and endemic equilibrium points and showed that their model was locally and asymptotically stable when the threshold quantity is less than one and unstable otherwise. Their simulation proved that at the absence of control measures there will be a corresponding increase in the number of people who use drugs or are exposed to drugs. White and Comiskey (2007) developed a model to ascertain the impact of treatment on heroin use. They show that it is better to focus on preventing drug use in the first place rather than treatment, because drug users are prone to relapse even after treatment.

Nyabadza et al. (2013) formulated and discussed the system of drug abuse with crystal meth "Tik" that is influenced by drug-supply chains. They revealed that control of "Tik" can be one of the suitable measures to increase the number of light drug users that cease from drug usage. Bae (2014) discussed a model that involved the relationship between drug and tobacco usage. He established that the tobacco model displaced a periodic motion showing stages of recovery and re-addiction. Yakubu et al (2022) developed a five-compartment model that led to a system of ODEs. They showed that the drug-free equilibrium point is locally and globally asymptotically stable when the threshold quantity is less than one. They claimed that the illegal use of drugs can be eliminated from the community through adequate control measures. They further showed that if the basic reproduction number is greater than one then drug addiction can persist, and they proved that the model is locally and globally asymptotically stable.

In this study, we develop a drug addiction model and apply it to the current situation of Nigeria. Two types of drug users were considered, namely light users and heavy users. Both users were subjected to treatment before recovery. This model differs from Binuyo et al. (2021) in the following ways: presence of a treatment class, different technique for analysing GAS, construction of Lyapunov function, and sensitivity analysis. The dynamical systems of the model's behavior were also treated through numerical simulation.

2. Mathematical Model

2.1 Model Framework

The formulation of the model focuses on the dynamics of drug users in a mixed population. This is classified into five compartments: the susceptible represented by $S(t)$, lighter drug users $L(t)$, heavy drug users $H(t)$, those under treatment $T(t)$, and those who have recovered from drug usage $R(t)$, where t is time. The entire population size $N(t)$ is given by:

$$N(t) = S(t) + L(t) + H(t) + T(t) + R(t) \quad (1)$$

The SLHTR drug model is based on some major assumptions. We assume there is a frequent homogeneous mixing between individuals, and individuals enter the susceptible population at a constant rate of φ because of birth and immigration. We assume a susceptible individual becomes a drug user through interaction with other drug users. It is further assumed that an interaction function is responsible for this relationship analogous to the usual epidemic model known as the force of infection, and the per capita contact rate is β. Figure 1 gives the compartmental diagram of the model describing how individuals are faced with the likelihood of been initiated into drug addiction (i.e., drug infected as in a disease epidemic model). All populations experience the same natural death rate and possess equal chances of becoming drug users. Also, we assume that lighter drug users die at the rate of δ_L when using a drug for the first time in life, and heavy drug users die due to drug usage at the rate of δ_H.We also assume that susceptible populations become users of drugs through a social contact process alone, which produces an interaction function that is analogous to the usual epidemiological model known as the force of infection. Susceptible individuals acquire both light and heavy addictions through effective social contact with these drug users at the rate of λ. The force of social interaction (infection) related with this drug addiction is described by

$$\lambda = \beta \frac{(L + \psi H)}{N}$$

where β is the effective contact rate for both light (L) and heavy (H) drug users, and ψ is the modification parameter that lies between 0 and 1. This accounts for the relative infectiousness of persons with heavy drug users. The susceptible persons become lightly and heavily addicted to drugs at the rates of $(1 - \varepsilon)\lambda$ and $\varepsilon\lambda$, respectively. The lighter drug users become heavily addicted, taken to a rehabilitation center (treatment), and recover at the rates of α_1, α_2 and α_3 respectively. Those who recover from drug addictions have the chances of returning to light and heavy addictions at the rates of θ_1 and θ_2 respectively.

2.2 Model Equations

It follows from the assumptions and the schematic diagram shown below, that the ordinary differential equations for the model are given as:

$$\left.\begin{aligned} \dot{S} &= \varphi - \lambda S - \mu_S S \\ \dot{L} &= \varepsilon\lambda S + \theta_1 R - AL \\ \dot{H} &= (1-\varepsilon)\lambda S + \alpha_1 L + \theta_2 R - BH \\ \dot{T} &= \rho H + \alpha_2 L - CT \\ \dot{R} &= \gamma T + \alpha_3 L - DR \end{aligned}\right\} \quad (2)$$

where

$A = \mu_L + \delta_L + \alpha_1 + \alpha_2 + \alpha_3$

$B = \mu_H + \delta_H + \rho$

$C = \mu_T + \rho$

$D = \mu_R + \theta_l + \theta_2$

with initial conditions $\left(S(0), L(0), H(0), T(0), R(0)\right) \in \mathbb{R}^5{}_+$.

2.3 Model Diagram

Figure 1 below represents the transmission of drug addiction denoted by arrows which describe the movement of individuals to compartments.

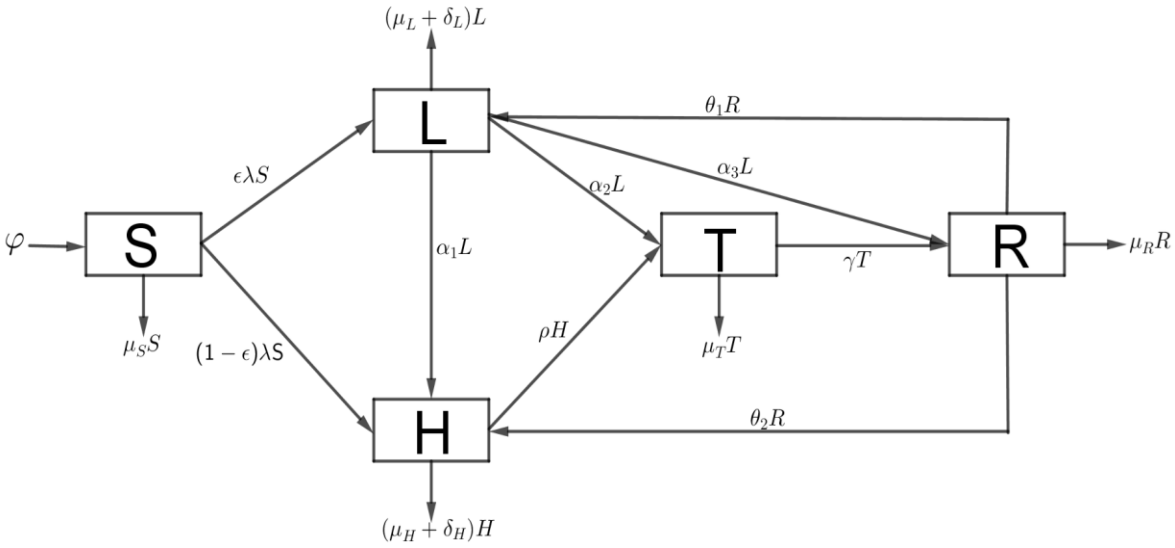

Figure 1. Schematic Diagram of Drug Addiction

Table 1 below lists the variables and parameters used in the model.

Table 1. Variables and Parameters Description with Values

Variables	Description	Values	Reference
$S(t)$	Susceptible population	5000	Yakubu et al. (2022)
$L(t)$	Lighter drug users	3000	Yakubu et al. (2022)
$H(t)$	Heavy drug users	2500	Yakubu et al. (2022)
$T(t)$	Treated population	500	Yakubu et al. (2022)
$R(t)$	Recovered Population	270	Yakubu et al. (2022)
Parameters			
φ	The recruitment rate	1500	Mohammed et al. (2019)
ϵ	The fraction of susceptible becoming H.	0.2	Estimate
$\mu_S, \mu_L, \mu_H, \mu_T, \mu_R$	Natural death rate of S, L, H, T and R	0.02	Yakubu et al. (2022)
δ_L, δ_H	Death rate due L and H	0.001	Estimate
$\alpha_1, \alpha_2, \alpha_3$	Proportional of L becoming H, T and R	0.3, 0.35, 0.45	Yakubu et al. (2022)/Estimate
θ_1, θ_2	The rate at which R becomes H and L	0.002, 0.005	Estimate
γ	The rate at which H moves to T	0.5	Mohammed et al. (2022)

2.4 Basic Properties of the Model

For the drug users transmissions for equation 2 to be suitable and meaningful like the usual diseases epidemiological models it is imperative to show that all the state variables are non-negative at time t. This implies that for all time $t \geq 0$ the solutions of equation (2) remain non-negative.

2.4.1 Non-negativity of the Solution

Theorem 1. If $S(0) \geq 0,\ L(0) \geq 0,\ H(0) \geq 0, T(0) \geq 0$ and $R(0) \geq 0$, then model (2) remains non-negative for all $t > 0$.

Proof:

Consider that $S(0) \geq 0$ from model (2), then the first ODE, written as:

$$\frac{d}{dt}[S(t)\omega(t)] = \varphi\omega(t)$$

where $\omega(t) = exp\left(\int_0^t \left[\frac{\beta(L+\psi H)S}{N} + \mu_S\right] dS\right) > 0$ which is the I.F. Therefore, integrating with respect to t, gives

$$S(t) = \left[S(0) + \int_0^t \varphi\omega(s)ds\right] \times \omega^{-1}(s) > 0$$

This same theorem can be used to show that $L(t) \geq 0, H(t) \geq 0, T(t) \geq 0,$ and $R(t) \geq 0$.

2.4.2 Invariant Region

Theorem 2. All the solutions of the SLHTR model (2) with initial values in $\mathbb{R}_+^5$ are bounded and converge into a region $\mathbb{X} = \left\{(S, L, H, T, R)\epsilon\ \mathbb{R}_+^5 : 0 < N(t) \leq \frac{\varphi}{\mu_S}\right\}$.

Proof: Considering equation (2) above where $\big(S(t), L(t), H(t), T(t), R(t)\big)$ gives the SLHTR model (2). Now differentiating equation (1) with respect to t gives

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dL(t)}{dt} + \frac{dH(t)}{dt} + \frac{dT(t)}{dt} + \frac{dR(t)}{dt}$$

Substituting the values of the ODEs and simplifying, we have

$$\frac{dN(t)}{dt} = \varphi - \mu_S S - \mu_L L - \mu_H H - \mu_T T - \mu_R R - (\delta_L L + \delta_H H)$$

Adopting the technique used by Nkamba et al. (2019), the equation reduces to

$$\frac{dN(t)}{dt} = \varphi - \mu N - (\delta_L L + \delta_H H)$$

where $\mu = \min\{\mu_S, \mu_L, \mu_H, \mu_T, \mu_R\}$. In absence of drug interaction with drug users there will be no deaths due to light and heavy addictions, so, $\delta_L = \delta_H = 0$. The equation above becomes

$$\frac{dN(t)}{dt} = \varphi - \mu N \tag{3}$$

From equation (3), it follows that

$$\lim_{t \to \infty} \text{Sup}\, N(t) \leq \frac{\varphi}{\mu}$$

Also, from model (2), the first ODE

$$\frac{dS(t)}{dt} \leq \varphi - \mu_S S$$

thus,

$$\lim_{t \to \infty} Sup\, S(t) \leq \frac{\varphi}{\mu_S}.$$

This completes the required proof.

Next, relating Theorem 2 with the trivial existence and uniqueness of a local stability, it is obvious that the theorem establishes the mathematical and biological feasibility of the model.

Theorem 3. The system of model 2 is considered a biologically feasible dynamical system of a compact set when $\Gamma = \left\{(S, L, H, T, R) \epsilon\, \mathbb{R}_+^5 : 0 \leq S \leq \frac{\varphi}{\mu_S}, N(t) \leq \frac{\varphi}{\mu}\right\}$.

3. Drug-Free Equilibrium (DFE) and the Basic Reproduction Number (BRN)

The drug-free equilibrium of the proposed SLHTR model is obtained by considering L = H = T = R = 0. Therefore, the DFE is $E_0 = \left(\frac{\varphi}{\mu_S}, 0, 0, 0, 0\right)$. We proceed to compute the basic reproduction number using the next generation matrix technique as found in Nkamba et al. (2019). Let $\mathrm{X} = (L, H)^T$ and rewrite equation 2 as $\mathrm{X} = \mathbb{F}(\mathrm{X}) - \mathbb{W}(\mathrm{X})$ where $\mathbb{F}(\mathrm{X}) = (\varepsilon\lambda S, (1 - \varepsilon)\lambda S)^T$ and $\mathbb{W}(\mathrm{X}) = (AL - \theta_1 R, BH - \theta_2 R)^T$. These equations represent the vector of new generated drug interaction and the transfer between the populations, respectively. Then the Jacobian matrices of $\mathbb{F}(\mathrm{X})$ and $\mathbb{W}(\mathrm{X})$ are given as:

$$F = \beta \frac{S_0}{N_0} \begin{pmatrix} \varepsilon & \psi\varepsilon \\ (1-\varepsilon) & \psi(1-\varepsilon) \end{pmatrix} \quad \text{and} \quad W = \begin{pmatrix} A & 0 \\ -\alpha_1 & B \end{pmatrix}$$

Therefore, the next generation matrix is represented by FV^{-1} of the system SLHTR. The basic reproduction is denoted by $\mathfrak{R}_0 = \rho(FV^{-1})$ where ρ denotes the spectral radius of the next generation matrix FV^{-1} of the model given by

$$\mathfrak{R}_0 = \rho(FV^{-1}) = \beta \frac{[\varepsilon(B + \alpha_1\psi) + A\varphi(1-\varepsilon)]}{AB} \tag{4}$$

Equation (4) describes the drug habit transmission potential of SLHTR governed by eleven parameters. This number shows the new addiction that occurs by the presence of one addicted person in an entire susceptible population. The elimination of those using drugs in the population is only possible when $\mathfrak{R}_0 < 1$, and the drug-using population increases exponentially if $\mathfrak{R}_0 > 1$.

3.1 Stability of Drug – Free Equilibrium

In this section we shall examine the nature of stability for the drug-free equilibrium of the SLTHR model at E_0.

Theorem 4. The SLHTR model at $E_0 = \left(\frac{\varphi}{\mu_S}, 0,0,0,0\right)$ is locally asymptotically stable when $\mathfrak{R}_0 < 1$ and unstable when $\mathfrak{R}_0 > 1$.

Proof: It follows that Jacobian matrix of equation 2 at E_0 is represented as:

$$J_{E_0} = \begin{pmatrix} -\mu_S & 0 & \beta\psi & 0 & 0 \\ 0 & -A & \beta\psi & 0 & \theta_1 \\ 0 & \alpha_1 & -B & 0 & \theta_2 \\ 0 & \alpha_2 & \rho & -C & 0 \\ 0 & \alpha_3 & 0 & \gamma & -D \end{pmatrix} \quad (5)$$

The characteristics equation of the matrix J_{E_0} is represented by $|J_{E_0} - \lambda I| = 0$ where λ is any eigenvalue of equation (5). Hence the eigenvalues of equation (5) are: $\lambda_1 = -\mu_S, \lambda_2 = -A, \lambda_3 = -B, \lambda_4 = -C, \lambda_5 = -D$ Since all the eigenvalues are negative, it follows that $\Re_0 < 1$, and the system is said to be locally asymptotically stable.

Theorem 5. The SLHTR model at $E_0 = \left(\frac{\varphi}{\mu_S}, 0,0,0,0\right)$ is considered globally asymptotically stable when $\Re_0 < 1$ and unstable when $\Re_0 > 1$.

Proof: We adopt the technique used in Pal et al. (2020) to prove Theorem 5. We categorized our model ODE into two groups: X_1 represents the population that is free from drugs, and X_2 represents the population of drug users. The model ODE is written as:

$$\begin{cases} \frac{dX_1}{dt} = F(X_1, X_2) \\ \frac{dX_2}{dt} = G(X_1, X_2), G(X_1, 0) = 0 \end{cases} \quad (6)$$

where $X_1 = (S, R) \in \mathbb{R}^2_+$ and $X_2 = (L, H, T) \in \mathbb{R}^3_+$ as $\left(\frac{\varphi}{\mu_S}, 0, 0, 0, 0\right)$ is the drug addiction free equilibrium of the dynamics (2). The DFE is globally stable when the following conditions are satisfied:

1. For $\frac{dX_1}{dt} = F(X_1, X_2), X_1^*$ is globally asymptotically stable
2. $G(X_1, X_2) = ZX_1 - \check{G}(X_1, X_2)$, $\check{G}(X_1, X_2) \geq 0$ for $(X_1, X_2) \in \Omega$ where
   $Z = D_{X_2} G(X_1^*, 0)$, is a Metzler matrix and Ω is positively invariant set with respect to the model (2).

Now adopting Castillo-Chavez et al. (2002) we shall examine the features mentioned above.

$$F(X_1, 0) = \begin{vmatrix} \varphi - \mu_S \\ 0 \end{vmatrix}, \; Z = \begin{vmatrix} -A & 0 & 0 \\ \alpha_1 & -B & 0 \\ \alpha_2 & \rho & -C \end{vmatrix} \text{ and } \check{G}(X_1, X_2) = \begin{vmatrix} \varepsilon\lambda S \\ (1-\varepsilon)\lambda S \\ 0 \end{vmatrix}$$

It is very clear to see that $\check{G}(X_1, X_2) \geq 0$ if the stat variable remains within Ω at every time *t*. Therefore, $X_1^* = \left(\frac{\varphi}{\mu_S}, 0\right)$ is indeed the globally asymptotically stable equilibrium of the dynamic $\frac{dX_1}{dt} = F(X_1, 0)$. Thus the theorem is proved.

3.2 Global Stability of Drug-Free Equilibrium

We shall investigate the global stability of the drug-free equilibrium.

Lemma 3.2. The drug-free equilibrium of SLHTR model is globally asymptotically stable if $\Re_0 < 1$ and unstable if $\Re_0 > 1$.

Proof: Adopting the procedure in Pal et al. (2020), we define a Lyapunov function

$F = a_1 L + a_2 H$

Where both a_1 and $a_2 > 0$ and F is defined as

$\dot{F} = a_1 \dot{L} + a_2 \dot{H}$ (7)

Then substituting the values of $\dot{L}$ and $\dot{H}$ from equation (2) to have

$\dot{F} = [a_1\varepsilon + a_2(1-\varepsilon)]\lambda S - (a_1 A + a_2\alpha_1)L - a_2 BH$ (8)

Thus, to obtain the values of a_1 and a_2, we equate the coefficient of λS to the numerator of $\mathcal{R}_0$ without β and that of L to the denominator of $\mathcal{R}_0$ as shown below.

$a_1\varepsilon + a_2(1-\varepsilon) = \varepsilon(B + \alpha_1\psi) + A\varphi(1-\varepsilon)$

$a_1 A + a_2\alpha_1 = AB$

Solving these two equations, we obtain $a_1 = \alpha_1\psi + B > 0$ and $a_2 = A\psi > 0$. Substituting these values in equation (7) we have

$$\dot{F} = [\varepsilon(\alpha_1\psi + B\,) + A\psi(1-\varepsilon)]\lambda S - (A(\alpha_1\psi + B\,) + A\psi\alpha_1)L - AB\psi H$$

$$\dot{F} = [\varepsilon(\alpha_1\psi + B\,) + A\psi(1-\varepsilon)]\beta\frac{(L+\psi H)S}{N} - (A(\alpha_1\psi + B\,) + A\psi\alpha_1)L - AB\psi H$$

$$\dot{F} \leq AB(L+\psi H\,)\left[\beta\frac{S}{N}\left(\frac{\varepsilon(B+\alpha_1\psi)+A\varphi(1-\varepsilon)}{AB}\right) - 1\right] \tag{9}$$

For $S \leq N$ known to be in the variant set, it implies that

$$\dot{F} = AB(L+\psi H\,)(\mathcal{R}_0 - 1)$$

It follows that $\dot{F} < 0$ if $\mathcal{R}_0 < 1$; similarly, $\mathcal{R}_0 = 1$ implies that $L = H = 0$ and the equality holds. We conclude from Lasalle's invariance principle that the DFE is globally asymptotically stable since $S \to \frac{\varphi}{\mu_S}$ as $t \to \infty$ at $L = H = 0$.

4. Sensitivity Analysis for $\mathcal{R}_0$

Sensitivity analysis plays a crucial role in ascertaining the parameter that mostly influences the behavior of $\Re_c$ which can easily aid in predicting the intervention strategy of the dynamics. Relative changes can be measured in a variable when parameters vary noticeably in sensitivity indices. A parameter Ω is considered sensitive when a small change in its parameter value gives a certain behavior to the model Abegye et al. (2022), described by

$$\Gamma_{\Omega}^{\mathcal{R}_0} = \frac{\partial\mathcal{R}_0}{\partial\Omega}\times\frac{\Omega}{\mathcal{R}_0} \quad \ldots \quad \ldots \quad \ldots \tag{10}$$

$$\Gamma_{\beta}^{\mathcal{R}_0} = \frac{\partial\mathcal{R}_0}{\partial\beta}\times\frac{\beta}{\mathcal{R}_0} = 1$$

$$\Gamma_{\epsilon}^{\mathcal{R}_0} = \frac{\partial\mathcal{R}_0}{\partial\epsilon}\times\frac{\epsilon}{\mathcal{R}_0} = -\frac{\epsilon\{\varphi(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3) - (\mu_H+\delta_H+\rho+\alpha_1\psi)\}}{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)}$$

$$\Gamma_{\mu_H}^{\mathcal{R}_0} = -\frac{\mu_H\{\epsilon\alpha_1\psi+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}{(\mu_H+\delta_H+\rho)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\delta_H}^{\mathcal{R}_0} = -\frac{\delta_H\{\epsilon\alpha_1\psi+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}{(\mu_H+\delta_H+\rho)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\rho}^{\mathcal{R}_0} = -\frac{\rho\{\epsilon\alpha_1\psi+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}{(\mu_H+\delta_H+\rho)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\alpha_1}^{\mathcal{R}_0} = -\frac{\alpha_1\epsilon\{(\mu_H+\delta_H+\rho)-\psi(\mu_L+\delta_L+\alpha_2+\alpha_3)\}}{(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\psi}^{\mathcal{R}_0} = \frac{\psi\epsilon\alpha_1}{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)}$$

$$\Gamma_{\varphi}^{\mathcal{R}_0} = \frac{\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)}{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)}$$

$$\Gamma_{\mu_L}^{\mathcal{R}_0} = -\frac{\mu_L\epsilon\,(\mu_H+\delta_H+\rho+\alpha_1\psi)}{(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\delta_L}^{\mathcal{R}_0} = -\frac{\delta_L\epsilon\,(\mu_H+\delta_H+\rho+\alpha_1\psi)}{(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\alpha_2}^{\mathcal{R}_0} = -\frac{\alpha_2\epsilon\,(\mu_H+\delta_H+\rho+\alpha_1\psi)}{(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

$$\Gamma_{\alpha_3}^{\mathcal{R}_0} = -\frac{\alpha_3\epsilon\,(\mu_H+\delta_H+\rho+\alpha_1\psi)}{(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\{\epsilon(\mu_H+\delta_H+\rho+\alpha_1\psi)+\varphi(1-\epsilon)(\mu_L+\delta_L+\alpha_1+\alpha_2+\alpha_3)\}}$$

Note that the rate of contact β has a positive impact on the transmission of drug addiction. This means that as this value increases, more and more adolescents become drug addicts. Therefore, to control addiction among young people, we need $\beta < 1$. The parameters μ_L, δ_L, α_2, and α_3 presented a negative influence, which implies that increasing the values of these parameters would result in reducing the population of adolescents that would be addicted to drugs. Similarly, μ_H, δ_H, and ρ have a negative impact if $\epsilon < 1$, which implies that increasing their

values reduces the transmission of the addiction. Also, α_1 and ϵ have a negative impact as long as $\psi(\mu_L + \delta_L + \alpha_1 + \alpha_2) < (\mu_H + \delta_H + \rho)$ and $(\mu_H + \delta_H + \rho + \alpha_1\psi) < \varphi(\mu_L + \delta_L + \alpha_1 + \alpha_2 + \alpha_3)$, respectively. The parameters ψ and φ influence the transmission of drug addiction positively. Hence to control drug addiction among adolescents, interested parties should focus their attention on the modification parameter and number of youths recruited to the susceptible population.

5. Numerical Simulation of SLHTR model

In this section we shall present the behavior of the dynamical system by numerical simulation using the parameter values given above over a period of 20 years.

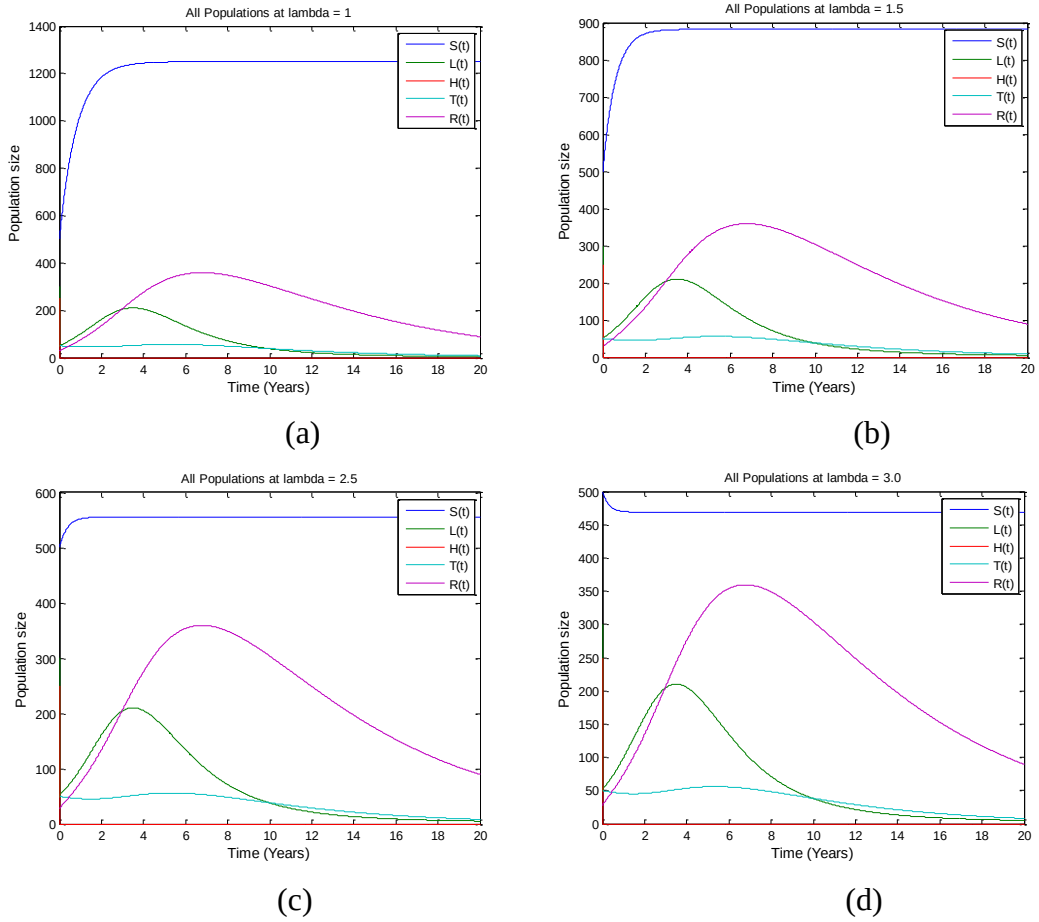

Figure 2. The Behavior of the Model (2) at $\lambda = 1, 1.5, 2.5$ and 3.0.

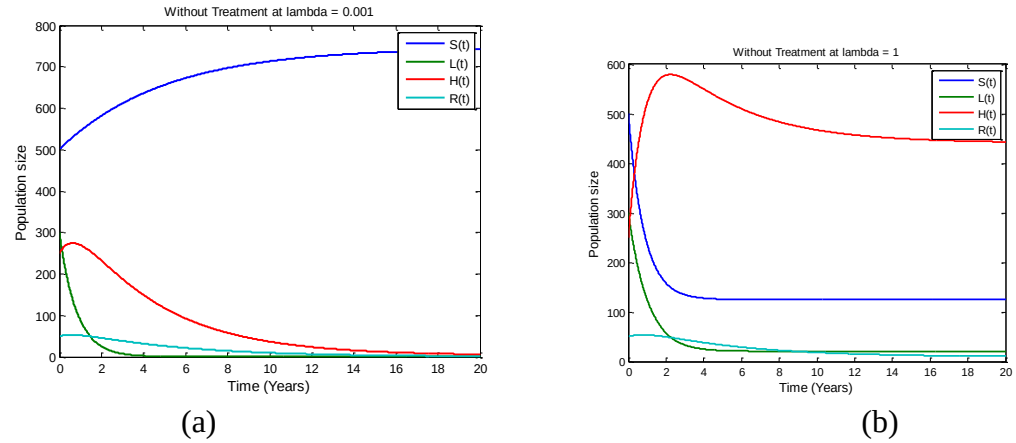

Figure 3. The Behaviour of the Dynamics Without Rehabilitation (Treatment) at $\lambda = 0.001$ and 1.

The simulation of the dynamics in Figure 2(a): shows that the susceptible population grows up to 12,000, which is its maximum population above the other populations, L, H, T and R (with population below 400) when $\lambda = 1$. This shows that the susceptible population flourishes well when there is no interaction with drug users and the contact rate is very low. On the other hand, when λ increases to 1.5, the susceptible population drops below 900. The interpretation is that when the infectious contact rate increased, some people did become infected and moved to the lighter and heavier addiction classes resulting in a decrease of the susceptible population. Similarly, Figures 2(c) and 2(d) present a situation that follows the same phenomenon – as λ increases to 2.5 and 3.0 the susceptible population drops below 600 and 500, respectively.

Figures 3(a) and 3(b) describe the behavior of the model without rehabilitation at $\lambda = 0.001$ and 1. When $\lambda = 0.001$, the susceptible population grows, and when $\lambda = 1$, the susceptible population collapses (dies). In Figure 3(b), the heavier drug addict population grows above all the other populations. This means that treatment becomes highly needed when addiction increases rapidly among adolescents. It will be dangerous if most of the adolescent population are heavy drug addicts. The need to adopt measures to reduce and eradicate the use of drugs becomes imperative by the Government, non-governmental bodies, and religious groups.

6. Discussion and Conclusion

Indeed, drug addiction has promoted many violent atrocities among adolescents; therefore, it is necessary to control drug use among the youth (Hafiruddin 2019). The numerical simulation conducted in this work shows that rehabilitation of drug addicts plays an indispensable role in controlling drug misuse among adolescents. Those who are rehabilitated should be completely separated from their previous mates or gangs whether light or heavy users; otherwise, they will keep cycling between drug users, treatment, recovery, and back to addiction again. Therefore, it becomes impossible to eliminate the drug abuse in the society and among the adolescent in Nigeria. The only chance to eliminate drug addiction is to ensure that those who are rehabilitated move permanently to the susceptible population. Attention should be given to those who have been treated by the Government, non-governmental organizations, and religious bodies to enhance their continuous freedom from drug misuse. Also, all stakeholders in the human population should intensify their public campaigns on the danger of drug usage without a physician prescription (Chinnadurai 2020).

We studied the behavior of drug addiction among Nigeria adolescents using a five-compartmental model that led to the formulation of a systems of ODEs. The drug addiction free equilibrium point was obtained, which had been proved to be locally and globally asymptotically stable especially if the model threshold quantity is less than one. The implication is that addiction to drugs can be eliminated from the human population when substantial measures are taken. In addition, the analysis indicated the possibility of the persistence of drug addiction when the threshold quantity is greater than one. The implication is that all the stakeholders must take the appropriate control measures to not allow drug addiction to persist in the human population.

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