Modelling Divorce Epidemic on Network

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Abstract

Most couples entering into marriage are happy and do not anticipate divorce as an option. However, in reality, marriage is complicated, and many factors can lead to divorce. Using a compartmental model, we simulated and analyzed transmission network of divorce spread in continuous time on networks (that is, we analyzed married-divorced-separated model on networks). According to our findings, divorced individuals or nodes can cause many marriage nodes to become separated. Again there were few divorced nodes at the peak and at the end of the epidemic, but most marriages resulted in separation at the end of the epidemic. At the end of the epidemic, 80% of married nodes became separated nodes, which is a very serious problem. This suggests that divorce or separation is contagious and can spread quickly through indirect contact. This makes it vital to examine the effect of divorce to the population so as reduce the number of broken homes at the end of the divorce epidemic.

Keywords: compartmental model, complex systems, Marriage networks, divorce epidemics

1. Introduction

Marriage is regarded in high esteem in most communities and most people see it as an added advantage. It is regarded as a symbol of love, stability, and dedication. Also, it is seen as a way of fortifying the relationship between partners, leaving a lasting legacy, strengthening the family, forging a solid link between them, and establishing a stable environment in which to raise children. Others also see it as a chance for two individuals to develop together and share their lives. Research has shown that matrimony can result in increased advantages for an individual's physical, mental, and emotional well-being (Waite, 1995). Along with increasing life happiness, financial security and social support, marriage can also lessen loneliness and foster a sense of camaraderie. It is a source of courage and consolation for couples going through a trying moment. Contrarily, most people do not plan for divorce, however occasionally it does reach a stage where it leaves a couple with no other option than to file for divorce. Any couple going through a divorce will find it tough, and the experience can have a lasting effect on all parties. Before making any judgments, it's crucial to take your time, carefully consider the circumstances, and speak with an expert.

Numerous studies have used mathematical modeling to examine dynamic systems in areas such as physics, chemistry, biology, and economics. It is an effective way for deciphering the dynamics and behavior of complex systems and for forecasting their future. The compartment models that Kermack and McKendrick (1932) established serve as the foundational models for the majority of investigations conducted today. According to these models, each subject at any given time is assumed to belong to a certain compartment of the population. These models also presume that the population's subjects are well-mixed, which implies that a complete graph represents the subjects' underlying contact network. Due to the stigma attached to divorce, there is growing interest in modeling divorce dynamics with compartmental models. Gambrah & Adzadu (2018) developed a mathematical model for the dynamics of divorce epidemic using a non-linear MSD. Tessema et al., (2022) used the stability theory of differential equations to analyze qualitatively a deterministic model for marriage and divorce in a population. Karaagac & Owolabi, (2023) developed the fractal-fractional marriage divorced model by the fractal fractional Caputo–Fabrizio derivative. Barnes, et al., (2023) proposed a mathematical model to examine the variations in the ratios of adulterers, married people, and divorcees. Hugo & Lusekelo, (2021) developed a model where counselling was used as control measure to ensure peace and harmony in marriages. In recent years, epidemic processes have been studied in more detail and on a wider range of realistic network structures (eg Wang et al., 2022; Hiraoka et al., 2022; Valdez et al., 2023).

Other studies have likewise used this idea to model other social problems like the spread of rumors on social media, the spread of misinformation, and many others (Zhu et al., 2021; Zhang et al., 2018). These models have proven to be very useful as scholars have gotten a better insight in the dynamics of these issues. Modeling divorce in likewise manner can demonstrate how behavioral shifts and rising divorce rates, can lead to an epidemic of divorce by simulating interpersonal interactions. It can also pinpoint the most effective ways to stop or lessen the effects of an epidemic of divorce. Research has indicated that current theories of an epidemic's spread entail a network. This could be a network of contacts between people. The edges show the ways of contact, while the nodes stand in for the people.

The spread of divorce can be explained using network theory. The application of network theory allows for the identification of variables such as social relations and the frequency of divorce among peers, family, and acquaintances that contribute to the divorce epidemic. Divorce is contagious (Chen et al., 2021) and can spread through social networks. According to the network theory, those who have friends and family who have divorced are more likely to get divorced themselves. Therefore, if a significant portion of the population in the area has divorced individuals, the divorce rate in that area may rise.

This paper examines marriage-divorce-separation (MDS) model on a network by means of simulation and inference of divorce spread in continuous time transmission network. When the diffusion is combined with the basic MDS dynamics, the necessary components to describe and comprehend divorce are obtained. The structure of the article is as follows: the materials and methods are presented in Section 2, along with an introduction to the model, a discussion of its key components, and the epidemic criterion. In Section 3, we analyzed and stimulated our model on a 1000 node network to obtain our results. Section 4 provides the discussion and conclusion of our study.

2. Materials and Methods

In this section we provide the mathematical model used in this study. Given a vector function of time $v(t)$, it's time-derivative is given as $\dot{v}(t)$. For two vectors $v_1, v_2 \in \mathbb{R}^n$, any of the following expressions could be true: $v_2 \leq v_1$, $v_2 < v_1, v_2 \neq v_1$ or $v_2 \ll v_1$. For a matrix $A \in \mathbb{R}^{n\times n}$, *ρ(A)* is the spectral radius and $s(A)$ represent its eigenvalues' largest real part. Also, a_{ij} shows i, j^{th} entry of *A* and the set $\{1,...,n\}$ is represented by $[n]$. A given Metzler matrix $M \in \mathbb{R}^{n\times n}$ is reducible if there exists a permutation matrix Q such that $Q^{-1} \times Q = \begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix}$, where T_1 is a square matrix and T_3 is also a square matrix, or if $n = 1$ and $Q = 0$.

Given the graph $G = (N, L)$, *N* represents the nodes and $L \subseteq N \times N$ represents the edges whiles (i, j) represents a link from node *i* to *j*. If *G* has *n* nodes, then $W \in \mathbb{R}^{n\times n}$ which is an adjacency matrix can be associate with it, where $w_{i,j} \in \mathbb{R}_{\geq 0}$ are its' entries, and $w_{i,j} = 0$ if $(i, j) \notin L$.

1.1 Marriage-Divorced-Separated(MDS) Networked model

Studies have shown that, marriage individuals who are connected to divorced individuals in one way or the other are likely also get divorced or go through the divorce process (McDermott et al., 2013). As such we modeled the marriage-divorced-separated(MDS) as:

$$\dot{M}(t) = -\beta M(t)D(t) + \delta S(t)$$

$$\dot{D}(t) = \beta M(t)D(t) - \gamma D(t) \quad (1)$$

$$\dot{S}(t) = \gamma D(t) - \delta S(t)$$

where

β= transmission rate,

γ=separation rate,

δ= receding rate after separation. On the other hand, setting δ = 0 provides us with an MDS model, from which separated individuals do not go back to marriage.

The MDS networked model used in this study represent the situation in which individuals as nodes are connected by way of a contact graph which is defined by the matrix, $A = \{a_{ij}\}$, where a_{ij} represents the presence or otherwise of a connection between *i* and *j*.

For our MDS process, the probability of individual i being divorced is represented by $p_i(t) \in [0,1]$ whiles that of individual i being separated is represented by $s_i(t) \in [0,1]$ at any time t, hence we have the equation:

$$\dot{p}_i = (1 - p_i - s_i)\beta_i \sum_{j=1}^{n} a_{ij} p_j - s_i p_i \quad (2)$$

$$\dot{s}_i = s_i p_i$$

where they remain in the range [0, 1].

Considering our n nodes network, there is the possibility for each node to be infected if one of its neighbors is divorced. We denote the number of married nodes, divorced nodes and separated nodes in subpopulation i, as M_i , D_i and S_i respectively. Assuming a constant subpopulation $\forall i$: $M_i(t) + D_i(t) + S_i(t) = N_i$ for all $t \geq 0$.

Hence equation (1) can be written as:

$$\dot{M}_i(t) = -\sum_{j=1}^{n} \eta_{ij} \frac{M_i(t)}{N_i} D_j(t) - \delta_i S_i(t)$$

$$\dot{D}_i(t) = \sum_{j=1}^{n} \eta_{ij} \frac{M_i(t)}{N_i} D_j(t) + \gamma_i D_i(t) \quad (3)$$

$$\dot{S}_i(t) = \gamma_i D_i(t) - \delta_i S_i(t)$$

By the application of Euler's method (Din, 2018) to equation (2) we obtain the following discrete time models:

$$p_i^{k+1} = p_i^k + y\left[(1 - p_i^k - s_i^k)\beta_i \sum_{j=1}^{n} a_{ij} p_j^k - \gamma_i p_i^k\right] \quad (4)$$

$$s_i^{k+1} = s_i^k - y\gamma_i p_i^k$$

where k =time step
y= sampling parameter.

1.1 Stability analysis

The Basic Reproduction Number (*R_0*), for networked epidemic models are crucial for characterizing the asymptotic stability behavior of the equilibrium state of the spread dynamics. For our case R_0 provides a stability threshold for the divorce-free equilibrium for the MDS model.

The proposition by Khanafer et al., 2016 and Fall et al., 2007 offers a sufficient and necessary condition for the divorce free equilibrium's stability.

Proposition: Assume $G=(N,L)$ is a digraph which is strongly connected. With $[0,1]^n$ as the domain of attraction, the divorce free equilibrium is asymptotically stable if and only if $R_0 \le 1$.

Considering the continuous time model above, we obtain

$$m_i(t)=1-p_i(t)-s_i(t), \forall i\in[n] \quad (5)$$

using the results of Mei et al. (2017).

Theorem (Mei et al., 2017): Consider the networked MDS model over a strongly connected digraph with $\beta>0$, $\gamma>0$, adjacency matrix *A*, $p_i(0)>0$ for some *i*, and $m_i(0)>0$ $\forall i\in[n]$. Let $\psi_{\max}(t)$ be the normalized left eigenvector and $\lambda_{\max}(t)$ be the dominant eigenvalue of the non-negative matrix $diag(m(t))A$ for $t\ge 0$. Thus, $\forall i\in[n]$,

i. $t\to m_i(t)$ is decreasing monotonically,

ii. the pairs $(m^*,0)$ is the set of equilibrium points, for all $m^*\in[0,1]^n$,

iii. $\lim_{t\to\infty} p_i(t)=0$,

iv. there is a $\bar{t}$: $\beta\lambda_{\max}(t)<\gamma$ $\forall t\ge\bar{t}$, and $t\to\psi_{\max}(t)^T v(t)$ $\forall t\ge\bar{t}$, which is the weighted average is exponentially and monotonically reducing to zero.

We now write our MDS discrete time model as:

$$m_i^{k+1}=m_i^k+y\left[-m_i^k\sum_{j=1}^{n}\beta_i a_{ij}p_j^k\right]$$

$$p_i^{k+1}=p_i^k+y\left[m_i^k\sum_{j=1}^{n}\beta_i w_{ij}p_j^k-\gamma_i p_i^k\right] \quad (6)$$

$$s_i^{k+1}=s_i^k-y\gamma_i p_i^k$$

3. Simulations

In this section, we perform simulations and analyze the impact of the parameter on the divorce spreading and the threshold. with 1000 nodes, where 999 are married and one individual is divorced. For consistency, the networks are the same throughout the paper.

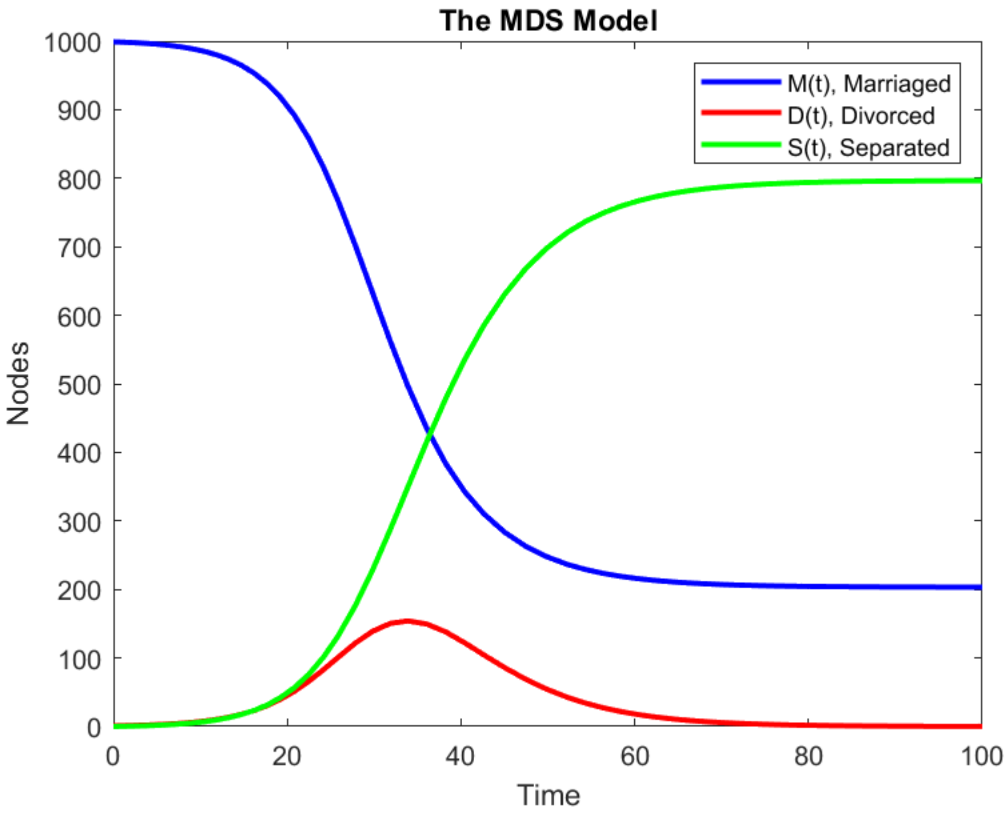

Figure 1:Networked MDS Epidemic Curve

Figure 1 shows the epidemic curve of the results where, blue, red, and green represent married, divorced, and separated, respectively. From the above figure we can see that the epidemic goes down at time greater than 30. Although the peak of the epidemic in not great (that is about 150 nodes) the aftermath sees a huge number of separation as compared with marriage. Which shows that the aftermath of this epidemics is more serious than we anticipated.

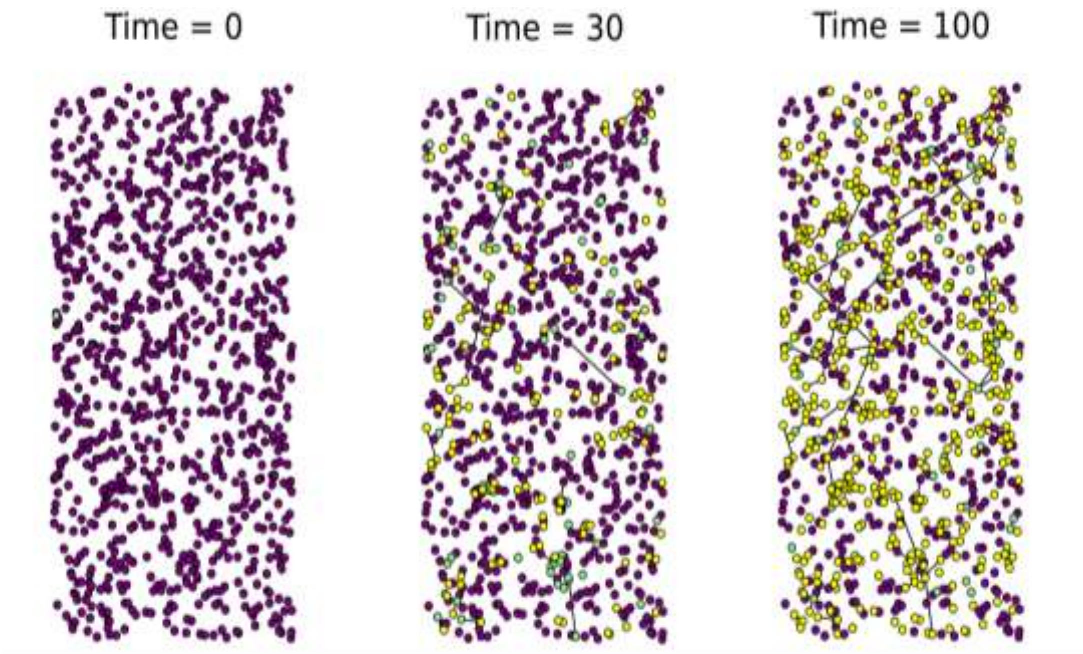

Figure 2:Epidemic state in the network at 3 different time points. Purple is the marriage node, green divorced node and yellow is the separation node.

Figure 2 also shows the epidemic states from the beginning when there is only one divorced individual (node) among the 1000 nodes that is when time is 0 and then at the peak of the epidemic when time is 30 and lastly at the end of the epidemic when time is 100. In the above diagram, we can see that divorced nodes are not as many at the peak and end of the epidemic, but at the end of the epidemic, separation nodes are more than all other nodes. This is likely because divorced nodes are more of a one-time event, whereas separation nodes are an ongoing process. As the epidemic progresses, more people are affected, and more people decide to get divorce or separation. This shows that we have a big issue at hand as studies have shown that the percentage of separation marriages that ends in divorce is greater than those that go back to the same marriage.

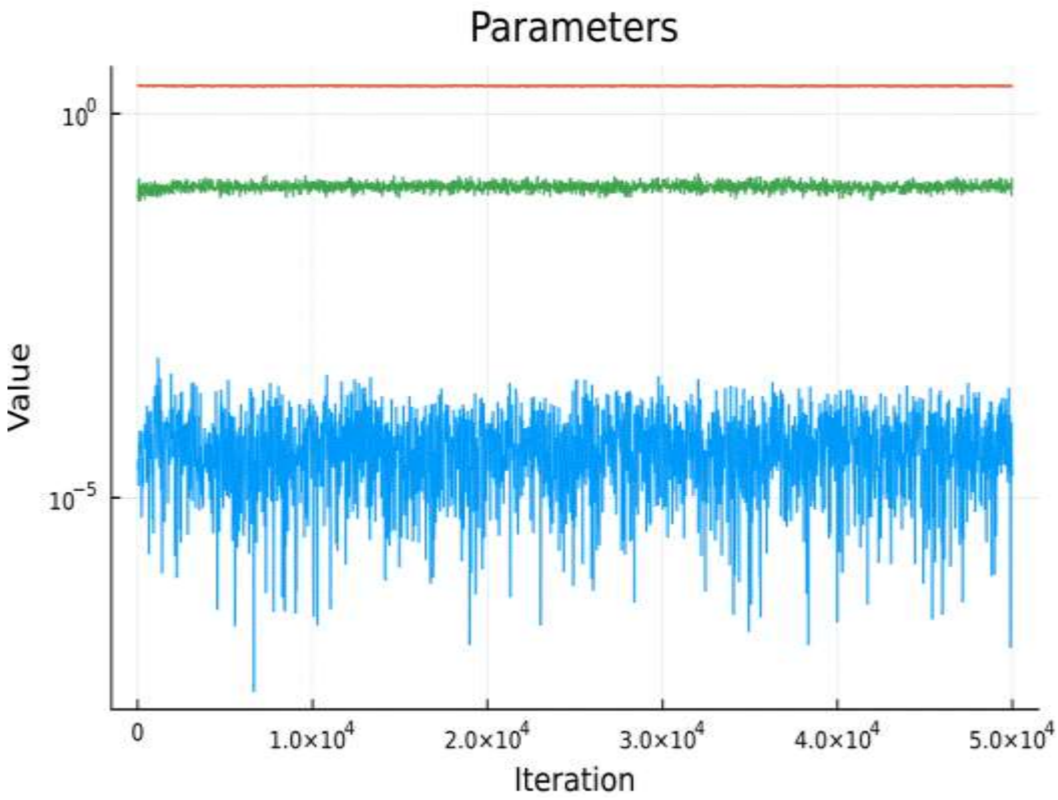

Figure 3:MCMC results for the parameters used.

Figure 3 shows the MCMC results for the parameters used in this study. It indicates the values of the parameters over 50,000 iterations.

4. Conclusion

In this paper we have presented MDS mathematical model on network where stability analysis results were presented for the model. We found out that, during the peak period, couples may be more likely to remain together due to the fear of the unknown. But as the epidemic wears on, they may find that their marriage isn't working and decide to separate. We therefore reaffirm the need for couples to stay away from divorced individuals as this can lead to their own marriage breakup. In sum, during this period of uncertainty, it is important to prioritize the health of marriages, and to remain vigilant of any potential risks. Again, it could be worthwhile if separation could be taken as a serious issue and steps could be taken to avoid it. Also, couples should be mindful of the strain that social isolation can put on their relationship. Without the physical and social contact that comes with normal marital interactions, couples may not be able to resolve issues that may be causing tension within the relationship which can ultimately lead to a breakdown in the marriage. Future studies can look at the effectiveness of couples counseling, virtual communication, and other strategies that can help couples manage the tension caused by social isolation (separation). Couples should also make time to check in with one another and communicate their thoughts and feelings. Finally, couples should plan quality time for each other, such as virtual date nights, which can help build and strengthen their relationship. One limitation of this study is that it only looked at married couples. It would be interesting to explore how the model works for other types of relationship, such as roommates or family members.

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