New Result of Fixed Points for Weakly Compatible Mappings with (CLR) Property in Intuitionistic Fuzzy Metric Space.

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Abstract

In this article we demonstrate new common fixed point theorems for weakly compatible mappings by using the (CLR) property in IFMS, as well as generalizing and improving the result of J.S.Park in the algebraic framework of fuzzy sets.

Keywords: CFP, CLRs and CLRt & $CLR_{(A,S),T,}$ property, weakly compatible, IFMS.

1. Introduction

Zadeh's highly innovative paper [3] introduced the idea of fuzzy sets(uncertain sets). This was followed by several other articles by Zadeh, which initiated the extension of fuzzy methods and ideas towards knowledge representation and artificial intelligence. Chronologically many famous authors defined several extensions of fuzzy set such as L-fuzzy sets, interval_valued fuzzy sets, rough sets, and the fourth extension was intuitionistic fuzzy sets, which was introduced by K.T.Atanassov [13,14,15] in 1983. In 1975 Kramosil and Michalek [4] gave the notion of fuzzy metric space, using the concept of fuzzy set. Afterward, George and Veeramani [1] has modified the concept of Kramosil and Michalek, and suggested a new definition of fuzzy metric space with t-norms, which has been frequently used in research in these decades. Subsequently in 2004, J.H.Park [5] introduced a generalized form of fuzzy metric space with the help of t-norms and t-conorms, called intuitionistic fuzzy metric space. In 2011, W. Sintunavarat and et-al [24] gave a visionary and innovative idea, known as CLRg property, which is an improvement of (EA) property [16].According to the examples (2.11) of [10] and (2.16), (2.17) of [19] we see that a pair of self – mappings (A, S) satisfying (EA) property along with closedness of subspace S(X) enjoy the (CLRs) property with respect to S. Similarly the pair (B, T) is also enjoy the (CLRt) property with respect to T. Keeping this logic in mind, M. Imdad and et-al [19], introduced an extended form of (CLR) property called (CLRst) property with respect to S & T. Afterward more extended form of (CLR) property, launched by V. POPA [23], which is known as $CLR_{(A,S),T}$.Many famous researchers established several results of coincidence point, fixed point and common fixed point theorems for self contraction mappings by using different notions like compatible mappings, weakly compatible, (EA) property, common (EA) property, (CLRg), (CLRst) property, conditionally compatible of type (E) etc. in intuitionistic fuzzy metric space, which can be seen in [5, 6,7, 8,9,20,21,22].

In this paper we improve and generalize the result of J.S. Park [8] in IFMS, with the help of algebra of fuzzy sets, and prove some new results of common fixed point theorems using $CLR_{(A,S),T}$ property.

2. Preliminaries.

Definition 2.1-[2] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm, if for all a, b, c, d $\in [0,1]$, * is satisfying following conditions.

- ***** is commutative and associative.
- ***** is continuous.
- $a * 1 = a$ for all $a \in [0,1]$.
- $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, for all a, b, c, d $\in [0,1]$.

Definition 2.2- [2] A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm, if for all a, b, c, d $\in$ [0,1], $\diamond$ is satisfying following conditions.

- $\diamond$ is commutative and associative.
- $\diamond$ continuous.
- $a \diamond 0 = a$ for all $a \in [0,1]$.
- $a \diamond b \geq c \diamond d$, whenever $a \leq c$ and $b \leq d$, for all a, b, c, d $\in [0,1]$.

Definition 2.3 [5] A 5-tuple (X, M, N, *, $\diamond$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

For all x, y $\in$ X, and s, t > 0;

[IFM-I] $M(x, y, t) + N(x, y, t) \leq 1$;

[IFM-II] $M(x, y, 0) = 0$;

[IFM-III] $M(x, y, t) = 1$, if and only if $x = y$;

[IFM-IV] $M(x, y. t) = M(y, x, t)$,

[IFM-V] $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

[IFM-VI] $M(x, y, \cdot) : [0, \infty) \to [0,1]$ is left continuous;

[IFM-VII] $\lim_{t\to\infty} M(x, y, t) = 1$;

[IFM-VIII] $N(x, y, 0) = 1$;

[IFM-IX] $N(x, y, t) = 0$, if and only if $x = y$;

[IFM-X] $N(x, y. t) = N(y, x, t)$,

[IFM-XI] $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$;

[IFM-I] $N(x, y, \cdot) : [0, \infty) \to [0,1]$ is right continuous;

[IFM-I] $\lim_{t\to\infty} N(x, y, t) = 0$;

Then (M. N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) represent the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.1 In IFMS form (X, M, N, *, $\diamond$), continuous t-norm * and continuous t-conorm $\diamond$ defined by $t*t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0,1]$.

Definition 2.4 [9] Let (X, M, N, *, $\diamond$) be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x $\in$ X, if

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0 \text{ for each } t > 0;$$

(b) A sequence $\{x_n\}$ in X is said to be Cauchy sequence, iff

$$\lim_{n\to\infty} M\left(x_{n+p}, x_n, t\right) = 1 \text{ and } \lim_{n\to\infty} N\left(x_{n+p}, x_n, t\right) = 0, \text{ for } t > 0, p > 0;$$

Example 2.1 [22] Let (X, d) be metric space. Define t-norm $a*b = \min\{a, b\}$, and t-co-norm $a \diamond b = \max\{a, b\}$, for all x, y $\in$ X, and s, t > 0, $M(x, y, t) = \frac{t}{t + d(x,y)}$, $N(x, y, t) = \frac{\frac{d(x,y)}{d(x,y)}}{t + d(x,y)}$. Then (X, M, N, *, $\diamond$) is an IFMS, and the intuitionistic fuzzy metric (M, N) induced by the metric d is often referred to as the standard IFM.

Definition 2.5 [11,22] Let (X, M, N, *, $\diamond$) be an intuitionistic fuzzy metric space. Then two self maps A and S, are said to be weakly compatible if they commute at their coincidence points, i.e.

$$Ax = Sx \implies ASx = SAx$$

Definition 2.6 [23] Let A, S, and T be self-mappings of a metric space (X,d). The pair (A, S) is said to satisfy common limit range property with respect to T, denoted $CLR_{(AS)T}$ if there exists a sequence $\{x_n\}$ in X such that,

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t, \text{ for some } t \in S(X) \cap T(X)$$

Remark 2.2. Let A, B, S and T be self-mappings of a metric space (X, d). If (A, S) and (B, T) satisfy the common limit range property with respect to S and T, then (A, S) satisfy the common limit range property with respect to T. The converse is not true.

Now we demonstrate following lemma in IFMS, inspired by [12].

Lemma 2.1 Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space. Then for each $n \in N$, and $t > 0$,

(a) $M(x, y, t) \geq M(x, y, \frac{t}{n})$.

(b) $N(x, y, t) \leq N(x, y, \frac{t}{n})$

Proof This lemma is automatically prove by properties (V) and (XI).

Lemma 2.2 [17,21] Let A, and S be two weakly compatible self mappings of a nonempty set X. If A and S have a unique point of coincidence w = Ax = Sx, for some x ∈ X , then w is the unique common fixed point of A and S .

In (2016), J.S.Park [8] demonstrated the algebraic sum and difference of fuzzy sets, as follows.

Definition 2.7 Let $\mathcal{F} = \{\mu, \vartheta : X^2 \times [0, \infty) \to [0, 1]\}$ be the set of all fuzzy sets defined on $X^2 \times [0, \infty)$.Suppose that $\mu, \vartheta \in \mathcal{F}$, then the algebraic sum and difference $\mu \oplus \mu$, and $\vartheta \ominus \vartheta$ of fuzzy sets is defined by

$$\mu(x_1, y_1, t) \oplus \mu(x_2, y_2, t) = \sup_{t_1 + t_2 = t} \min\{ \mu(x_1, y_1, t_1), \mu(x_2, y_2, t_2)\}$$

$$\vartheta (x_1, y_1, t) \ominus \vartheta (x_2, y_2, t) = \inf_{t_1 + t_2 = t} \max\{ \vartheta (x_1, y_1, t_1), \vartheta (x_2, y_2, t_2)\}$$

The output of above algebraic relationship is

(1) $\mu(x, y, t) \oplus \mu(x, y, t) \geq \mu(x, y, \frac{t}{2})$

(2) $\vartheta (x, y, t) \ominus \vartheta (x, y, t) \leq \vartheta (x, y, \frac{t}{2})$

(3) $\mu(x, y, t) \oplus 1 \geq \min\{ \mu(x, y, t - \varepsilon), \mu(x, x, \varepsilon)\} = \mu(x, y, t - \varepsilon)$

(4) $\vartheta(x, y, t) \ominus 0 \leq \max\{ \vartheta(x, y, t - \varepsilon), \vartheta(x, x, \varepsilon)\} = \vartheta(x, y, t - \varepsilon)$

If we take limit $\varepsilon \to 0$, we get

$\mu(x, y, t) \oplus 1 \geq \mu(x, y, t)$ and $\vartheta(x, y, t) \ominus 0 \leq \vartheta(x, y, t)$

J.S. Park, improved and generalized the result of [16] in IFMS, by using the above algebraic structure as follows.

Let Φ denotes a family of maps such that for each $\phi, \psi \in \Phi$, with $\phi, \psi : [0,1]^3 \to [0, 1]$, where ϕ and ψ are continuous & increasing and decreasing in each co-ordinate variable respectively. Also

$\phi(t, t, t) \geq t$ and $\psi(t, t, t) \leq t$ for every $t \in [0, 1]$.

Theorem 2.1 Let A, B, S and T be the self maps from an IFMS X, satisfying for all $x, y \in X$, and $t > 0$, $\phi, \psi \in \Phi$, and some $0 \leq k < 2$,

$$A(X) \subseteq T(X), B(X) \subseteq S(X),$$

$$M(Ax, By, t) \geq \phi\{M (Sx, Ty, \tfrac{2t}{k}), M (Ax, Sx, \tfrac{2t}{k}) \oplus M (By, Ty, \tfrac{2t}{k}), M (Ax, Ty, \tfrac{4t}{k}) \oplus M (Sx, By, \tfrac{4t}{k})\}$$

$$N(Ax, By, t) \leq \psi\{N (Sx, Ty, \tfrac{2t}{k}), N (Ax, Sx, \tfrac{2t}{k}) \ominus N (By, Ty, \tfrac{2t}{k}), N (Ax,Ty, \tfrac{4t}{k}) \ominus N (Sx, By, \tfrac{4t}{k})\}$$

Suppose that

(a) One of the pairs (A, S) and (B , T) satisfies the property (E.A),

(b) (A, S) and (B, T) are weakly compatibla,

(c) One of A(X), B(X), S(X) and T(X) is a complete subspace of X.

Then A, B, S and T have a unique common fixed point in IFMS.

Remark 2.3 In above result , for k = 0, all calculation are futile exercise because $\lim_{n\to\infty} \left(\frac{2t}{k}\right)^n$ is an indeterminate quantity .

$\phi, \psi \in \Phi$ are not well-defined, because the range of both functions is [0,1].

3. Main Results.

In this paper, first of all we modify and extend the definition of $\phi, \psi \in \Phi$, and then improve and generalize the result of the Park [8], by using a new type of common limit range property.

Definition 3.1 Let Φ denotes a family of maps such that for each $\phi, \psi \in \Phi$, with $\phi, \psi : [0,1]^4 \rightarrow [0, 1]$, where ϕ and ψ are continuous & increasing and decreasing in each co-ordinate variable respectively. Also

$$\phi(t, t, t, t) \geq t \text{ for every } t \in [0, 1) \text{ and } \phi(t, t, t, t) = t \text{ for } t = 1 .$$

and

$$\psi(t, t, t, t) \leq t \text{ for every } t \in (0, 1] \text{ and } \psi(t, t, t, t) = t \text{ for } t = 0 .$$

Theorem 3.1 Let A, B, S and T be the self maps from an IFMS X, satisfying for all $x, y \in X$, and $t > 0$, $\phi, \psi \in \Phi$, and some $0 < k < 2$,

$$M(Ax, By, t) \geq \phi \left\{ \begin{matrix} M(Sx, Ty, \frac{2t}{k}), \\ M(Ax, Sx, \frac{2t}{k}) \oplus M(By, Ty, \frac{2t}{k}), \\ M(Ax, Ty, \frac{4t}{k}) \oplus M(Sx, By, \frac{4t}{k}), \\ M(Ax, By, \frac{6t}{k}) \oplus M(Sx, Ty, \frac{6t}{k}) \end{matrix} \right\}$$

$$N(Ax, By, t) \leq \psi \left\{ \begin{matrix} N(Sx, Ty, \frac{2t}{k}), \\ N(Ax, Sx, \frac{2t}{k}) \ominus N(By, Ty, \frac{2t}{k}), \\ N(Ax, Ty, \frac{4t}{k}) \ominus N(Sx, By, \frac{4t}{k}), \\ N(Ax, By, \frac{6t}{k}) \ominus N(Sx, Ty, \frac{6t}{k}) \end{matrix} \right\} \qquad (1)$$

If A, S, and T satisfy $CLR_{(A,S)T}$ property, then $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$,

Moreover (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in IFMS.

Proof : Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, and A, B, S and T be the self maps defined on X.

Suppose that maps A, S, and T satisfy $CLR_{(A,S)T}$ property, then there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in S(X) \cap T(X)$$

Since $z \in T(X)$ $\exists$ $z_1 \in X$ such that $z = Tz_1$. Then by (1) we have

$$M(Ax_n, Bz_1, t) \geq \phi \left\{ \begin{matrix} M(Sx_n, Tz_1, \frac{2t}{k}), \\ M(Ax_n, Sx_n, \frac{2t}{k}) \oplus M(Bz_1, Tz_1, \frac{2t}{k}), \\ M(Ax_n, Tz_1, \frac{4t}{k}) \oplus M(Sx_n, Bz_1, \frac{4t}{k}), \\ M(Ax_n, Bz_1, \frac{6t}{k}) \oplus M(Sx_n, Tz_1, \frac{6t}{k}) \end{matrix} \right\}$$

$$N(Ax_n, Bz_1, t) \leq \psi \left\{ \begin{matrix} N(Sx_n, Tz_1, \frac{2t}{k}), \\ N(Ax_n, Sx_n, \frac{2t}{k}) \ominus N(Bz_1, Tz_1, \frac{2t}{k}), \\ N(Ax_n, Tz_1, \frac{4t}{k}) \ominus N(Sx_n, Bz_1, \frac{4t}{k}), \\ N(Ax_n, Bz_1, \frac{6t}{k}) \ominus N(Sx_n, Tz_1, \frac{6t}{k}). \end{matrix} \right\}$$

Now on taking limit n tends to infinity in both case like $\lim_{n\to\infty} \inf$ and $\lim_{n\to\infty} \sup$ we get,

$$M(z, Bz_1, t) \geq \phi \left\{ \begin{array}{ll} M(z, z, \frac{2t}{k}), & M(z, z, \frac{2t}{k}) \oplus M(Bz_1, z, \frac{2t}{k}), \\ M(z, z, \frac{4t}{k}) \oplus M(z, Bz_1, \frac{4t}{k}), & M(z, Bz_1, \frac{6t}{k}) \oplus M(z, z, \frac{6t}{k}), \end{array} \right\}$$

$$\geq \phi\{ 1, 1 \oplus M(B z_1, z, \frac{2t}{k}), 1 \oplus M(z, Bz_1, \frac{4t}{k}), M(z, Bz_1, \frac{6t}{k}) \oplus 1,\},$$

And

$$N(z, Bz_1, t) \leq \psi \left\{ \begin{array}{ll} N(z, z, \frac{2t}{k}), & N(z, z, \frac{2t}{k}) \ominus N(Bz_1, z, \frac{2t}{k}), \\ N(z, z, \frac{4t}{k}) \ominus N(z, Bz_1, \frac{4t}{k}), & N(z, Bz_1, \frac{6t}{k}) \ominus N(z, z, \frac{6t}{k}) \end{array} \right\}$$

$$\leq \psi\{ 0, 0 \ominus N(B z_1, z, \frac{2t}{k}), 0 \ominus M(z, Bz_1, \frac{4t}{k}), M(z, Bz_1, \frac{6t}{k}) \ominus 0,\} \quad (2)$$

Since

$$1 \oplus M(z, Bz_1, \frac{4t}{k}) \geq M(z, Bz_1, \frac{4t}{k}) \geq M(z, Bz_1, \frac{2t}{k}) * M(Bz_1, Bz_1, \frac{2t}{k})$$

$$\geq M(z, Bz_1, \frac{2t}{k}) * 1 = M(z, Bz_1, \frac{2t}{k})$$

$$0 \ominus N(z, Bz_1, \frac{4t}{k}) \leq N(z, Bz_1, \frac{4t}{k}) \leq N(z, Bz_1, \frac{2t}{k}) \diamond N(Bz_1, Bz_1, \frac{2t}{k})$$

$$\leq N(z, Bz_1, \frac{2t}{k}) \diamond 0 = N(z, Bz_1, \frac{2t}{k})$$

Same way

$$1 \oplus M(z, Bz_1, \frac{6t}{k}) \geq M(z, Bz_1, \frac{6t}{k}) \geq M(z, Bz_1, \frac{2t}{k}) * M(Bz_1, Bz_1, \frac{4t}{k})$$

$$\geq M(z, Bz_1, \frac{2t}{k}) * 1 = M(z, Bz_1, \frac{2t}{k}),$$

$$0 \ominus N(z, Bz_1, \frac{6t}{k}) \leq N(z, Bz_1, \frac{6t}{k}) \leq N(z, Bz_1, \frac{2t}{k}) \diamond N(Bz_1, Bz_1, \frac{4t}{k})$$

$$\leq N(z, Bz_1, \frac{2t}{k}) \diamond 0 = N(z, Bz_1, \frac{2t}{k}),$$

Then from (2)

$$M(z, Bz_1, t) \geq \phi\{1, M(B z_1, z, \frac{2t}{k}), M(z, Bz_1, \frac{2t}{k}), M(z, Bz_1, \frac{2t}{k})\},$$

$$N(z, Bz_1, t) \leq \psi\{0, N(Bz_1, z, \frac{2t}{k}), N(z, Bz_1, \frac{2t}{k}), N(z, Bz_1, \frac{2t}{k})\}.$$

$\Longrightarrow$ $$M(z, Bz_1, t) \geq M(z, Bz_1, \frac{2t}{k}) \geq \ldots M(z, Bz_1, \left(\frac{2}{k}\right)^n t) \to 1$$

$$N(z, Bz_1, t) \leq N(z, Bz_1, \frac{2t}{k}) \leq \ldots N(z, Bz_1, \left(\frac{2}{k}\right)^n t) \to 0, \text{ as } n \to \infty$$

$\Longrightarrow$ $Bz_1 = z \Longrightarrow z = Bz_1 = Tz_1$ thus $z \in C(B, T)$

Since $z \in S(X)$ $\exists$ $z_2 \in X$ such that $z = Sz_2$. Then by (1) we have

$$M(Az_2, Bz_1, t) \geq \phi\left\{\begin{matrix} M(Sz_2, Tz_1, \frac{2t}{k}), & M(Az_2, Sz_2, \frac{2t}{k}) \oplus M(B\,z_1, Tz_1, \frac{2t}{k}), \\ M(Az_2, Tz_1, \frac{4t}{k}) \oplus M(Sz_2, B\,z_1, \frac{4t}{k}), & M(Az_2, Bz_1, \frac{6t}{k}) \oplus M(Sz_2, Tz_1, \frac{6t}{k}), \end{matrix}\right\}$$

$$\geq \phi\left\{\begin{matrix} M(z, z, \frac{2t}{k}), & M(Az_2, z, \frac{2t}{k}) \oplus M(z, z, \frac{2t}{k}), \\ M(Az_2, z, \frac{4t}{k}) \oplus M(z, z, \frac{4t}{k}), & M(Az_2, z, \frac{6t}{k}) \oplus M(z, z, \frac{6t}{k}), \end{matrix}\right\}$$

$$M(Az_2, z, t) \geq \phi\ \{1,\ M(Az_2, z, \tfrac{2t}{k}) \oplus 1,\ M(Az_2, z, \tfrac{4t}{k}) \oplus 1,\ M(Az_2, z, \tfrac{6t}{k}) \oplus 1\},$$
$$\geq \phi\ \{1,\ M(Az_2, z, \tfrac{2t}{k}),\ M(Az_2, z, \tfrac{2t}{k}),\ M(Az_2, z, \tfrac{2t}{k})\},$$

$$\Longrightarrow \quad M(Az_2, z, t) \geq M(Az_2, z, \tfrac{2t}{k}) \geq \ldots \quad M(Az_2, z, \left(\tfrac{2}{k}\right)^n t) \rightarrow 1, \text{ as } n \rightarrow \infty$$

And

$$N(Az_2, Bz_1, t) \leq \psi\left\{\begin{matrix} N(Sz_2, Tz_1, \frac{2t}{k}), & N(Az_2, Sz_2, \frac{2t}{k}) \ominus N(B\,z_1, Tz_1, \frac{2t}{k}), \\ N(Az_2, Tz_1, \frac{4t}{k}) \ominus N(Sz_2, B\,z_1, \frac{4t}{k}), & N(Az_2, Bz_1, \frac{6t}{k}) \ominus N(Sz_2, Tz_1, \frac{6t}{k}), \end{matrix}\right\}$$

$$\leq \psi\left\{\begin{matrix} N(z, z, \frac{2t}{k}), & N(Az_2, z, \frac{2t}{k}) \ominus N(z, z, \frac{2t}{k}), \\ N(Az_2, z, \frac{4t}{k}) \ominus N(z, z, \frac{4t}{k}), & N(Az_2, z, \frac{6t}{k}) \ominus N(z, z, \frac{6t}{k}), \end{matrix}\right\}$$

$$N(Az_2, z, t) \leq \psi\ \{0,\ N(Az_2, z, \tfrac{2t}{k}) \ominus 0,\ N(Az_2, z, \tfrac{4t}{k}) \ominus 0,\ N(Az_2, z, \tfrac{6t}{k}) \ominus 0\},$$
$$\leq \phi\ \{0,\ N(Az_2, z, \tfrac{2t}{k}),\ N(Az_2, z, \tfrac{2t}{k}),\ N(Az_2, z, \tfrac{2t}{k})\},$$

$$\Longrightarrow \quad N(Az_2, z, t) \leq N(Az_2, z, \tfrac{2t}{k}) \leq \ldots \quad N(Az_2, z, \left(\tfrac{2}{k}\right)^n t) \rightarrow 0, \text{ as } n \rightarrow \infty$$

$\Longrightarrow$ $Az_2 = z \Longrightarrow z = Az_2 = Sz_2$ thus $z \in C(A, S)$,

$\Longrightarrow$ $z = Az_2 = Sz_2 = Bz_1 = Tz_1$. Hence $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$.

Therefore z is a point of coincidence of (A, S) and (B, T).

Since (A, S) and (B, T) are weakly compatible, Then by Lemma (2.2), to prove the rest of the theorem it is sufficient to show that the point of coincidence of (A, S) and (B, T), is unique.

Suppose that w is another point of coincidence of (A, S) and (B, T),

Case – I : Firstly suppose $w \in C(A, S)$, then there exist $w_1 \in X$ such that $w = Aw_1 = Sw_1$.

$$M(w, z, t) = M(Aw_1, Bz_1, t)$$

$$\geq \phi\left\{\begin{matrix} M(Sw_1, Tz_1, \frac{2t}{k}), & M(Aw_1, Sw_1, \frac{2t}{k}) \oplus M(Bz_1, Tz_1, \frac{2t}{k}), \\ M(Aw_1, Tz_1, \frac{4t}{k}) \oplus M(Sw_1, Bz_1, \frac{4t}{k}), & M(Aw_1, Bz_1, \frac{6t}{k}) \oplus M(Sw_1, Tz_1, \frac{6t}{k}), \end{matrix}\right\}$$

$$\geq \phi\Big\{M(w, z, \tfrac{2t}{k}),\ M(w, w, \tfrac{2t}{k}) \oplus M(z, z, \tfrac{2t}{k}), M(w, z, \tfrac{4t}{k}) \oplus M(w, z, \tfrac{4t}{k}),$$
$$M(w, z, \tfrac{6t}{k}) \oplus M(w, z, \tfrac{6t}{k})\Big\},$$

And

$$N(w, z, t) = N(Aw_1, Bz_1, t)$$

$$\leq \psi \left\{ \begin{matrix} N(Sw_1, Tz_1, \frac{2t}{k}), & N(Aw_1, Sw_1, \frac{2t}{k}) \ominus N(Bz_1, Tz_1, \frac{2t}{k}), \\ N(Aw_1, Tz_1, \frac{4t}{k}) \ominus N(Sw_1, Bz_1, \frac{4t}{k}), & N(Aw_1, Bz_1, \frac{6t}{k}) \ominus N(Sw_1, Tz_1, \frac{6t}{k}), \end{matrix} \right\}$$

$$\leq \psi \Big\{ N(w, z, \tfrac{2t}{k}), \quad N(w, w, \tfrac{2t}{k}) \ominus N(z, z, \tfrac{2t}{k}), N(w, z, \tfrac{4t}{k}) \ominus N(w, z, \tfrac{4t}{k}),$$

$$N(w, z, \tfrac{6t}{k}) \ominus N(w, z, \tfrac{6t}{k}) \Big\}, \qquad (3)$$

Since, for $\varepsilon \in (0, \frac{2t}{k})$

$M(w, w, \frac{2t}{k}) \oplus 1 \geq \min\{ M(w, w, \frac{2t}{k} - \varepsilon), M(z, z, \varepsilon)\} = M(w, w, \frac{2t}{k} - \varepsilon) = 1$, as $\varepsilon \to 0$

$M(w, z, \frac{4t}{k}) \oplus M(w, z, \frac{4t}{k}) \geq \min \{ M(w, z, \frac{2t}{k}), M(w, z, \frac{2t}{k})\} = M(w, z, \frac{2t}{k})$,

$$\begin{aligned} M(w, z, \tfrac{6t}{k}) \oplus M(w, z, \tfrac{6t}{k}) &\geq \min \{ M(w, z, \tfrac{4t}{k}), M(w, z, \tfrac{2t}{k})\}, \\ &\geq \min \{ M(w, z, \tfrac{2t}{k}) * M(z, z, \tfrac{2t}{k}), M(w, z, \tfrac{2t}{k})\}, \\ &= \min \{ M(w, z, \tfrac{2t}{k}) * 1, M(w, z, \tfrac{2t}{k})\}, \\ &= M(w, z, \tfrac{2t}{k}), \end{aligned}$$

And

$N(w, w, \frac{2t}{k}) \ominus 0 \leq \max\{ N(w, w, \frac{2t}{k} - \varepsilon), N(z, z, \varepsilon)\} = N(w, w, \frac{2t}{k} - \varepsilon) = 0$, as $\varepsilon \to 0$

$N(w, z, \frac{4t}{k}) \ominus N(w, z, \frac{4t}{k}) \leq \max\{ N(w, z, \frac{2t}{k}), N(w, z, \frac{2t}{k})\} = N(w, z, \frac{2t}{k})$,

$$\begin{aligned} N(w, z, \tfrac{6t}{k}) \ominus N(w, z, \tfrac{6t}{k}) &\leq \max\{ N(w, z, \tfrac{4t}{k}), N(w, z, \tfrac{2t}{k})\}, \\ &\leq \max\{ N(w, z, \tfrac{2t}{k}) \diamond N(z, z, \tfrac{2t}{k}), N(w, z, \tfrac{2t}{k})\}, \\ &= \max\{ N(w, z, \tfrac{2t}{k}) \diamond 1, N(w, z, \tfrac{2t}{k})\}, \\ &= N(w, z, \tfrac{2t}{k}), \end{aligned}$$

From equation (3),

$$M(w, z, t) \geq \phi\{ M(w, z, \tfrac{2t}{k}), 1, M(w, z, \tfrac{2t}{k}), M(w, z, \tfrac{2t}{k})\},$$

$\Rightarrow$ $M(w, z, t) \geq M(w, z, \frac{2t}{k}) \geq \ldots \quad M(w, z, \left(\frac{2}{k}\right)^n t) \to 1$, as $n \to \infty$,

And

$$N(w, z, t) \leq \phi\{ N(w, z, \tfrac{2t}{k}), 0, N(w, z, \tfrac{2t}{k}), N(w, z, \tfrac{2t}{k})\},$$

$\Rightarrow$ $N(w, z, t) \leq N(w, z, \frac{2t}{k}) \leq \ldots \quad N(w, z, \left(\frac{2}{k}\right)^n t) \to 0$, as $n \to \infty$.

$\Rightarrow$ $w = z$.

Thus z is unique point of coincidence of (A, S).

Case – II: Similarly, we can show that z is unique point of coincidence of (B, T).

Hence z is unique common fixed point of A, B, S and T.

Remark 3.1 Our result modifies and generalizes the result of Park [8] in the structure of IFMS, as per the following.

1. For the desired result, the completeness of the range of any self-mapping is not required.

2. Containment of range of all self-mappings is totally removed.

3. Range of k is replace by $k \in (0, 2)$.

4. We replace (E.A.) property by common limit in the range property with respect to T, which eliminates closeness and completeness of range of A, B, S and T.

Corollary 3.1 Let A, B, S and T be the self maps from an IFMS X, satisfying for all x, y $\in$ X, $t > 0$, $\phi, \psi \in \Phi$, and some $0 < k < 2$, and for each $p \in N$,

$$M(Ax, By, \tfrac{t}{p}) \geq \phi \left\{ \begin{array}{c} M(Sx, Ty, \frac{2t}{k}), \\ M(Ax, Sx, \frac{2t}{k}) \oplus M(By, Ty, \frac{2t}{k}), \\ M(Ax, Ty, \frac{4t}{k}) \oplus M(Sx, By, \frac{4t}{k}), \\ M(Ax, By, \frac{6t}{k}) \oplus M(Sx, Ty, \frac{6t}{k}) \end{array} \right\}$$

$$N(Ax, By, \tfrac{t}{p}) \leq \psi \left\{ \begin{array}{c} N(Sx, Ty, \frac{2t}{k}), \\ N(Ax, Sx, \frac{2t}{k}) \ominus N(By, Ty, \frac{2t}{k}), \\ N(Ax, Ty, \frac{4t}{k}) \ominus N(Sx, By, \frac{4t}{k}), \\ N(Ax, By, \frac{6t}{k}) \ominus N(Sx, Ty, \frac{6t}{k}) \end{array} \right\}$$

If A, S, and T satisfy $CLR_{(A,S)T}$, then $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$

Moreover (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in IFMS

Proof : By using Lemma (2.1) in the calculation of Theorem (3.1), we can obtain the desired result.

Now to prove the above result in a different manner, we define increasing and decreasing functions ϕ, ψ. as follows.

Let Φ denotes a family of maps such that for each $\phi, \psi \in \Phi$, with $\phi, \psi : [0,1] \to [0, 1]$, where ϕ and ψ are continuous & increasing and decreasing functions respectively.

$$\phi(t) \geq t \quad \text{for every} \quad t \in [0, 1) \quad \text{and} \quad \phi(t) = t \quad \text{for} \quad t = 1,$$

And

$$\psi(t) \leq t \quad \text{for every} \quad t \in (0, 1] \quad \text{and} \quad \psi(t) = t \quad \text{for} \quad t = 0.$$

Remark 3.2 If S = T, Then $CLR_{(A,S)T}$, property converts to CLR_S, property.

Theorem 3.2 Let A, and S be the self maps from an IFMS X, satisfying for all x, y $\in$ X, and $t > 0$, $\phi, \psi \in \Phi$, and some $0 < k < 2$, and for each $p \in N$,

$$M(Ax, Ay, \tfrac{t}{p}) \geq \phi \left\{ \min \left\{ \begin{array}{c} M(Sx, Sy, \frac{2t}{k}), \\ M(Ax, Sx, \frac{2t}{k}) \oplus M(Ay, Sy, \frac{2t}{k}), \\ M(Ax, Sy, \frac{4t}{k}) \oplus M(Sx, Ay, \frac{4t}{k}), \\ M(Ax, Ay, \frac{6t}{k}) \oplus M(Sx, Sy, \frac{6t}{k}) \end{array} \right\} \right\}$$

$$N(Ax, Ay, \tfrac{t}{p}) \leq \psi\left\{\max\left\{\begin{array}{c} N(Sx, Sy, \frac{2t}{k}), \\ N(Ax, Sx, \frac{2t}{k}) \ominus N(Ay, Sy, \frac{2t}{k}), \\ N(Ax, Sy, \frac{4t}{k}) \ominus N(Sx, Ay, \frac{4t}{k}), \\ N(Ax, Ay, \frac{6t}{k}) \ominus N(Sx, Sy, \frac{6t}{k}) \end{array}\right\}\right\} \quad (4)$$

If the pair (A, S), satisfy CLR_S property, then $C(A, S) \neq \phi$,

Moreover (A, S) weakly compatible, then A and S have a unique common fixed point in IFMS X.

Proof: Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space, and A, S be the self maps defined on X. Suppose that pair (A, S), satisfy CLR_S property, then there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \qquad \text{for some } z \in S(X)$$

Since $z \in S(X)$ $\exists$ $z_1 \in X$ such that $z = Sz_1$. Suppose that $Az_1 \neq Sz_1$, now put $x = z_1$ and $y = x_n$, in (4) we have

$$M(Az_1, Ax_n, \tfrac{t}{p}) \geq \phi\left\{\min\left\{\begin{array}{ll} M(Sz_1, Sx_n, \frac{2t}{k}), & M(Az_1, Sz_1, \frac{2t}{k}) \oplus M(Ax_n, Sx_n, \frac{2t}{k}), \\ M(Az_1, Sx_n, \frac{4t}{k}) \oplus M(Sz_1, Ax_n, \frac{4t}{k}), & M(Az_1, Ax_n, \frac{6t}{k}) \oplus M(Sz_1, Sx_n, \frac{6t}{k}), \end{array}\right\}\right\}$$

And

$$N(Az_1, Ax_n, \tfrac{t}{p}) \leq \psi\left\{\max\left\{\begin{array}{ll} N(Sz_1, Sx_n, \frac{2t}{k}), & N(Az_1, Sz_1, \frac{2t}{k}) \ominus N(Ax_n, Sx_n, \frac{2t}{k}), \\ N(Az_1, Sx_n, \frac{4t}{k}) \ominus N(Sz_1, Ax_n, \frac{4t}{k}), & N(Az_1, Ax_n, \frac{6t}{k}) \ominus N(Sz_1, Sx_n, \frac{6t}{k}), \end{array}\right\}\right\}$$

Now on taking limit n tends to infinity

$$M(Az_1, Sz_1, \tfrac{t}{p}) \geq \phi\left\{\min\left\{\begin{array}{ll} M(Sz_1, Sz_1, \frac{2t}{k}), & M(Az_1, Sz_1, \frac{2t}{k}) \oplus M(Sz_1, Sz_1, \frac{2t}{k}), \\ M(Az_1, Sz_1, \frac{4t}{k}) \oplus M(Sz_1, Sz_1, \frac{4t}{k}), & M(Az_1, Sz_1, \frac{6t}{k}) \oplus M(Sz_1, Sz_1, \frac{6t}{k}), \end{array}\right\}\right\}$$

And

$$N(Az_1, Sz_1, \tfrac{t}{p}) \leq \psi\left\{\max\left\{\begin{array}{ll} N(Sz_1, Sz_1, \frac{2t}{k}), & N(Az_1, Sz_1, \frac{2t}{k}) \ominus N(Sz_1, Sz_1, \frac{2t}{k}), \\ N(Az_1, Sz_1, \frac{4t}{k}) \ominus N(Sz_1, Sz_1, \frac{4t}{k}), & N(Az_1, Sz_1, \frac{6t}{k}) \ominus N(Sz_1, Sz_1, \frac{6t}{k}), \end{array}\right\}\right\}$$

$\Longrightarrow$

$$\begin{aligned} M(Az_1, Sz_1, \tfrac{t}{p}) &\geq \phi\left\{\min\left\{1, M(Az_1,Sz_1, \tfrac{2t}{k}) \oplus 1, M(Az_1,Sz_1, \tfrac{4t}{k}) \oplus 1, M(Az_1,Sz_1, \tfrac{6t}{k}) \oplus 1\right\}\right\} \\ &\geq \phi\left\{\min\left\{1, \ M(Az_1,Sz_1, \tfrac{2t}{k}), \ M(Az_1,Sz_1, \tfrac{4t}{k}), \ M(Az_1,Sz_1, \tfrac{6t}{k})\right\}\right\} \\ &= \phi\left\{ M(Az_1,Sz_1, \tfrac{2t}{k})\right\} \\ &\geq M(Az_1,Sz_1, \tfrac{2t}{k}), \end{aligned}$$

$$\Longrightarrow \quad M(Az_1, Sz_1, t) \geq M(Az_1, Sz_1, \tfrac{2t}{k}) \geq \ldots \quad M(Az_1, Sz_1, \left(\tfrac{2}{k}\right)^n t) \to 1, \text{ as } n \to \infty,$$

And

$$N(Az_1, Sz_1, \tfrac{t}{p}) \leq \psi\left\{\max\left\{0, N(Az_1,Sz_1, \tfrac{2t}{k}) \ominus 0, N(Az_1,Sz_1, \tfrac{4t}{k}) \ominus 0, N(Az_1,Sz_1, \tfrac{6t}{k}) \ominus 0\right\}\right\}$$

$$\leq \psi\{\max\{1,\ N(Az_1,Sz_1,\ \tfrac{2t}{k}),\ N(Az_1,Sz_1,\ \tfrac{4t}{k}),\ N(Az_1,Sz_1,\ \tfrac{6t}{k})\}\}$$
$$= \psi\{\ N(Az_1,Sz_1,\ \tfrac{2t}{k})\}$$
$$\leq\ N(Az_1,Sz_1,\ \tfrac{2t}{k}),$$

$$\Longrightarrow\qquad N(Az_1, Sz_1, t) \geq N(Az_1, Sz_1,\ \tfrac{2t}{k}\ ) \geq \ldots\qquad N(Az_1, Sz_1,\ \left(\tfrac{2}{k}\right)^n t\ ) \rightarrow 0, \text{ as } n \rightarrow \infty,$$

A contradiction, hence $z = Az_1 = Sz_1$.Thus $C(A, S) \neq \phi$

Therefore z is a point of coincidence of (A, S), now like theorem 3.1, we can show that point of coincidence is unique.

Since pair of self-mapping (A, S) is weakly compatible, then by Lemma (2.2), z is a unique common fixed point of A and S..

4. Application.

Definition 4.1 [18] Two families of self-mappings $\{A_I\}_{i=1}^{m}$ and $\{B_j\}_{j=}^{n}$ are said to be pairwise commuting, if :

(1) Ai Aj = Aj Ai, for all i, j = 1,2,3……..m,

(2) Bi Bj = Bj Bi, for all i, j = 1,2,3……..m,

(3) Ai Bj = Bj Ai, for all i= 1,2,3……..m, and j = 1,2,3…….n.

We use the above definition of commutativity as an application of our new result (3.1) for finite family of weakly compatible mappings in the framework of IFMS, as follows.

Theorem 4.1 Let $\{Ai\}_{i=1}^{m}$, $\{Bj\}_{j=1}^{n}$, $\{Sr\}_{r=1}^{p}$ and $\{Ts\}_{s=1}^{q}$ are four finite family of self-mappings defined on IFMS (X, M, N, *, ⋄), where * is continuous t-norm and ⋄ is continuous t-conorm, with

$A = A_1 A_2 A_3 \ldots\ldots A_m$, $B = B_1B_2B_3\ldots\ldots B_n$, $S = S_1S_2S_3\ldots\ldots S_p$ and $T = T_1T_2T_3\ldots\ldots T_q$
Satisfying contraction condition of Theorem (3.1) such that pair (A, S) share the new common limit range property with respect to T, i.e. $CLR_{(A,S)T}$, then $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$.
Moreover if the pairs of families ({Ai}, {Sr}) and ({Bj}, {Ts}) commute pair wise for each $0 \leq i \leq m$, $0 \leq j \leq n$, $0 \leq r \leq p$ and $0 \leq s \leq q$, then $\{Ai\}_{i=1}^{m}$, $\{Bj\}_{j=1}^{n}$, $\{Sr\}_{r=1}^{p}$ and $\{Ts\}_{s=1}^{q}$ have unique common fixed point.
Proof: Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space. According to the [18], pair wise commuting property of families of self-mappings provides weak compatibility of pairs (A, S) and (B, T).

Now by using the calculation of Theorem (3.1), we obtain that A, B, S and T have a unique common fixed point z in X. For the remaining part of the proof we follow [18].

Now by choosing $A_1 = A_2 = A_3\ldots\ldots = A_m = A$, $B_1 = B_2 = B_3\ldots\ldots = B_n = B$, $S_1 = S_2 = S_3\ldots\ldots S_p = S$ and $T_1 = T_2 = T_3\ldots\ldots = T_q = T$ in Theorem (4.1), we get the following result.

Corollary 4.1 Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space. And A, B, S and T, are self-mappings defined on X. where * is continuous t-norm and ⋄ is continuous t-co norm. And satisfying the following conditions.

For all $x, y \in X$, and $t > 0$, there exist $\phi, \psi \in \Phi$, $0 < k < 2$, and $m, n, p, q \in N$, such that.

$$M(A^m x, B^n y, t) \geq \phi\left\{\begin{array}{c} M(S^p x, T^q y, \tfrac{2t}{k}), \\ M(A^m x, S^p x, \tfrac{2t}{k}) \oplus M(B^n y, T^q y, \tfrac{2t}{k}), \\ M(A^m x, T^q y, \tfrac{4t}{k}) \oplus M(S^p x, B^n y, \tfrac{4t}{k}), \\ M(A^m x, B^n y, \tfrac{6t}{k}) \oplus M(S^p x, T^q y, \tfrac{6t}{k}), \end{array}\right\}$$

$$N(A^m x, B^n y, t) \leq \psi\left\{\begin{array}{c} N(S^p x, T^q y, \tfrac{2t}{k}), \\ N(A^m x, S^p x, \tfrac{2t}{k}) \ominus N(B^n y, T^q y, \tfrac{2t}{k}), \\ N(A^m x, T^q y, \tfrac{4t}{k}) \ominus N(S^p x, B^n y, \tfrac{4t}{k}), \\ N(A^m x, B^n y, \tfrac{6t}{k}) \ominus N(S^p x, T^q y, \tfrac{6t}{k}), \end{array}\right\} \qquad (5)$$

If the pair (A^m, B^n) satisfy new common limit range property with respect to T^q, then $C(A^m, S^p) \neq \phi$ and $C(B^n, T^q) \neq \phi$.

Moreover if the pairs (A^m, S^p) and (B^n, T^q) commute pair wise, then A, B, S and T have a unique common fixed point.

Our next result is a partial generalization and an application of Theorem (3.1) with minimal restrictions for six pair wise commuting self-mappings in IFMS.

Theorem 4.2 Let A, B, S, T, P and Q are six self-mappings defined on IFMS (X, M, N, $*$, $\diamond$), where $*$ is continuous t-norm and $\diamond$ is continuous t-co norm. And satisfying the following conditions.

For all x, y $\in$ X, and t $>$ 0, $\phi, \psi \in \Phi$, and some 0 < k < 2,

$$M(Ax, By, t) \geq \phi\left\{\begin{array}{c} M(SPx, TQy, \frac{2t}{k}), \\ M(Ax, SPx, \frac{2t}{k}) \oplus M(By, TQy, \frac{2t}{k}), \\ M(Ax, TQy, \frac{4t}{k}) \oplus M(SPx, By, \frac{4t}{k}), \\ M(Ax, By, \frac{6t}{k}) \oplus M(SPx, TQy, \frac{6t}{k}) \end{array}\right\}$$

$$N(Ax, By, t) \leq \psi\left\{\begin{array}{c} N(SPx, TQy, \frac{2t}{k}), \\ N(Ax, SPx, \frac{2t}{k}) \ominus N(By, TQy, \frac{2t}{k}), \\ N(Ax, TQy, \frac{4t}{k}) \ominus N(SPx, By, \frac{4t}{k}), \\ N(Ax, By, \frac{6t}{k}) \ominus N(SPx, TQy, \frac{6t}{k}) \end{array}\right\} \quad (6)$$

If A, SP, and TQ satisfy new common limit in the range property, i.e. $CLR_{(A,\, SP)TQ}$,

then $C(A, SP) \neq \phi$ and $C(B, TQ) \neq \phi$.

Moreover if the pairs (A, SP) and (B, TQ) commute pair wise, (i.e. AS = SA, AP = PA, SP = PS, BT = TB, BQ = QB and TQ = QT,) then A, B, S, T, P and Q have a unique common fixed point.

Proof: Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. By Theorem (3.1) z is a unique common fixed point of A, B, SP and TQ in IFMS X,(i.e. Az = Bz = SPz = TQz = z). Now we show that z is unique common fixed point of A, S and P. For this we show that Pz = z,

Now put x =Pz, y = z in (6), we get

$$M(APz, Bz, t) \geq \phi\left\{\begin{array}{c} M(SPPz, TQz, \frac{2t}{k}), \\ M(APz, SPPz, \frac{2t}{k}) \oplus M(Bz, TQz, \frac{2t}{k}), \\ M(APz, TQz, \frac{4t}{k}) \oplus M(SPPz, Bz, \frac{4t}{k}), \\ M(APz, Bz, \frac{6t}{k}) \oplus M(SPPz, TQz, \frac{6t}{k}) \end{array}\right\}$$

$$N(APz, Bz, t) \leq \psi\left\{\begin{array}{c} N(SPPz, TQz, \frac{2t}{k}), \\ N(APz, SPPz, \frac{2t}{k}) \ominus N(Bz, TQz, \frac{2t}{k}), \\ N(APz, TQz, \frac{4t}{k}) \ominus N(SPPz, Bz, \frac{4t}{k}), \\ N(APz, Bz, \frac{6t}{k}) \ominus N(SPPz, TQz, \frac{6t}{k}) \end{array}\right\}$$

$\Longrightarrow$

$$\left\{\begin{array}{ll} M(Pz, z, \frac{2t}{k}), & M(Pz, Pz, \frac{2t}{k}) \oplus M(z, z, \frac{2t}{k}), \end{array}\right.$$

$$M(Pz, z, t) \geq \phi \quad M(Pz, z, \tfrac{4t}{k}) \oplus M(Pz, z, \tfrac{4t}{k}), \quad M(Pz, z, \tfrac{6t}{k}) \oplus M(Pz, z, \tfrac{6t}{k}),$$

$$N(Pz, z, t) \leq \psi \begin{Bmatrix} N(Pz, z, \frac{2t}{k}), & N(Pz, Pz, \frac{2t}{k}) \ominus N(z, z, \frac{2t}{k}), \\ N(Pz, z, \frac{4t}{k}) \ominus N(Pz, z, \frac{4t}{k}), & N(Pz, z, \frac{6t}{k}) \ominus N(Pz, z, \frac{6t}{k}), \end{Bmatrix} \tag{7}$$

Where

$$M(Pz, z, \tfrac{4t}{k}) \oplus M(Pz, z, \tfrac{4t}{k}) \geq \min\left\{ M(Pz, z, \tfrac{2t}{k}), \; M(Pz, z, \tfrac{2t}{k})\right\} = M(Pz, z, \tfrac{2t}{k})$$

$$N(Pz, z, \tfrac{4t}{k}) \ominus N(Pz, z, \tfrac{4t}{k}) \leq \max\left\{ N(Pz, z, \tfrac{2t}{k}), \; N(Pz, z, \tfrac{2t}{k})\right\} = N(Pz, z, \tfrac{2t}{k})$$

and

$$\begin{aligned} M(Pz, z, \tfrac{6t}{k}) \oplus M(Pz, z, \tfrac{6t}{k}) &\geq \min\{ M(Pz, z, \tfrac{4t}{k}), \; M(Pz, z, \tfrac{2t}{k})\} \\ &\geq \min\{ M(Pz, z, \tfrac{2t}{k}) * M(z, z, \tfrac{2t}{k}), M(Pz, z, \tfrac{2t}{k})\} \\ &= \min\{ M(Pz, z, \tfrac{2t}{k}), M(Pz, z, \tfrac{2t}{k})\} = M(Pz, z, \tfrac{2t}{k}) \end{aligned}$$

$$\begin{aligned} N(Pz, z, \tfrac{6t}{k}) \ominus N(Pz, z, \tfrac{6t}{k}) &\leq \max\left\{ N(Pz, z, \tfrac{4t}{k}), N(Pz, z, \tfrac{2t}{k})\right\} \\ &\leq \max\left\{ N(Pz, z, \tfrac{2t}{k}) \diamond N(z, z, \tfrac{2t}{k}), \; N(Pz, z, \tfrac{2t}{k})\right\} \\ &= \max\left\{ N(Pz, z, \tfrac{2t}{k}), \; N(Pz, z, \tfrac{2t}{k})\right\} = N(Pz, z, \tfrac{2t}{k}) \end{aligned}$$

Then from equation (7)

$$M(Pz, z, t) \geq \phi\{M(Pz, z, \tfrac{2t}{k}), \; 1, \; M(Pz, z, \tfrac{2t}{k}), \; M(Pz, z, \tfrac{2t}{k})\}$$

$$\Longrightarrow \quad M(Pz, z, t) \geq M(Pz, z, \tfrac{2t}{k}) \geq \ldots \; M(Pz, z, \left(\tfrac{2}{k}\right)^n t) \to 1 \quad \text{as } n \to \infty$$

and

$$N(Pz, z, t) \leq \psi\{N(Pz, z, \tfrac{2t}{k}), \; 0, \; N(Pz, z, \tfrac{2t}{k}), \; N(Pz, z, \tfrac{2t}{k})\},$$

$$\Longrightarrow \quad N(Pz, z, t) \leq N(Pz, z, \tfrac{2t}{k}) \leq \ldots \; N(Pz, z, \left(\tfrac{2}{k}\right)^n t) \to 0, \text{ as } n \to \infty$$

$\Longrightarrow$ Pz = z, therefore SPz = z $\Longrightarrow$ Sz = z.

Similarly we can show that Qz = z and Tz = z.

Hence z is unique common fixed point of A, B, S, T, P and Q in X.

5. Conclusion:

First of all we fixed the range of integer k for valid calculation in our outcomes. Afterward we improved and generalized the result of the Park, by using a new common limit range property and the algebra of fuzzy sets in the framework of IFMS. and relaxed in both containment and completeness of range of self-mappings. Finally, we use our result as an application for finite families of pair wise commuting self-mappings in the structure of IFMS.

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