A Mathematical Model Formulation and Analysis of Sexual Violence Trends in Nigeria

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Abstract

This study proposes a novel approach to analyzing and predicting the trends of sexual violence in Nigeria by employing mathematical modeling techniques commonly used in epidemiology. It aims to adapt epidemic models, like the Susceptible-Infectious-Recovered (SIR) model, to understand the spread and impact of sexual violence within the population. The research uses national surveys, reports, and official records data to establish model parameters and validate their accuracy. By simulating the dynamics of sexual violence while considering factors such as demographics, cultural norms, socioeconomic conditions, and interventions, the study provides insights into the effectiveness of preventive measures. The outcomes aim to inform policymakers, public health professionals, and stakeholders with evidence-based strategies to combat sexual violence and create safer communities in Nigeria.

Keywords: Mathematical Model, Sexual Violence, Positivity Analysis, Basic Reproduction Number, Equilibrium Points, Stability Analysis, Sensitivity Analysis, Numerical Solution.

1. Introduction

Sexual assault leads to a host of detrimental health and social repercussions such as school drop-out, physical injury, and psychological trauma (Heise, 1998). Although sexual assault can sometimes result in unintended pregnancy, numerous risk factors are common to both outcomes, such as low status for women, poverty, and drug and/or alcohol misuse (WHO, 2005). Sexual assault is a major problem in many African nations; between 10% and 59% of girls and women report experiencing sexual assault at least once in their lifetime, which often happened while they were young (Xianguo, 2023). In Nigeria, specifically, about 34.4% of women report sexual assault in childhood; however, these numbers vary by source (David, 2018). Gender-based violence (GBV), particularly sexual violence, is a significant vice that cuts beyond ethnic, social, economic, and regional barriers, affecting 1 in 3 women worldwide (World Bank, 2019). This issue in terms of their health, growth, and development negatively affects women and girls all around the world (Krishnan 2007). Studies like the ones cited above are growing awareness of the prevalence of gender-based violence around the world. Because of the prevalence of sexual assault and its many harmful consequences, it is imperative to create solutions that are more efficient and more precisely targeted at reducing sexual assault.

The risk of exposure to sexually transmitted diseases during forced intercourse has always been a significant concern, but this concern is heightened in Nigeria, which has the highest Human Immunodeficiency Virus (HIV) burden in Sub-Saharan Africa (Bassey, 2023). Sexual assault not only exposes victims to a variety of sexually transmitted diseases such as Syphilis, HIV (AIDS), and gonorrhea, but it can also result in the victims becoming pregnant. These unintended pregnancies are typically adolescent pregnancies.

Mathematical modeling provides a powerful tool to analyze complex phenomena, such as the trends of sexual violence, by capturing the interplay of several factors and their dynamic nature. By employing mathematical models, we can gain insights into the underlying dynamics and patterns of sexual violence incidents in Nigeria. This research aims to develop a comprehensive mathematical model that will enable the exploration and understanding of sexual violence trends in Nigeria. We will consider factors such as demographics, socio-economic indicators, cultural influences, and law enforcement activities. Similar studies have been conducted on domestic violence in Ghana (Otoo, 2014), Spain (Poza 2016), Bangladesh (Islam 2020).

The outcomes of this research will have significant implications for policymakers, law enforcement agencies, and social organizations involved in combating sexual violence in Nigeria. The mathematical model will serve as a decision-support tool, providing evidence-based insights that can guide the allocation of resources, development of prevention programs, and enhancement of victim support services. Additionally, the findings of this research will contribute to the existing body of knowledge on sexual violence and help researchers and academics deepen their understanding of the phenomenon and its underlying dynamics.

2. Model Formulation

To investigate the dynamics of sexual violence in two interacting populations of violent persons and victims, we developed a model that separates the violent population into potentially violent $S_1(t)$, violent $V_1(t)$, infected violent $I_1(t)$, not infected violent $H_1(t)$ and recovered violent $R_1(t)$, while the victim population is split into susceptible victim $S_2(t)$, victim $V_2(t)$, infected victim $I_2(t)$, not infected victim $H_2(t)$, and recovered victim $R_2(t)$. Next, we consider δ as the rate of natural death, μ_1 as the recruitment rate of the population becoming potentially violent, μ_2 as the recruitment rate of the population becoming a susceptible victim, α_1 as the rate of potentially violent becoming violent, α_2 as the rate of the susceptible victim becoming a victim, β_1 as the infection rate of the violent population, β_2 as the infection rate of the victim population, λ_1 as the not infection rate of the violent population, λ_2 as the not infection rate of the victim population, θ_1 as the recovery rate of the infected violent population by quarantine and punishment, θ_2 as the recovery rate of the infected victim population by treatment and counseling, ξ_1 as the recovery rate of not infected violent population by punishment, ξ_2 as the recovery rate of not infected victim population by treatment and counseling, γ_1 as the rate of the potentially violent becoming recovered, γ_2 as the rate of the recovered victim becoming susceptible, ω as the violence induced death rate, ϕ_1 as the contact rate between infected violent individuals and susceptible victim and ϕ_2 as the contact rate between not infected violent individuals and susceptible victim. We denote the total violent population by N_R and the total victim population by N_V.

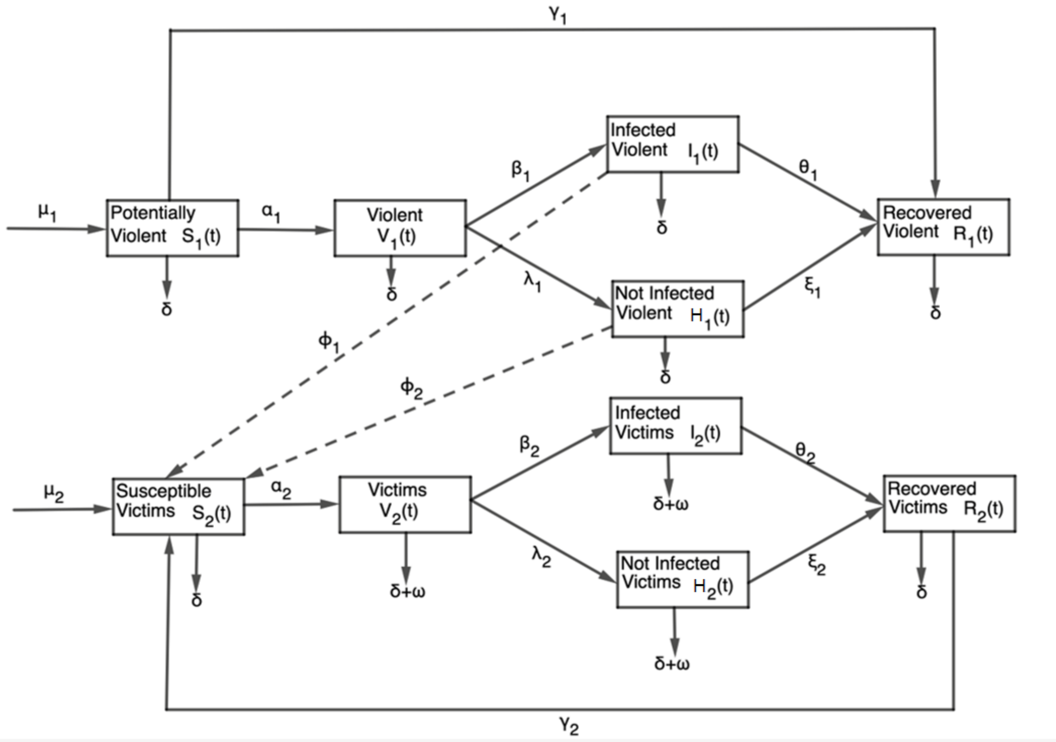

Figure 1: Compartmental Model of Sexual Violence Transmission

2.1 Model Equations

$$\frac{dS_1}{dt} = \mu_1 - \alpha_1 S_1 V_1 - (\delta + \gamma_1) S_1$$
$$\frac{dV_1}{dt} = \alpha_1 S_1 V_1 - (\delta + \beta_1 + \lambda_1) V_1$$
$$\frac{dI_1}{dt} = \beta_1 V_1 I_1 - (\delta + \theta_1 + \phi_1) I_1$$
$$\frac{dH_1}{dt} = \lambda_1 V_1 H_1 - (\delta + \xi_1 + \phi_2) H_1$$

$$\frac{dR_1}{dt} = \theta_1 I_1 + \gamma_1 S_1 + H_1 \xi_1 - \delta R_1 \quad (1)$$
$$\frac{dS_2}{dt} = \mu_2 + \gamma_2 R_2 - S_2(\phi_1 \alpha_2 I_1 + \phi_2 \alpha_2 H_1 + \delta)$$
$$\frac{dV_2}{dt} = \alpha_2 S_2 - (\delta + \omega + \beta_2 + \lambda_2) V_2$$
$$\frac{dI_2}{dt} = \beta_2 V_2 - (\delta + \omega + \theta_2) I_2$$
$$\frac{dH_2}{dt} = \lambda_2 V_2 - (\delta + \omega + \xi_2) H_2$$
$$\frac{dR_2}{dt} = \theta_2 I_2 + \xi_2 H_2 - (\delta + \gamma_2) R_2$$
$$N_R = S_1(t) + V_1(t) + I_1(t) + H_1(t) + R_1(t)$$
$$N_V = S_2(t) + V_2(t) + I_2(t) + H_2(t) + R_2(t)$$

3. Model Analysis

To verify the effectiveness of this model, we performed the following mathematical analysis with support of some relevant theorems.

3.1 Positivity Analysis

Theorem 1:

If $S_1(t),\ V_1(t),\ I_1(t),\ H_1(t),\ R_1(t),\ S_2(t),\ V_2(t),\ I_2(t),\ H_2(t)$, and $R_2(t)$ are all strictly positive, then the solutions $\left(S_1(t),\ V_1(t),\ I_1(t),\ H_1(t), R_1(t),\ S_2(t),\ V_2(t),\ I_2(t),\ H_2(t), R_2(t)\right)$ of the model (Eq. (1)) are non-negative.

Proof:

From model (1), we have that $\frac{dS_1(t)}{dt} \geq \mu_1 - (\delta + \gamma_1) S_1$, which implies that

$$\frac{dS_1(t)}{dt} + (\delta + \gamma_1) S_1 \geq \mu_1 \quad (2)$$

From Eq. (2), the integrating factor is I.F. $= e^{\int(\delta+\gamma_1)dt} = e^{(\delta+\gamma_1)t}$.

Multiply both sides of Eq. (2) with the integrating factor (I.F.) to get

$$e^{(\delta+\gamma_1)t} \frac{dS_1(t)}{dt} + e^{(\delta+\gamma_1)t}(\delta + \gamma_1) S_1 \geq \mu_1 e^{(\delta+\gamma_1)t}$$

which implies that $\frac{d}{dt}\left(S_1 e^{(\delta+\gamma_1)t}\right) \geq \mu_1 e^{(\delta+\gamma_1)t}$. From this we get

$$d\left(S_1 e^{(\delta+\gamma_1)t}\right) \geq \mu_1 e^{(\delta+\gamma_1)t} dt \quad (3)$$

Integrating Eq. (3), we have

$$S_1 e^{(\delta+\gamma_1)t} \geq \frac{\mu_1 e^{(\delta+\gamma_1)t}}{\delta+\gamma_1} + C \quad (4)$$

where *C* is a constant.

Applying the initial condition at $t = 0,\ S_1(t) \geq S_1(0)$.

Hence from Eq. (4), we have

$$S_1(0) \geq \frac{\mu_1}{(\delta + \gamma_1)} + C$$

or equivalently,

$$S_1(0) - \frac{\mu_1}{(\delta + \gamma_1)} \geq C$$

Putting this upper bound of *C* into Eq. (4) yields

$$S_1 e^{(\delta+\gamma_1)t} \geq \frac{\mu_1 e^{(\delta+\gamma_1)t}}{\delta + \gamma_1} + \left(S_1(0) - \frac{\mu_1}{(\delta + \gamma_1)}\right)$$

Dividing through by $e^{(\delta+\gamma_1)t}$, we arrive at

$$S_1(t) \geq \frac{\mu_1}{\sigma+\gamma_1} + e^{-(\delta+\gamma_1)t}\left(S_1(0) - \frac{\mu_1}{(\delta+\gamma_1)}\right) \qquad (5)$$

Hence, $S_1(t) > 0$ at $t = 0$ and as $t \to \infty$. Similarly, we can find the positivity of $V_1(t)$, $I_1(t)$, $H_1(t), R_1(t)$, $S_2(t)$, $V_2(t)$, $I_2(t), H_2(t)$, $R_2(t)$ under the initial conditions.

Therefore, we have shown that $S_1(t)$, $V_1(t)$, $I_1(t)$, $H_1(t)$, $R_1(t)$, $S_2(t)$, $V_2(t)$, $I_2(t)$, $H_2(t)$, and $R_2(t)$ are all strictly positive for all $t \geq 0$.

3.2 Equilibrium Points

3.2.1 Sexual Violence Free Equilibrium Points

Let $E_{SVFE}(S_1^*, V_1^*, I_1^*, H_1^*, R_1^*, S_2^*, V_2^*, I_2^*, H_2^*, R_2^*)$ be the sexual violence free equilibrium point of the model (Eq. (1)). We need to solve $\frac{dS_1^*}{dt} = \frac{dV_1^*}{dt} = \frac{dI_1^*}{dt} = \frac{dH_1^*}{dt} = \frac{dR_1^*}{dt} = \frac{dS_2^*}{dt} = \frac{dV_2^*}{dt} = \frac{dI_2^*}{dt} = \frac{dH_2^*}{dt} = \frac{dR_2^*}{dt} = 0$ of the model (Eq. (1)) to find the sexual violence free equilibrium point. This gives us the following system of equations, which we denote as Eq. (6).

$$\frac{dS_1^*}{dt} = \mu_1 - \alpha_1 S_1^* V_1^* - (\delta + \gamma_1) S_1^* = 0$$
$$\frac{dV_1^*}{dt} = \alpha_1 S_1^* V_1^* - (\delta + \beta_1 + \lambda_1) V_1^* = 0$$
$$\frac{dI_1^*}{dt} = \beta_1 V_1^* I_1^* - (\delta + \theta_1 + \phi_1) I_1^* = 0$$
$$\frac{dH_1^*}{dt} = \lambda_1 V_1^* H_1^* - (\delta + \xi_1 + \phi_2) H_1^* = 0$$
$$\frac{dR_1^*}{dt} = \theta_1 I_1^* + \gamma_1 S_1^* + \xi_1 H_1^* - \delta R_1^* = 0$$
$$\frac{dS_2^*}{dt} = \mu_2 + \gamma_2 R_2^* - S_2^*(\phi_1 \alpha_2 I_1^* + \phi_2 \alpha_2 H_1^* + \delta) = 0$$
$$\frac{dV_2^*}{dt} = \alpha_2 S_2^* - (\delta + \omega + \beta_2 + \lambda_2) V_2^* = 0$$
$$\frac{dI_2^*}{dt} = \beta_2 V_2^* - (\delta + \omega + \theta_2) I_2^* = 0$$
$$\frac{dH_2^*}{dt} = \lambda_2 V_2^* - (\delta + \omega + \xi_2) H_2^* = 0$$
$$\frac{dR_2^*}{dt} = \theta_2 I_2^* + \xi_2 H_2^* - (\delta + \gamma_2) R_2^* = 0$$

At the sexual violence free equilibrium point, we assume the absence of sexual violence. That is, we assume $V_1^* = V_2^* = 0$. Under this assumption, we obtain the following from Eq. (6):

$$S_1^* = \frac{\mu_1}{\delta + \gamma_1}, \quad I_1^* = 0, \quad H_1^* = 0, \quad R_1^* = \frac{\gamma_1 \mu_1}{\delta(\delta + \gamma_1)}, \quad S_2^* = \frac{\mu_2}{\delta}, \quad I_2^* = 0, \quad H_2^* = 0, \quad R_2^* = 0$$

Then, the sexual violence free equilibrium point is:

$$E_{SVFE}(S_1^*, V_1^*, I_1^*, H_1^*, R_1^*, S_2^*, V_2^*, I_2^*, H_2^*, R_2^*) = \left(\frac{\mu_1}{\delta + \gamma_1}, 0,0,0, \frac{\gamma_1 \mu_1}{\delta(\delta + \gamma_1)}, \frac{\mu_2}{\delta}, 0,0,0,0\right) \qquad (7)$$

3.2.2 Sexual Violence Endemic Equilibrium Points

Introducing a violent individual to the population can alter the total population dynamics. For endemic equilibrium, sexual violence still exists. Hence, we cannot assume $V_1^* = V_2^* = 0$.

Let $E_{SVEE}(S_1^*, V_1^*, I_1^*, H_1^*, R_1^*, S_2^*, V_2^*, I_2^*, H_2^*, R_2^*)$ be the sexual violence endemic equilibrium point. To find the sexual violence endemic equilibrium point, we must solve the systems of equations Eq. (6). Therefore, we obtain the following which we denote as Eq. (8):

$$S_1^* = \frac{\mu_1}{\delta+\gamma_1},$$

$$V_1^* = 0$$
$$I_1^* = 0$$
$$H_1^* = 0$$
$$R_1^* = \frac{\gamma_1 \mu_1}{\delta(\delta + \gamma_1)}$$
$$S_2^* = \frac{\delta + \omega + \beta_2 + \lambda_2}{\alpha_2} V_2^*$$
$$I_2^* = \frac{\beta_2}{\delta + \omega + \theta_2} V_2^*$$
$$H_2^* = \frac{\lambda_2}{\delta + \omega + \xi_2} V_2^*$$
$$R_2^* = \left(\frac{\theta_2 \beta_2}{(\delta + \gamma_2)(\delta + \omega + \theta_2)} + \frac{\xi_2 \lambda_2}{(\delta + \gamma_2)(\delta + \omega + \xi_2)}\right) V_2^*$$
$$V_2^* = \frac{\alpha_2 \mu_2 (\delta t_1 - \gamma_2 t_2 + \gamma_2 \mu_2 t_2)}{(\beta_2 + \lambda_2 + \delta + \omega)(\delta t_1 - \gamma_2 t_2)}$$

where

$$t_1 = (\theta_2 + \delta + \omega)(\delta + \gamma_2)(\beta_2 + \lambda_2 + \delta + \omega)(\xi_2 + \delta + \omega)$$
$$t_2 = (\theta_2 \beta_2 \alpha_2)(\xi_2 + \delta + \omega) + (\xi_2 \alpha_2 \lambda_2)(\theta_2 + \delta + \omega).$$

3.3 Basic Reproduction Number

In this section, the Basic Reproduction Number is a measure of the potential for the spread of sexual violence in a population. We will use the next generation matrix method to determine the basic reproduction number of the model (Eq. 1), by considering the violence and victims compartments of the system. Let F_i be the rate of appearance of sexual violence in compartment i, and let W_i be the rate of transfer of individuals out of compartment i, given the sexual violence free equilibrium; then R_0 is the largest eigenvalue of the next generation matrix donated by

$$G = J_F J_W^{-1}$$

Here we have 6 compartments where sexual violence may occur: $V_1, V_2, I_1 I_2, H_1$, and H_2. The rate of appearance of sexual violence into these compartments is given by F_1, F_2, F_3, F_4, F_5, and F_6 respectively, and the rate of transfer of individuals out of these compartments is given by W_1, W_2, W_3, W_4, W_5, and W_6 respectively. Thus, we have

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} \alpha_1 S_1 V_1 \\ \alpha_2 S_2 \\ \beta_1 V_1 I_1 \\ \beta_2 V_2 \\ \lambda_1 V_1 H_1 \\ \lambda_2 V_2 \end{bmatrix} \text{ and } W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \end{bmatrix} = \begin{bmatrix} (\delta + \beta_1 + \lambda_1) V_1 \\ (\delta + \omega + \beta_2 + \lambda_2) V_2 \\ (\delta + \theta_1 + \phi_1) I_1 \\ (\delta + \omega + \theta_2) I_2 \\ (\delta + \xi_1 + \theta_2) H_1 \\ (\delta + \omega + \xi_2) H_2 \end{bmatrix}.$$

Evaluating the Jacobian matrix of F, we have that

$$J_F = \begin{pmatrix} \frac{\partial F_1}{\partial V_1} & \frac{\partial F_1}{\partial V_2} & \frac{\partial F_1}{\partial I_1} & \frac{\partial F_1}{\partial I_2} & \frac{\partial F_1}{\partial H_1} & \frac{\partial F_1}{\partial H_2} \\ \frac{\partial F_2}{\partial V_1} & \frac{\partial F_2}{\partial V_2} & \frac{\partial F_2}{\partial I_1} & \frac{\partial F_2}{\partial I_2} & \frac{\partial F_2}{\partial H_1} & \frac{\partial F_2}{\partial H_2} \\ \frac{\partial F_3}{\partial V_1} & \frac{\partial F_3}{\partial V_2} & \frac{\partial F_3}{\partial I_1} & \frac{\partial F_3}{\partial I_2} & \frac{\partial F_3}{\partial H_1} & \frac{\partial F_3}{\partial H_2} \\ \frac{\partial F_4}{\partial V_1} & \frac{\partial F_4}{\partial V_2} & \frac{\partial F_4}{\partial I_1} & \frac{\partial F_4}{\partial I_2} & \frac{\partial F_4}{\partial H_1} & \frac{\partial F_4}{\partial H_2} \\ \frac{\partial F_5}{\partial V_1} & \frac{\partial F_5}{\partial V_2} & \frac{\partial F_5}{\partial I_1} & \frac{\partial F_5}{\partial I_2} & \frac{\partial F_5}{\partial H_1} & \frac{\partial F_5}{\partial H_2} \\ \frac{\partial F_6}{\partial V_1} & \frac{\partial F_6}{\partial V_2} & \frac{\partial F_6}{\partial I_1} & \frac{\partial F_6}{\partial I_2} & \frac{\partial F_6}{\partial H_1} & \frac{\partial F_6}{\partial H_2} \end{pmatrix} = \begin{pmatrix} \alpha_1 S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I_1 & 0 & \beta_1 V_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 \\ \lambda_1 H_1 & 0 & 0 & 0 & \lambda_1 V_1 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then at the sexual violence free equilibrium, we have

$$J_F = \begin{pmatrix} \frac{\alpha_1\mu_1}{\delta+\gamma_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Also evaluating the Jacobian matrix of $V(x)$, we have that

$$J_W = \begin{pmatrix} \frac{\partial W_1}{\partial V_1} & \frac{\partial W_1}{\partial V_2} & \frac{\partial W_1}{\partial I_1} & \frac{\partial W_1}{\partial I_2} & \frac{\partial W_1}{\partial H_1} & \frac{\partial W_1}{\partial H_2} \\ \frac{\partial W_2}{\partial V_1} & \frac{\partial W_2}{\partial V_2} & \frac{\partial W_2}{\partial I_1} & \frac{\partial W_2}{\partial I_2} & \frac{\partial W_2}{\partial H_1} & \frac{\partial W_2}{\partial H_2} \\ \frac{\partial W_3}{\partial V_1} & \frac{\partial W_3}{\partial V_2} & \frac{\partial W_3}{\partial I_1} & \frac{\partial W_3}{\partial I_2} & \frac{\partial W_3}{\partial H_1} & \frac{\partial W_3}{\partial H_2} \\ \frac{\partial W_4}{\partial V_1} & \frac{\partial W_4}{\partial V_2} & \frac{\partial W_4}{\partial I_1} & \frac{\partial W_4}{\partial I_2} & \frac{\partial W_4}{\partial H_1} & \frac{\partial W_4}{\partial H_2} \\ \frac{\partial W_5}{\partial V_1} & \frac{\partial W_5}{\partial V_2} & \frac{\partial W_5}{\partial I_1} & \frac{\partial W_5}{\partial I_2} & \frac{\partial W_5}{\partial H_1} & \frac{\partial W_5}{\partial H_2} \\ \frac{\partial W_6}{\partial V_1} & \frac{\partial W_6}{\partial V_2} & \frac{\partial W_6}{\partial I_1} & \frac{\partial W_6}{\partial I_2} & \frac{\partial W_6}{\partial H_1} & \frac{\partial W_6}{\partial H_2} \end{pmatrix}$$

Now,

$$J_W = \begin{pmatrix} \delta+\beta_1+\lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta+\omega+\beta_2+\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta+\theta_1+\phi_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta+\omega+\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta+\xi_1+\phi_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta+\omega+\xi_2 \end{pmatrix}$$

Next, we evaluate the inverse of J_W.

$$J_W^{-1} = \begin{pmatrix} \frac{1}{\delta+\beta_1+\lambda_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\delta+\omega+\beta_2+\lambda_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\delta+\phi_1+\theta_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\delta+\omega+\theta_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\delta+\xi_1+\phi_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta+\omega+\xi_2} \end{pmatrix}$$

Evaluating $G = J_F J_W^{-1}$, we have

$$G = \begin{pmatrix} \frac{\alpha_1\mu_1}{(\delta+\gamma_1)(\delta+\beta_1+\lambda_1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_2}{\delta+\omega+\lambda_2+\beta_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\lambda_2}{\delta+\omega+\lambda_2+\beta_2} & 0 & 0 & 0 & 0 \end{pmatrix}$$

Now, finding the eigenvalues of G we have

$$\begin{vmatrix} \frac{\alpha_1\mu_1}{(\delta+\gamma_1)(\delta+\beta_1+\lambda_1)} - \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0-\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0-\lambda & 0 & 0 & 0 \\ 0 & \frac{\beta_2}{\delta+\omega+\lambda_2+\beta_2} & 0 & 0-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0-\lambda & 0 \\ 0 & \frac{\lambda_2}{\delta+\omega+\lambda_2+\beta_2} & 0 & 0 & 0 & 0-\lambda \end{vmatrix} = 0$$

Which gives us the following eigenvalues

$$\lambda_1 = \frac{\alpha_1\mu_1}{(\delta+\gamma_1)(\delta+\beta_1+\lambda_1)} \qquad \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0$$

Since the basic reproduction number R_0 is the eigenvalue with the largest absolute value, we have

$$R_0 = \frac{\alpha_1\mu_1}{(\delta+\gamma_1)(\delta+\beta_1+\lambda_1)}$$

3.4 Stability Analysis

3.4.1 Local Stability of the Sexual Violence Free Equilibrium

Theorem 2:

The sexual violence-free equilibrium point of the model (Eq. (1)) is locally asymptotically stable.

Proof:

To check for the stability of the sexual violence-free equilibrium of the model, we will show that all eigenvalues of the Jacobian matrix are negative. We need to obtain the Jacobian matrix of the model at the sexual violence free equilibrium point, namely J_{SVFE}. First, we find the Jacobian matrix of the model, call it J_E.

$$J_E = \begin{pmatrix} -\alpha_1V_1-(\delta+\gamma_1) & \alpha_1S_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1V_1 & \alpha_1S_1-(\delta+\beta_1+\lambda_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1I_1 & \beta_1V_1-(\delta+\phi_1+\theta_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1H_1 & 0 & \lambda_1V_1-(\delta+\phi_2+\xi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & \theta_1 & \xi_1 & -\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_1\alpha_2S_2 & -\phi_2\alpha_2S_2 & 0 & -(\phi_1\alpha_2I_1+\phi_2\alpha_2H_1+\delta) & 0 & 0 & 0 & \gamma_2 \\ 0 & 0 & 0 & 0 & 0 & \alpha_2 & -(\delta+\omega+\beta_2+\lambda_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & -(\delta+\omega+\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & -(\delta+\omega+\xi_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_2 & \xi_2 & -(\delta+\gamma_2) \end{pmatrix}$$

At Sexual Violence Free Equilibrium state, we have

$$J_{SVFE} = \begin{pmatrix} -(\delta+\gamma_1) & \frac{\alpha_1\mu_1}{\delta+\gamma_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha_1\mu_1}{\delta+\gamma_1}-(\delta+\beta_1+\lambda_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\delta+\phi_1+\theta_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\delta+\phi_2+\xi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & \theta_1 & \xi_1 & -\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\phi_1\alpha_2\mu_2}{\delta} & \frac{-\phi_2\alpha_2\mu_2}{\delta} & 0 & -\delta & 0 & 0 & 0 & \gamma_2 \\ 0 & 0 & 0 & 0 & 0 & \alpha_2 & -(\delta+\omega+\beta_2+\lambda_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & -(\delta+\omega+\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & -(\delta+\omega+\xi_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_2 & \xi_2 & -(\delta+\gamma_2) \end{pmatrix}$$

To determine the stability of the SVFE point, we find the eigenvalues of J_{SVFE}. Using computer software, we find that the eigenvalues are

$$\lambda_1 = -(\delta + \gamma_1)$$
$$\lambda_2 = \frac{\alpha_1\mu_1}{\delta + \gamma_1} - (\delta + \beta_1 + \lambda_1)$$
$$\lambda_3 = -(\delta + \phi_1 + \theta_1)$$
$$\lambda_4 = -(\delta + \phi_2 + \xi_1)$$
$$\lambda_5 = -\delta$$

Note that $\lambda_2 < 0$ whenever $R_0 < 1$, while $\lambda_1, \lambda_3, \lambda_4$, and λ_5 are always negative.

Moreover, the remaining eigenvalues are roots of the polynomial $\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, where

$$a_4 = \delta + k_1 + k_2 + k_3 + k_4$$
$$a_3 = \delta(k_1 + k_2 + k_3 + k_4) + k_1k_2 + k_1k_3 + k_1k_4 + k_2k_3 + k_2k_4 + k_3k_4$$
$$a_2 = \delta(k_1k_2 + k_1k_3 + k_1k_4 + k_2k_3 + k_2k_4 + k_3k_4) + k_1k_2k_3 + k_1k_2k_4 + k_1k_3k_4 + k_2k_3k_4$$
$$a_1 = \delta(k_1k_2k_3 + k_1k_2k_4 + k_1k_3k_4 + k_2k_3k_4) + k_1k_2k_3k_4 - \theta_2\beta_2\alpha_2\gamma_2 - \xi_2\lambda_2\alpha_2\gamma_2$$
$$a_0 = \delta k_1k_2k_3k_4 - k_3\theta_2\beta_2\alpha_2\gamma_2 - k_2\xi_2\lambda_2\alpha_2\gamma_2$$

with

$$k_1 = \delta + \omega + \beta_2 + \lambda_2, \qquad k_2 = \delta + \omega + \theta_2, \qquad k_3 = \delta + \omega + \xi_2, \qquad k_4 = \delta + \gamma_2.$$

The key observation beneath this notation is that each coefficient of this polynomial is positive, so we can conclude that each real root is negative. Since each root is an eigenvalue of J_{SVFE}, we can conclude that the eigenvalues of J_{SVFE} are negative. Therefore, the sexual violence free equilibrium point is locally asymptotically stable.

3.4.2 Local Stability of the Sexual Violence Endemic Equilibrium

Theorem 3: The sexual violence endemic equilibrium point of the model (Eq. (1)) is locally asymptotically stable.

Proof:

We found the Jacobian matrix of the model in the previous proof. Evaluating the Jacobian at the sexual violence endemic equilibrium point, call it J_{SVEE}, we have

$$J_{SVEE} = \begin{pmatrix} -(\delta+\gamma_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\delta+\beta_1+\lambda_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\delta+\phi_1+\theta_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\delta+\phi_2+\xi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & \theta_1 & \xi_1 & -\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_1\alpha_2 & -\phi_2\alpha_2 & 0 & -\delta & 0 & 0 & 0 & \gamma_2 \\ 0 & 0 & 0 & 0 & 0 & \alpha_2 & -(\delta+\omega+\beta_2+\lambda_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & -(\delta+\omega+\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & -(\delta+\omega+\xi_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_2 & \xi_2 & -(\delta+\gamma_2) \end{pmatrix}$$

To determine the stability of the SVEE point, we find the eigenvalues of J_{SVEE}. A quick comparison between J_{SVEE} and J_{SVFE} reveals that all the eigenvalues of J_{SVFE} are eigenvalues of J_{SVEE} except for the second eigenvalue λ_2. The second eigenvalue of J_{SVEE} is $\lambda_2 = -(\delta + \beta_1 + \lambda_1)$. It follows that $\lambda_i < 0$ for $i = 1,2,3,\cdots,10$ when $R_0 < 1$. Therefore, the sexual violence endemic equilibrium state is locally asymptotically stable.

3.5 Sensitivity Analysis

We can use sensitivity analysis to determine how changes to the different rates in the model affect the model output. The sensitivity indices describe the impact of the different rates on the output. The formula for calculating the sensitivity analysis is as follows:

$$S_{parameter} = \frac{\partial R_0}{\partial parameter} * \frac{parameter}{R_0}$$

where

$$R_0 = \frac{\alpha_1 \mu_1}{(\sigma + \gamma_1)(\sigma + \beta_1)}$$

Table 1: Sensitivity Analysis Table

Parameter	Description	Indices
α_1	Rate at which potentially violent becomes violent	Positive
μ_1	Recruitment rate into violent population	Positive
δ	Natural mortality rate	Negative
γ_1	Rate at which potentially violent becomes recovered	Negative
β_1	Rate of violent individuals becoming effected	Negative
λ_1	Rate of individuals moving from violent to not infected violent	Negative

4. Numerical Solutions

We conducted numerical experiments using an ode analyzer from MAPLE 2019 to study the behavior of the system. We present the initial condition for each plot and parameter in the following table.

Many of the values in this table are estimates because we do not have the true values. However, we believe the work of this paper is still useful to help advance the understanding of sexual violence trends in Nigeria. The estimates will help demonstrate the stability and behavior of our model.

Some values are taken directly from a study of sexual violence in Bangladesh (Islam 2020) and are supported by a study of sexual violence in Nigeria (Abreu 2025). By comparing data from Nigeria (Abreu 2025) and data from Bangladesh (Al Mamun 2021) we can see that the prevalence of sexual violence is similar between these two countries. Note that even though we chose the parameter values based on previous research, we ran extra computations at various parameter values to see how those parameters affect our model (see Computations 10 through 18).

Table 2: Variable and Parameter Values Used in the Computation of Results

Variables	Description	Values	Source
$S_1(t)$	Potentially violent population at time (t)	1000	Estimate
$V_1(t)$	Violent population at time (t)	550	Estimate
$I_1(t)$	Infected violent population at time (t)	80	Estimate
$H_1(t)$	Not infected violent population at time (t)	100	Estimate
$R_1(t)$	Recovered violent population at time (t)	280	Estimate
$S_2(t)$	Susceptible victim population at time (t)	1200	Estimate
$V_2(t)$	Victim population at time (t)	720	Estimate
$I_2(t)$	Infected victim population at time (t)	160	Estimate
$H_2(t)$	Not infected victim population at time (t)	180	Estimate
$R_2(t)$	Recovered victim population at time (t)	400	Estimate

μ_1	Recruitment rate into violent population	0.29	Islam (2020), Abreu (2025)
μ_2	Recruitment rate into victim population	0.6	Islam (2020), Abreu (2025)
α_1	Rate at which potentially violent becomes violent	0.004	Islam (2020), Abreu (2025)
α_2	Rate at which susceptible victim becomes victim	0.0032	Islam (2020), Abreu (2025)
ϕ_1	Contact rate between infected violent individuals and susceptible victim	0.0020	Estimate
ϕ_2	Contact rate between not infected violent individuals and susceptible victim	0.0030	Estimate
β_1	Rate of violent individuals becoming infected	0.00005	Estimate
β_2	Rate of victim individuals becoming infected	0.0123	Estimate
λ_1	Rate of violent individuals becoming not infected	0.00015	Estimate
λ_2	Rate of victim individuals becoming not infected	0.0133	Estimate
θ_1	Recovery rate of infected violent by quarantine and punishment	0.0166	Islam (2020), Abreu (2025)
θ_2	Recovery rate of infected victim by treatment and counselling	0.043	Islam (2020), Abreu (2025)
ξ_1	Recovery rate of not infected violent by punishment	0.0186	Estimate
ξ_2	Recovery rate of not infected victim by treatment and counselling	0.063	Estimate
γ_1	Rate at which potentially violent becomes recovered	0.00066	Islam (2020), Abreu (2025)
γ_2	Rate at which recovered victim becomes susceptible	0.0014	Islam (2020), Abreu (2025)
δ	Natural mortality rate	0.0124	Islam (2020), Abreu (2025)
ω	Violence induced death rate	0.003	Islam (2020), Abreu (2025)

We performed a numerical simulation from time t=0 to t=10, for each of the compartments in consideration.

Computation 1:

We plot the behavior of the potentially violent population without altering any of the parameter and variable values.

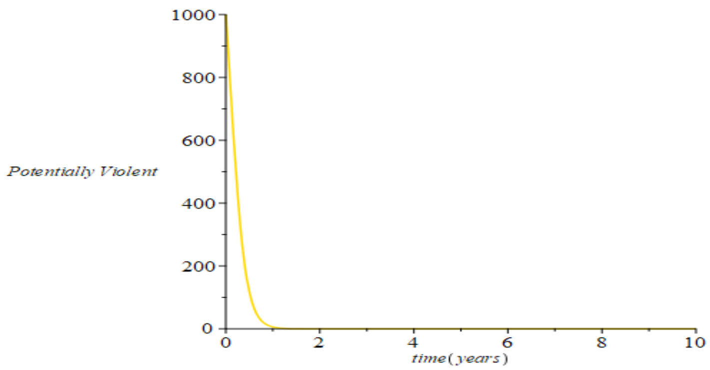

Figure 2: Result of Violent Population with Constant Variable and Parameter

Computation 2:

We plot the behavior of the violent population without altering any of the parameter and variable values.

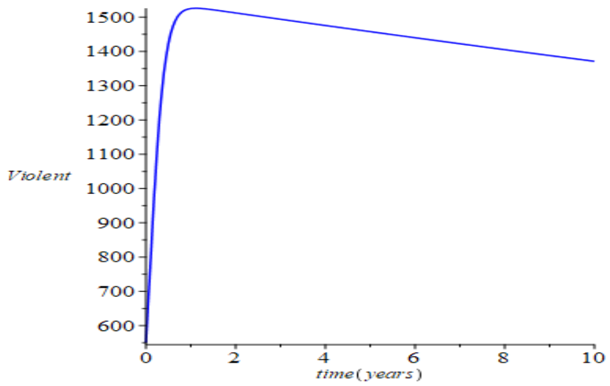

Figure 2: Result of Violent Population with Constant Variable and Parameter

Computation 3:

We plot the behavior of the infected violent population without altering any of the parameter and variable values.

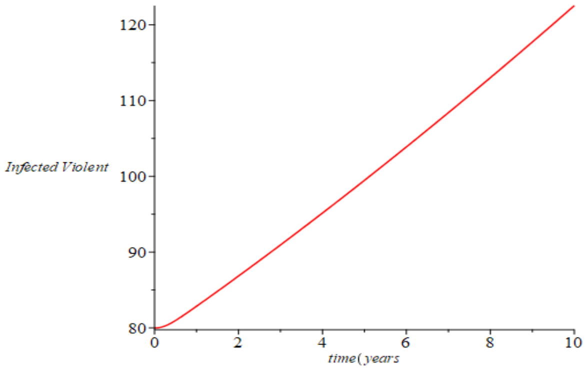

Figure 3: Result of Infected Violent Population with Constant Variable and Parameter

Computation 4:

We plot the behavior of the recovered violent population without altering any of the parameters and variable values.

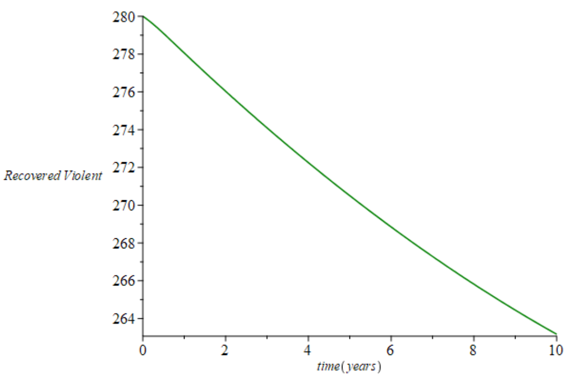

Figure 4: Result of Recovered Violent Population with Constant Variable and Parameter.

Computation 5:

We plot the behavior of the susceptible victim population without altering any of the parameter and variable values.

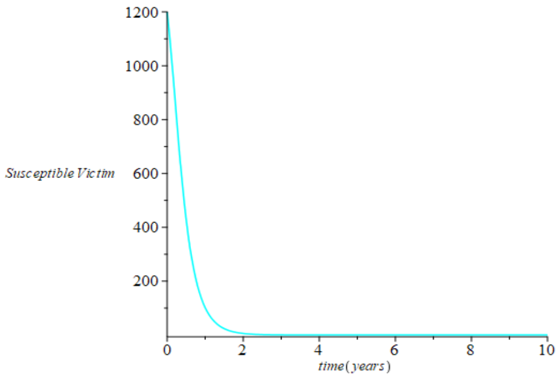

Figure 5: Result of Susceptible Victim Population with Constant Variable and Parameter

Computation 6:

We plot the behavior of the victim population without altering any of the parameters and variable values.

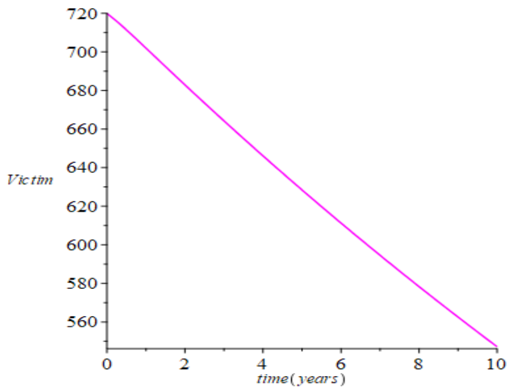

Figure 6: Result of Victim Population with Constant Variable and Parameter

Computation 7:

We plot the behavior of the infected victim population without altering any of the parameter and variable values.

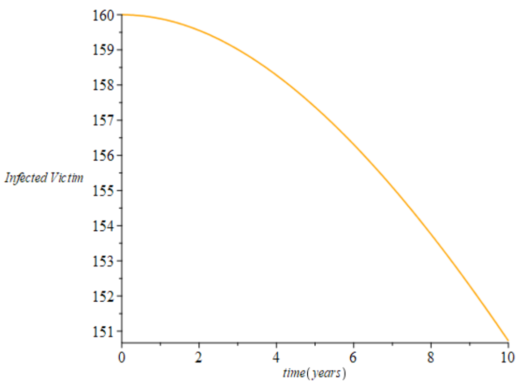

Figure 7: Result of Infected Victim Population with Constant Variable and Parameter

Computation 8:

We plot the behavior of the recovered victim population without altering any of the parameter and variable values.

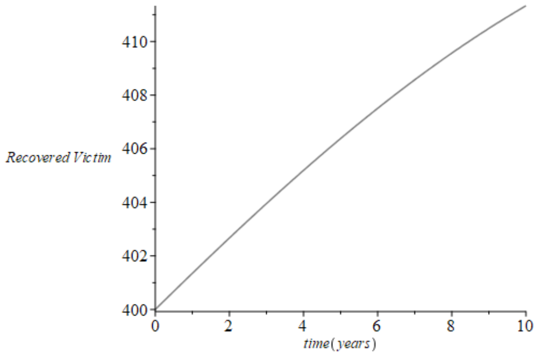

Figure 8: Result of Recovered Victim Population with Constant Variable and Parameter

Computation 9:

We plot the behavior of the entire population without altering any of the parameter and variable values.

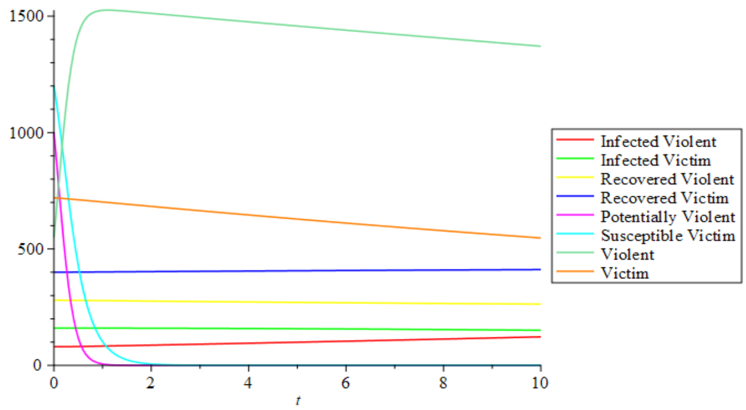

Figure 9: Result for the Entire Population with Constant Variable and Parameter

Computation 10:

We plot the behavior of the potentially violent population where all the variables and parameters remain constant except α_1. The simulated values for $\alpha_1 = 0.0002,\ 0.0004$, and 0.004.

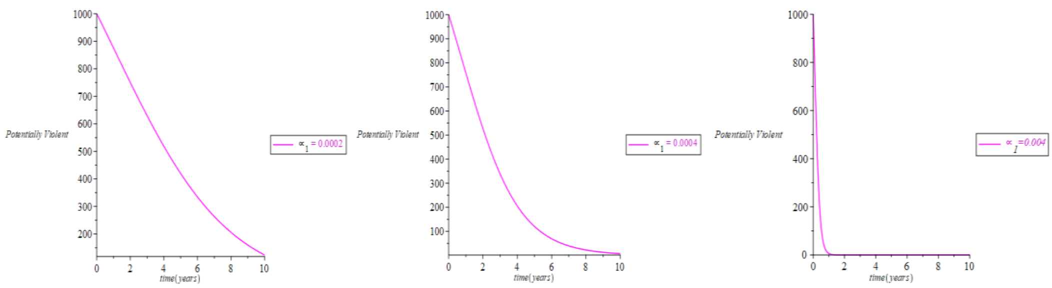

Figure 10: Result of Potentially Violent Population with Different Parameter Values of α_1

Computation 11:

We plot the behavior of the violent population where all the variables and parameters remain constant except α_1 and β_1. The simulated values are $\alpha_1 = 0.004$ and $\beta_1 = 0.00005$, also $\alpha_1 = 0.002$ and $\beta_1 = 0.0003$, and $\alpha_1 = 0.001$ and $\beta_1 = 0.0001$.

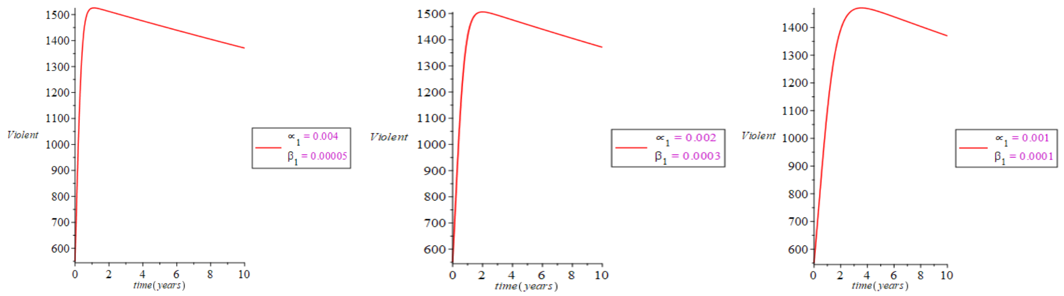

Figure 11: Result of Violent Population for Different Parameter Values of α_1 and β_1

Computation 12:

We plot the behaviour of the infected violent population where all the variables and parameters remain constant except β_1 and θ_1. The simulated values are for when $\beta_1 = 0.00005$ and $\theta_1 = 0.0166$, also when $\beta_1 = 0.0001$ and $\theta_1 = 0.1166$, and when $\beta_1 = 0.0002$ and $\theta_1 = 0.266$.

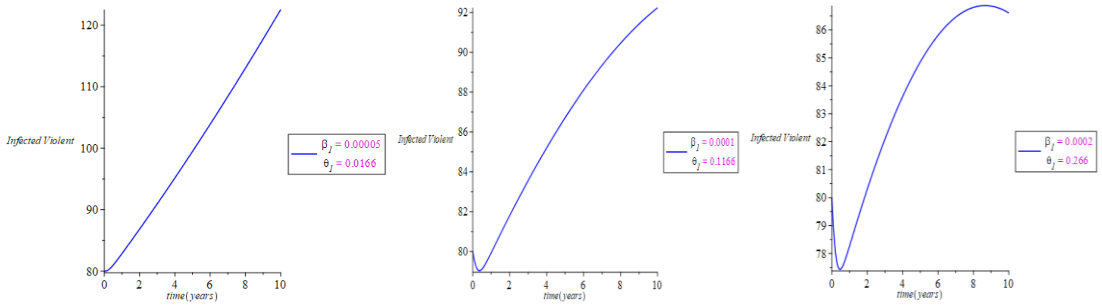

Figure 12: Result of Infected Violent Population for Different Parameter Values of β_1 and θ_1

Computation 13:

We plot the behavior of the recovered violent population where all the variables and parameters remain constant except θ_1. The simulated values for $\theta_1 = 0.0166,\ 0.0266,$ and 0.0366.

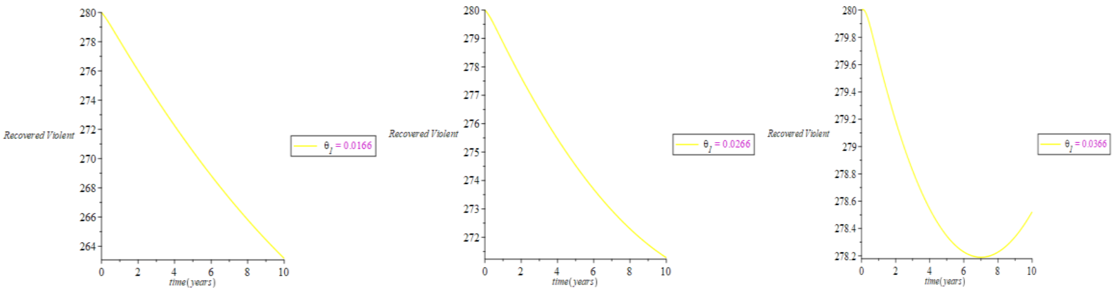

Figure 13: Result of Recovered Violent Population for Different Parameter Values of θ_1

Computation 14:

We plot the behavior of the susceptible victim population where all the variables and parameters remain constant except δ. The simulated values for $\delta = 0.03,\ 0.13,$ and 0.6.

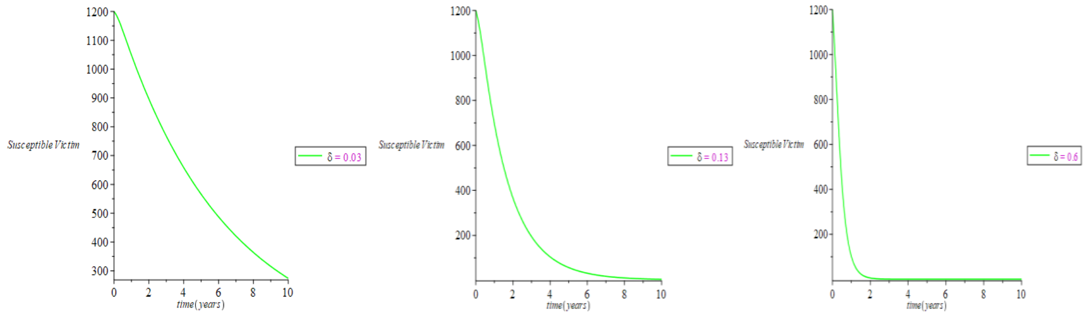

Figure 14: Result of Susceptible Victim Population for Different Parameter Values of δ

Computation 15:

We plot the behavior of the victim population where all the variables and parameters remain constant except β_2 and ω. The simulated values are for when $\beta_2 = 0.0123$ and $\omega = 0.003$ when $\beta_2 = 0.03$ and $\omega = 0.01$, and when $\beta_2 = 0.3$ and $\omega = 0.3$.

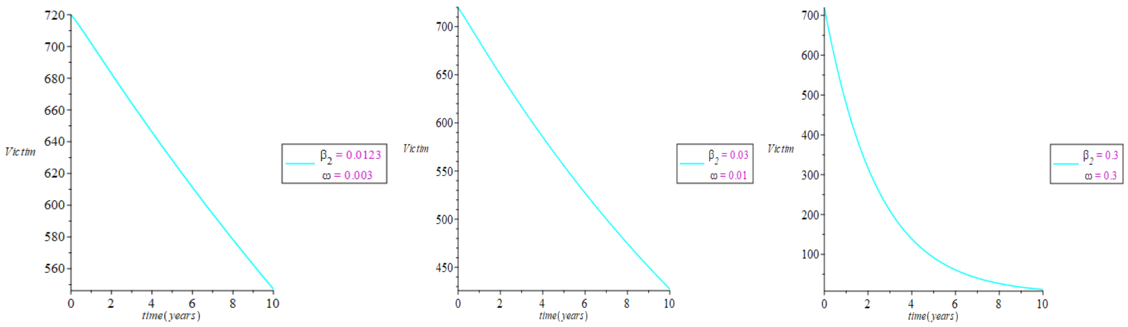

Figure 15: Result of Victim Population for Different Parameter Values of β_2 and ω

Computation 16:

We plot the behavior of the infected victim population where all the variables and parameters remain constant except β_2 and θ_2. The simulated values are $\beta_2 = 0.0123$ and $\theta_2 = 0.043$, and $\beta_2 = 0.03$ and $\theta_2 = 0.13$, and when $\beta_2 = 0.04$ and $\theta_2 = 0.23$.

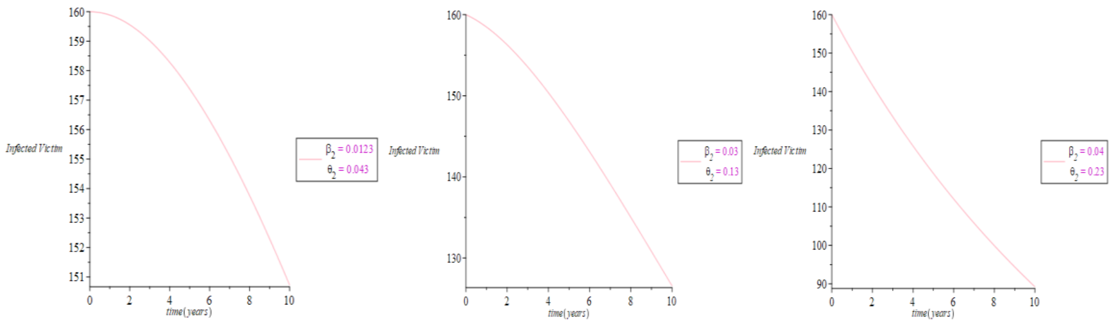

Figure 16: Result of Infected Victim Population for Different Parameter Values of β_2 and θ_2

Computation 17:

We plot the behavior of the recovered victim population where all the variables and parameters remain constant except θ_2. The simulated values for $\theta_2 = 0.043,\ 0.063,$ and 0.083.

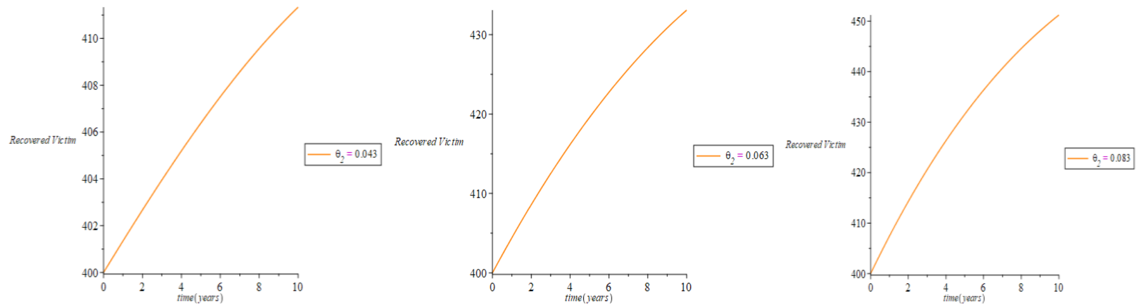

Figure 17: Result of Recovered Victim Population for Different Parameter Values of θ_2

Computation 18:

We plot the behavior of the entire population where all the variables and parameters remain constant except α_1, θ_1, and θ_2. Here $\alpha_1 = 0.001$, $\theta_1 = 0.08$, and $\theta_2 = 0.053$.

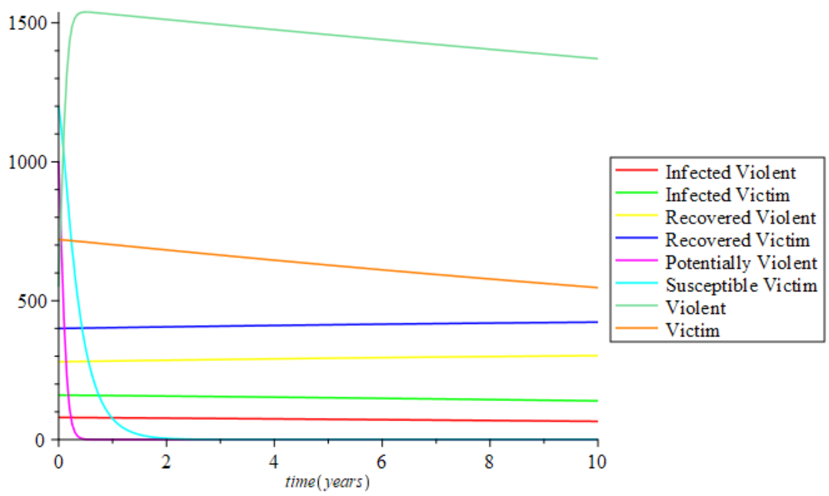

Figure 18: Result of the Entire Population for $\alpha_1 = 0.001$, $\theta_1 = 0.08$, and $\theta_2 = 0.053$

5. Discussion of Results

The Sexual violence model comprises of a set of ordinary differential equations (ODE) that exist in eight dimensions. For the model, we conducted the positivity analysis as given by equation (5). We established the equilibrium points (i.e., the Sexual Violence Free Equilibrium Points and the Sexual Violence Endemic Equilibrium Points) for the model system as given by equation (7) and equation (8), respectively. Next, we obtained the Reproduction number (R_0) using the next generation matrix. We also established the stability of the model i.e., the Local Stability of the Sexual Violence Free Equilibrium and the Local Stability of the Sexual Violence Endemic Equilibrium. We observed that all the eigenvalues are negative, that is $\lambda_i < 0$ for $i = 1,2,3, \ldots, 8$ when $R_0 < 1$. This implies that the sexual violence free equilibrium state and the sexual violence endemic equilibrium state is locally asymptotically stable. Next, we consider the following computational result: In Computation 1, the result shows that there is a decrease in the potentially violent population, and it gradually approaches zero. In Computation 2, the result shows that the violent population increases from the initial state to 1500 for the first year and then begins to decrease gradually. In Computation 3, the result shows that the infected violent population experiences a gradual increase from its initial state. In Computation 4, the result shows that the recovered violent population decreases rapidly until it reaches zero. In Computation 5, the result shows that there is a decrease in the susceptible victim population for the first two years, and it approaches zero. In Computation 6, the result shows that the victim population decreases rapidly for the whole ten years. In Computation 7, the result of the infected victim population shows that there is a gradual decrease from its initial state. Computation 8 shows that there is a gradual increase in the recovered victim population from its initial state from the result shown.

Computation 9 shows the result of the entire population without altering any of the variables or parameters. We can see that the potentially violent population decreases for the first year and it approaches zero. The violent population increases from its initial population i.e., 550 to 1500 for the first year and then gradually decreases. The infected violent population has a constant population until the fourth year, after which it increases slightly. The population of the recovered violent is constant. The susceptible victim population decreases for the first two years, and it approaches zero. The victim population decreases slightly from its initial state. The infected victim population has a constant population. The population of the recovered victim increases slightly. From Computation 10, we observe that as the parameter α_1 increases, the potentially violent population gradually decreases. From Computation 11, the result shows that the violent population increases for a period, and then decreases as the parameter α_1 decreases and the parameter β_1 increases. In Computation 12, the result shows that for $\beta_1 = 0.00005$ then $\theta_1 = 0.0166$, the infected violent population increases from its initial state. For $\beta_1 = 0.0001$ then $\theta_1 = 0.1166$, the infected violent population decreases slightly from its initial state and then increases. For $\beta_1 = 0.0002$ then $\theta_1 = 0.266$, the infected violent population decreases slightly and then increases up to when the population is 87 before it starts to decrease. In Computation 13, as the parameter θ_1 increases, the recovered violent population decreases. But when the parameter $\theta_1 = 0.0366$, the recovered violent population decrease until the 7^{th} year and then increases slightly. From Computation 14, we observe that as the parameter δ increases, the susceptible victim population gradually decreases. From Computation 15, the result shows that the victim population decreases as the parameter β_2 increases and the parameter ω also increases. In Computation 16, we observe that the infected victim population decreases gradually as there is an increase in the simulated parameter β_2 increases and the parameter θ_2 also increases. From Computation 17, as the parameter θ_2 increases, the recovered victim population increases rapidly for the period, $t = 10$ years. Computation 18 shows the graph for the values of the parameter $\alpha_1 = 0.001$, $\theta_1 = 0.08$, and $\theta_2 = 0.053$ on the entire population. It shows that when the values of the parameter α_1 decreases and the parameters θ_1 and θ_2 increase, then the recovered violent and the recovered victim population increases slightly while the violent and the victim population reduced slightly.

The practical application of these results is that efforts should be made to increase the recovery rate of infected violent individuals and infected victims (i.e., increase θ_1 and θ_2) and to decrease the rate at which potentially violent become violent (i.e., decrease α_1). The recovery of infected violent could be thought of as rehabilitation of these individuals into non-violent members of society. This rehabilitation could take place through a variety of means, from punishment to education. More research should be done to determine the best methods of rehabilitation. The recovery of infected victims includes both physical and mental recovery. More resources should be allocated for mental health counseling of victims, while hospitals and doctor offices should be adequately equipped and trained to treat victims with care and sensitivity. Lastly, both the violent and victim populations will decrease if the rate at which potentially violent become violent decreases. This could be done through widespread social awareness campaigns and more government assistance to keep youth out of poverty and in school. As found in a previous study (Abreu 2025), adolescents who are in poverty and/or out of school are more likely to be victims of sexual violence.

6. Conclusion

This study developed a mathematical model to analyze and predict sexual violence trends in Nigeria, using ordinary differential equations to capture the dynamics among potentially violent individuals, violent individuals, susceptible victims, and infected victims. Stability analysis confirmed the equilibrium states' local asymptotic stability. Simulations indicated a decrease in the potentially violent and violent populations over time, while the infected violent population gradually increased. The susceptible and victim populations showed significant reductions, and the recovered victim population increased, highlighting effective preventive and recovery measures. Sensitivity analysis demonstrated that changes in intervention and recovery rates significantly impact population dynamics. These findings underscore the importance of sustained, comprehensive approaches to combating sexual violence. The model offers a valuable framework for informing policy decisions and improving interventions to reduce sexual violence and support victims' recovery in Nigeria. Continued research and application of such models are essential for effective violence reduction strategies.

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