Contra $\hat{\omega}$ -Mappings

M.Lellis Thivagar
School Of Mathematics, Madurai Kamaraj University Madurai-625021, Tamil Nadu,INDIA,
mlthivagar@yahoo.co.in
M.Anbuchelvi (Corresponding author)
Department of Mathematics, V.V.Vanniaperumal College For Women Virudhunar-626001, Tamil Nadu, INDIA.
rsanbuchelvi@gmail.com

Abstract: In this paper we intend to introduce contra $\hat{\omega}$ -closed (resp.open) functions, contra $\hat{\omega}$ -quotient functions and contra $\hat{\omega}$ -irresolute functions, by utilizing $\hat{\omega}$ -closed sets. Also we find their basic properties and some applications.

Keywords and Phrases: contra $\hat{\omega}$ -closed function, contra $\hat{\omega}$ -quotient and $\hat{\omega}$ -quotient functions.
AMS **subject classification 2010** : 57C05

1. Introduction

[1] The notion of contra continuity was introduced by J.Dontchev. In 1991, M.Lellis Thivagar [7] extended the notion of quotient functions on α -open sets, semi-open and pre-open sets in topological spaces. In this paper we introduce and investigate the properties of contra $\hat{\omega}$ -quotient functions, contra $\hat{\omega}$ -closed functions and contra $\hat{\omega}$ -open functions, by utilizing $\hat{\omega}$ -closed sets. Also we find some applications of $\hat{\omega}$ - quotient functions.

2. Preliminaries

Throughout this paper a "space" means a topological space which lacks any separation axioms unless explicitly stated. For a subset A of X, $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A respectively.The family of all $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) sets of X is denoted by $\hat{\omega}O(X)$. (resp. $\hat{\omega}C(X)$)

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1 A subset A of a space X is called a

i) a -open set [3] if $A \subset int(cl(int_{\delta}(A)))$.

ii) $\alpha\hat{g}$ - closed set [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ -open in (X,τ).

iii) $\hat{\omega}$ -closed set [5] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\alpha\hat{g}$ -open in (X,τ).

Definition 2.2 A function $f:(X,\tau) \to (Y,\sigma)$ is called

i) contra $\hat{\omega}$ -continuous [8] if $f^{-1}(V)$ is $\hat{\omega}$ -closed subset of (X,τ) for every open set V of (Y,σ).

ii) $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) [8] if $f(V)$ is $\hat{\omega}$ -open(resp. $\hat{\omega}$ -closed) set of (Y,σ) for every open (resp.closed) set V of (X,τ).

iii) Strongly $\hat{\omega}$ -open or $(\hat{\omega})^*$ -open (resp. Strongly $\hat{\omega}$ -closed or $(\hat{\omega})^*$ -closed) [8] if $f(V)$ is $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) set of (Y,σ) for every $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) set V of (X,τ).

iv) $\hat{\omega}$ -continuous [8] if $f^{-1}(V)$ is $\hat{\omega}$ -open set of (X,τ) for every open set V of (Y,σ).

3. Contra $\hat{\omega}$ -Closed Functions

Definition 3.1 A function $f:(X,\tau) \to (Y,\sigma)$ is said to be contra $\hat{\omega}$ -closed (resp.contra $\hat{\omega}$ -open) if image of every closed (resp.open) subset of (X,τ) is $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) subset of (Y,σ).

Definition 3.2 A function $f:(X,\tau) \to (Y,\sigma)$ is said to be strongly contra $\hat{\omega}$ -closed map (resp.strongly contra $\hat{\omega}$ -open map) if image of every a -closed (resp. a -open) subset of (X,τ) is $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) subset of (Y,σ).

Definition 3.3 A function $f:(X,\tau) \to (Y,\sigma)$ is said to be completely contra $\hat{\omega}$ -closed map (resp.completely contra $\hat{\omega}$ -open map) if the image of every $\hat{\omega}$ -closed (resp. $\hat{\omega}$ -open) subset of (X,τ) is $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) subset of (Y,σ). Also f is said to be $\hat{\omega}$ -irresolute iff inverse image of every $\hat{\omega}$ -open set is $\hat{\omega}$ -open. (equivalently, inverse image of every $\hat{\omega}$ -closed set is $\hat{\omega}$ -closed.)

Theorem 3.4 Every completely contra $\hat{\omega}$ -closed map (resp.completely contra $\hat{\omega}$ -open map) is strongly contra $\hat{\omega}$ - closed map.(resp.strongly contra $\hat{\omega}$ -open map)

Proof. Let F -be any a -closed (resp. a -open) subset of (X,τ). By [5] Proposition 3.2 (i), F is a $\hat{\omega}$ -closed (resp. $\hat{\omega}$ -closed) subset of (X,τ) and hence by hypothesis, $f(F)$ is $\hat{\omega}$ -open (resp. $\hat{\omega}$ -closed) subset of (Y,σ). Thus, f is strongly contra $\hat{\omega}$ -closed map.(resp.strongly contra $\hat{\omega}$ -open).

Remark 3.5 Reversible implication is not always true as seen from the following example.

Example 3.6 *Let* $X = \{a,b,c,d\} = Y$, $\tau = \{\emptyset,\{a\},\{a,b\},X\}$, $\sigma = \{\emptyset,\{b,c\},\{a,b,c\},\{b,c,d\},Y\}$. Define f as an identity function. Then, f is strongly contra $\hat{\omega}$ -closed map but not completely contra $\hat{\omega}$ -closed map as $f(\{b,c,d\}) = \{b,c,d\}$ is not a $\hat{\omega}$ -open subset of (Y,σ) whereas $\{b,c,d\}$ is a $\hat{\omega}$ -closed subset of (X,τ).

Remark 3.7 The notion of contra $\hat{\omega}$ -closed map and strongly contra $\hat{\omega}$ -closed map (resp.completely contra $\hat{\omega}$ -closed map) are independent is understood from the following examples.

Example 3.8 Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset,\{a\},\{a,b\},X\}$, $Y = \{a,b,c,d\}$ $\sigma = \{\emptyset,\{b,c\},\{a,b,c\},\{b,c,d\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=a, f(b)=b, f(c)=c$ and $f(d)=d$. Then, f is strongly contra $\hat{\omega}$ -closed map, but not contra $\hat{\omega}$ -closed map .

Example3.9 Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},X\}$, $Y = \{a,b,c,d\}$ $\sigma = \{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\},Y\}$ .Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=c, f(b)=c, f(c)=b$ and $f(d)=a$. Then, f is contra $\hat{\omega}$ -closed map, but a strongly contra $\hat{\omega}$ -closed map.

Example 3.10 Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset,\{b,c\},\{a,b,c\},\{b,c,d\},X\}$, $Y = \{a,b,c,d\}$ $\sigma = \{\emptyset,\{a\},\{c\},\{a,b\}\{a,c\},\{a,b,c\},\{a,c,d\},Y\}$.

Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=a, f(b)=c, f(c)=c$ and $f(d)=b$. Then f is completely contra $\hat{\omega}$ -closed map, but not contra $\hat{\omega}$ -closed map as $f(\{d\}) = \{b\}$ is not a $\hat{\omega}$ open subset of (Y,σ) whereas $\{d\}$ is a closed subset of (X,τ).

Remark 3.11 From the following Examples, it is known that composition of contra $\hat{\omega}$ -closed (resp.strongly contra $\hat{\omega}$ -closed, completely contra $\hat{\omega}$ -closed contra $\hat{\omega}$ -closed) mappings is not always contra $\hat{\omega}$ -closed (resp.strongly contra $\hat{\omega}$ -closed, completely contra $\hat{\omega}$ -closed contra $\hat{\omega}$ -closed) mapping.

Example 3.12 Let X = Y = Z = {a,b,c,d} and topologies endowed on them are $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},X\}$, $\sigma = \{\emptyset,\{a\},Y\}$, $\eta = \{\emptyset,\{b,c\},\{a,b,c\},\{b,c,d\},Z\}$ respectively. Define $f:(X,\tau)\to(Y,\sigma)$ as f(x) = a for all x in X and define $g:(Y,\sigma)\to(Z,\eta)$ by g(a) = a, g(b) = b, g(c) = g(d) = c. Then, f and g are contra $\hat{\omega}$ -closed maps. If $g\circ f:(X,\tau)\to(Z,\eta)$ is defined by ($g\circ f$)(x) = g(f(x)) for all x in X, then $g\circ f$ is not a contra $\hat{\omega}$ -closed map.

Example 3.13 Let X = Y = Z = {a,b,c,d}, $\tau = \{\emptyset,\{a\},\{a,b\},X\}$, $\sigma = \{\emptyset,\{a\},\{b\},\{a,b\},Y\}$ and $\eta = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},Z\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as f(x) = a for all x in X and define $g:(Y,\sigma)\to(Z,\eta)$ by g(a) = c, g(b) = c, g(c) =a, g(d) = b. Then, f and g are strongly contra $\hat{\omega}$ -closed maps (resp.completely contra $\hat{\omega}$ -closed). If $g\circ f:(X,\tau)\to(Z,\eta)$ is defined by ($g\circ f$)(x) = g(f(x)) for all x in X, then $g\circ f$ is not a strongly contra $\hat{\omega}$ -closed map (resp.completely contra $\hat{\omega}$ -closed).

Theorems on Compositions.

Theorem 3.14 If $f:(X,\tau)\to(Y,\sigma)$ is completely contra $\hat{\omega}$ -closed map and $g:(Y,\sigma)\to(Z,\eta)$ is strongly $\hat{\omega}$ -open map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is completely contra $\hat{\omega}$ -closed map (resp.strongly contra $\hat{\omega}$ -closed map)

Proof. Let F be any $\hat{\omega}$ -closed (resp. a -closed) subset of (X,τ). By [5] Proposition 3.2 (i), every a-closed subset is $\hat{\omega}$ -closed subset of (X,τ) and since f is completely contra $\hat{\omega}$ -closed map, f(F) is $\hat{\omega}$ -open subset of (Y,σ). Since g is strongly $\hat{\omega}$ -open map, $(g\circ f)(F) = g(f(F))$ is $\hat{\omega}$ -open subset of (Z,η). Thus, $g\circ f$ is completely contra $\hat{\omega}$ -closed map (resp.strongly contra $\hat{\omega}$ -closed map)

Theorem 3.15 If $f:(X,\tau)\to(Y,\sigma)$ is strongly contra $\hat{\omega}$ -closed map and $g:(Y,\sigma)\to(Z,\eta)$ is strongly $\hat{\omega}$ -open map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is strongly contra $\hat{\omega}$ -closed map.

Proof. Let F -be any a -closed subset of (X,τ). Since f is strongly contra $\hat{\omega}$ -closed map, $f(F)$ is $\hat{\omega}$ -open subset of (Y,σ). Since g is strongly $\hat{\omega}$ -open map, $(g\circ f)(F) = g(f(F))$ is $\hat{\omega}$ -open in (Z,η). Thus, $g\circ f$ is strongly contra $\hat{\omega}$ -closed map.

Theorem 3.16 If $f:(X,\tau)\to(Y,\sigma)$ is $\hat{\omega}$ -continuous, surjective map and $g:(Y,\sigma)\to(Z,\eta)$ is any map such that $g\circ f:(X,\tau)\to(Z,\eta)$ is completely contra $\hat{\omega}$ -closed map, then $g:(Y,\sigma)\to(Z,\eta)$ is contra $\hat{\omega}$ -closed map.

Proof. Let F be any closed subset of (Y,σ). Since f is $\hat{\omega}$ -continuous, $f^{-1}(F)$ is $\hat{\omega}$ -closed subset (X,τ) and since $g\circ f$ is completely contra $\hat{\omega}$ -closed, $g(f(f^{-1}(F)))$ is $\hat{\omega}$ -open in (Z,η). Since f is surjective, g(F) is $\hat{\omega}$ -open subset of (Z,η). Thus, g is contra $\hat{\omega}$ -closed map.

Theorem 3.17 If $f:(X,\tau)\to(Y,\sigma)$ is surjective $\hat{\omega}$ -irresolute map and $g:(Y,\sigma)\to(Z,\eta)$ is any map such that

$g \circ f:(X,\tau)\to(Z,\eta)$ is completely contra $\hat{\omega}$-closed map, then $g:(Y,\sigma)\to(Z,\eta)$ is completely contra $\hat{\omega}$-closed map.

Proof. Let F-be any $\hat{\omega}$ closed subset of (Y,σ). Since f is $\hat{\omega}$-irresolute function, $f^{-1}(F)$ is $\hat{\omega}$-closed subset of (X,τ) and since $g\circ f$ is completely contra $\hat{\omega}$-closed map, $g(f(f^{-1}(F)))$ is $\hat{\omega}$-open subset of (Z,η). Since f is surjective, $g(F)$ is $\hat{\omega}$-open subset of (Z,η). Thus, g is completely contra $\hat{\omega}$-closed map.

Theorem 3.18 If $f:(X,\tau)\to(Y,\sigma)$ is any function and $g:(Y,\sigma)\to(Z,\eta)$ is an injective, $\hat{\omega}$-irresolute map such that $g\circ f:(X,\tau)\to(Z,\eta)$ is contra $\hat{\omega}$-closed map,then $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$-closed map.

Proof. Let F-be any closed subset of (X,τ). Since $g\circ f$ is contra $\hat{\omega}$-closed map, $g(f(F))$ is $\hat{\omega}$-open subset of (Z,η) and since g is injective $\hat{\omega}$-irresolute function, $g^{-1}(g(f(F)))=f(F)$ is $\hat{\omega}$-open subset of (Y,σ). Thus, f is contra $\hat{\omega}$-closed map.

Theorem 3.19 If $f:(X,\tau)\to(Y,\sigma)$ is any function and $g:(Y,\sigma)\to(Z,\eta)$ is an injective, $\hat{\omega}$-irresolute map such that $g\circ f:(X,\tau)\to(Z,\eta)$ is completely contra $\hat{\omega}$-closed map,then $f:(X,\tau)\to(Y,\sigma)$ is completely contra $\hat{\omega}$-closed map.

Proof. Let F be any $\hat{\omega}$-closed subset of (X,τ). Since $g\circ f$ is completely contra $\hat{\omega}$-closed map, $g(f(F))$ is $\hat{\omega}$-open subset of (Z,η) and since g is injective, $\hat{\omega}$-irresolute map, $g^{-1}(g(f(F)))=f(F)$ is $\hat{\omega}$-open subset of (Y,σ). Thus, f is contra $\hat{\omega}$-closed map.

Theorem 3.20 If $f:(X,\tau)\to(Y,\sigma)$ is $\hat{\omega}$-closed map and $g:(Y,\sigma)\to(Z,\eta)$ is completely contra $\hat{\omega}$-closed map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is contra $\hat{\omega}$-closed map.

Proof. Let F be any closed subset of (X,τ). Since f is $\hat{\omega}$-closed map, $f(F)$ is $\hat{\omega}$-closed subset of (Y,σ) and since g is completely contra $\hat{\omega}$-closed map, $g(f(F))$ is $\hat{\omega}$-open subset of (Z,η). Thus, $g\circ f$ is contra $\hat{\omega}$-closed map.

Theorem 3.21 If $f:(X,\tau)\to(Y,\sigma)$ is strongly $\hat{\omega}$-closed map and $g:(Y,\sigma)\to(Z,\eta)$ is completely contra $\hat{\omega}$-closed map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is completely contra $\hat{\omega}$-closed map.

Proof. Let F be any $\hat{\omega}$-closed subset of (X,τ). Since f is strongly $\hat{\omega}$-closed function, $f(F)$ is $\hat{\omega}$-closed subset of (Y,σ) and since g is completely contra $\hat{\omega}$-closed map, $g(f((F))$ is $\hat{\omega}$-open in (Z,η). Thus, $g\circ f$-is contra $\hat{\omega}$-closed map.

4 Contra $\hat{\omega}$-Quotient and Contra $\hat{\omega}$-irresolute Mappings

Definition 4.1 A surjective function $f:(X,\tau)\to(Y,\sigma)$ is said to be contra $\hat{\omega}$-quotient map if f is contra $\hat{\omega}$-continuous and $f^{-1}(V)$ is closed subset of (X,τ) implies that V is a $\hat{\omega}$-open subset of (Y,σ).

Definition 4.2 A surjective function $f:(X,\tau)\to(Y,\sigma)$ is said to be strongly contra $\hat{\omega}$-quotient map provided a set V is open subset of (Y,σ) iff $f^{-1}(V)$ is $\hat{\omega}$-closed subset of (X,τ).

Theorem 4.3 Every strongly contra $\hat{\omega}$-quotient function is contra $\hat{\omega}$-continuous map.

Proof. Let V be any open subset of (Y,σ). By hypothesis, $f^{-1}(V)$ is $\hat{\omega}$-closed subset of (X,τ). Therefore, f is contra $\hat{\omega}$-continuous map.

Remark 4.4 Reversible implication is not always true from the following example.

Example 4.5 Let $X=\{a,b,c,d\}$ $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$ $Y=\{p,q,r\}$ $\sigma=\{\varnothing,\{p,q\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=r, f(b)=r, f(c)=p$ and $f(d)=q$. Then f is contra $\hat{\omega}$-continuous function but not strongly contra $\hat{\omega}$-quotient.

Remark 4.6 The notion of contra $\hat{\omega}$-quotient and strongly contra $\hat{\omega}$-quotient are independent is understood from the following examples.

Example 4.7 Let $X=\{a,b,c,d\}$, $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$, $Y=\{p,q,r\}$ $\sigma=\{\varnothing,\{p,q\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=r, f(b)=r, f(c)=p$ and $f(d)=q$. Then f is contra $\hat{\omega}$-continuous,surjective and $f^{-1}(p,q)$ is the only closed subset of (X,τ) as well as $\{p,q\}$ is $\hat{\omega}$ open in (Y,σ). Therefore it is contra $\hat{\omega}$-quotient. Further more, $f^{-1}(\{p\})$ and $f^{-1}(\{q\})$ are $\hat{\omega}$-closed sets in (X,τ) but $\{p\}$ and $\{q\}$ are not open in (Y,σ). Therefore, it is not a strongly contra $\hat{\omega}$-quotient.

Example 4.8 Let $X=\{a,b,c,d\}$, $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$, $Y=\{p,q,r\}$ $\sigma=\{\varnothing,\{p\},\{p,q\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=q, f(b)=r, f(c)=p$ and $f(d)=q$. Then f is surjective and strongly contra $\hat{\omega}$-quotient map. Further more, $f^{-1}(p,q)$ is the only closed subset of (X,τ) and $\{p,q\}$ is not a $\hat{\omega}$ open in (Y,σ).

Therefore, it is not a contra $\hat{\omega}$ -quotient.

Theorem 4.9 Let $f:(X,\tau)\to(Y,\sigma)$ be any function. Then the following statements are equivalent.

i) $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$ -quotient map.

ii) If $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$ -continuous and surjective function,then $f^{-1}(V)$ is open subset of (X,τ) implies that V is $\hat{\omega}$ -closed subset of (Y,σ).

Proof. (i) $\Rightarrow$ (ii) Let $f:(X,\tau)\to(Y,\sigma)$ be a surjective and contra $\hat{\omega}$ -continuous map. Suppose that $f^{-1}(V)$ is any open subset of (X,τ). Then $X\setminus f^{-1}(V)=f^{-1}(Y\setminus V)$ is a closed subset of (X,τ) By hypothesis, $Y\setminus V$ is $\hat{\omega}$ -open subset of (Y,σ). Therefore, V is $\hat{\omega}$ -closed subset of (Y,σ).

(ii) $\Rightarrow$ (i) Let $f:(X,\tau)\to(Y,\sigma)$ be a surjective and contra $\hat{\omega}$ -continuous map. Suppose that V is any subset of Y such that $f^{-1}(V)$ is closed subset of (X,τ). Then $X\setminus f^{-1}(V)=f^{-1}(Y\setminus V)$ is open subset of (X,τ). By hypothesis, $Y\setminus V$ is $\hat{\omega}$ -closed subset of (Y,σ). Therefore, V is $\hat{\omega}$ -open subset of (Y,σ) and hence $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$ -quotient map.

Theorem 4.10 If $f:(X,\tau)\to(Y,\sigma)$ is a surjective, contra $\hat{\omega}$ -continuous and contra $\hat{\omega}$ -closed map,then f is contra $\hat{\omega}$ -quotient map.

Proof. Given that f is a surjective and contra $\hat{\omega}$ -continuous map.It suffices to show that for any subset V of Y, $f^{-1}(V)$ is closed subset of (X,τ) implies that V is $\hat{\omega}$ -open subset of (Y,σ). Suppose that V is any subset of Y such that $f^{-1}(V)$ is closed subset of (X,τ). Since f is contra $\hat{\omega}$ -closed and surjective, $V=f(f^{-1}(V))$ is $\hat{\omega}$ -open subset of (Y,σ) and hence f is contra $\hat{\omega}$ -quotient map.

Theorem 4.11 If $f:(X,\tau)\to(Y,\sigma)$ is an injective contra $\hat{\omega}$ -quotient map, then f is contra $\hat{\omega}$ -closed map.

Proof. Let V be any closed subset of (X,τ). Since f is injective by hypothesis, $V=f^{-1}(f((V)))$ is closed subset of (X,τ) and since f is contra $\hat{\omega}$ -quotient map, $f(V)$ is $\hat{\omega}$ -open subset of (Y,σ). Thus, f is contra $\hat{\omega}$ -closed map.

Theorem 4.12 Let $f:(X,\tau)\to(Y,\sigma)$ be surjective,closed and $\hat{\omega}$ -irresolute function. If $g:(Y,\sigma)\to(Z,\eta)$ is a contra $\hat{\omega}$ -quotient map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is a contra $\hat{\omega}$ -quotient map.

Proof. Let F be any closed subset of (Z,η). Since g is contra $\hat{\omega}$ -continuous map, $g^{-1}(F)$ is $\hat{\omega}$ -open subset of (Y,σ) and since f is $\hat{\omega}$ -irresolute map, $f^{-1}(g^{-1}(F))=(g\circ f)(F)$ is $\hat{\omega}$ -open subset of (X,τ). Thus $g\circ f$ is a contra $\hat{\omega}$ -continuous map. As f and g are surjective maps, $g\circ f$ is a surjective map. Suppose that V is any subset of (Z,η) such that $(g\circ f)^{-1}(V)$ is closed in (X,τ). Since f is surjective and closed function $g^{-1}(V)=f(f^{-1}(g^{-1}(V)))$ is closed subset of (Y,σ) and since g is contra $\hat{\omega}$ -quotient map, V is $\hat{\omega}$ -open subset of (Z,η).

Theorem 4.13 Assume that any union of $\hat{\omega}$ -open set is $\hat{\omega}$ -open. Let $\{A_\alpha : \alpha\in\Lambda\}$ be a covering of X by both pre open and closed subsets of X. If $f\,|\,A_\alpha$ is contra $\hat{\omega}$ -quotient map for each $\alpha\in\Lambda$, then $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$ -quotient map.

Proof. Since $f\,|\,A_\alpha$ is surjective, $f:(X,\tau)\to(Y,\sigma)$ is surjective. Since each $f\,|\,A_\alpha$ is contra $\hat{\omega}$ -continuous, f is contra $\hat{\omega}$ -continuous map. Suppose that F is any subset of Y such that $f^{-1}(F)$ is open subset of (X,τ). By relative topology, $(f\,|\,A_\alpha)^{-1}(F)$ is open subset of $(A_\alpha,\tau\,|\,A_\alpha)$. By hypothesis, F is $\hat{\omega}$ -closed subset of (X,τ). Thus, $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$ -quotient map.

Definition 4.14 $f:(X,\tau)\to(Y,\sigma)$ is said to be contra $\hat{\omega}$ -irresolute function if $f^{-1}(V)$ is $\hat{\omega}$ -closed subset of (X,τ) for every $\hat{\omega}$ -open set V of (Y,σ). Equivalently, inverse image of $\hat{\omega}$ -closed subset of (Y,σ) is $\hat{\omega}$ -open subset of (X,τ).

Example 4.15 Let $X=\{a,b,c,d\}$ $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$ $Y=\{p,q,r\}$ $\sigma=\{\varnothing,\{p,q\},Y\}$ .Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=r, f(b)=r, f(c)=p$ and $f(d)=q$. Then f is contra $\hat{\omega}$ -irresolute function.

Theorem 4.16 Suppose that $f:(X,\tau)\to(Y,\sigma)$ is contra $\hat{\omega}$ -irresolute map. Then the following statements hold.

i) If $g:(Y,\sigma)\to(Z,\eta)$ is contra $\hat{\omega}$ -irresolute map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is $\hat{\omega}$ -irresolute map.

ii) If $g:(Y,\sigma)\to(Z,\eta)$ is $\hat{\omega}$ -irresolute map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is contra $\hat{\omega}$ -irresolute map.

iii) If $g:(Y,\sigma)\to(Z,\eta)$ is contra $\hat{\omega}$ -continuous map, then $g\circ f:(X,\tau)\to(Z,\eta)$ is $\hat{\omega}$ -continuous map.

Proof. i) Let V be any $\hat{\omega}$-open subset of (Z,η). Since g is contra $\hat{\omega}$-irresolute map, $g^{-1}(V)$ is $\hat{\omega}$-closed subset of (Y,σ) and since f is contra $\hat{\omega}$-irresolute map, $f^{-1}(g^{-1}(V)) = (g \circ f)(V)$ is $\hat{\omega}$-open subset of (X,τ). Thus, $g \circ f$ is $\hat{\omega}$-irresolute map.

ii) Let V be any $\hat{\omega}$-open subset of (Z,η). Since g is $\hat{\omega}$-irresolute map, $g^{-1}(V)$ is $\hat{\omega}$-open subset of (Y,σ) and since f is contra $\hat{\omega}$-irresolute map, $f^{-1}(g^{-1}(V)) = (g \circ f)(V)$ is $\hat{\omega}$-closed subset of (X,τ). Thus, $g \circ f$ is contra $\hat{\omega}$-irresolute map.

iii) Let V be any open subset of (Z,η). Since g is contra $\hat{\omega}$-continuous map, $g^{-1}(V)$ is $\hat{\omega}$-closed subset of (Y,σ) and since f is contra $\hat{\omega}$-irresolute map, $f^{-1}(g^{-1}(V)) = (g \circ f)(V)$ is $\hat{\omega}$-open subset of (X,τ). Thus $g \circ f$ is $\hat{\omega}$-continuous map.

Theorem 4.17 If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ are such that

i) f is surjective strongly $\hat{\omega}$-closed map and $g \circ f:(X,\tau) \to (Z,\eta)$ is contra $\hat{\omega}$-irresolute map, then g is contra $\hat{\omega}$-irresolute map

ii) g is an injective contra $\hat{\omega}$-irresolute map and $g \circ f:(X,\tau) \to (Z,\eta)$ is strongly $\hat{\omega}$-open map, then f is completely contra $\hat{\omega}$-open map.

iii) g is an injective contra $\hat{\omega}$-irresolute map and $g \circ f:(X,\tau) \to (Z,\eta)$ is completely contra $\hat{\omega}$-open map, then f is strongly $\hat{\omega}$-closed map.

Proof. i) Let V be any $\hat{\omega}$-open subset of (Z,η). Since $g \circ f$ is contra $\hat{\omega}$-irresolute map, $(g \circ f)(V) = f^{-1}(g^{-1}(V))$ is $\hat{\omega}$-closed subset of (X,τ). Since f is surjective and strongly $\hat{\omega}$-closed map, $g^{-1}(V) = f(f^{-1}(g^{-1}(V)))$ is $\hat{\omega}$-closed subset of (Y,σ). Thus, g is contra $\hat{\omega}$-irresolute map.

ii) Let V be any $\hat{\omega}$-open subset of (X,τ). Since $g \circ f$ is strongly $\hat{\omega}$-open map, $(g \circ f)(V) = g(f(V))$ is $\hat{\omega}$-open subset of (Z,η). Since g is an injective contra $\hat{\omega}$-irresolute map $f(V) = g^{-1}(g(f(V)))$ is $\hat{\omega}$-closed subset of (Y,σ). Thus, f is completely contra $\hat{\omega}$-open map.

iii) Let V be any $\hat{\omega}$-closed subset of (X,τ). Since $g \circ f$ is completely contra $\hat{\omega}$-open map, $(g \circ f)(V) = g(f(V))$ is $\hat{\omega}$-open subset of (Z,η). Since g is an injective contra $\hat{\omega}$-irresolute map, $f(V) = g^{-1}(g(f(V)))$ is $\hat{\omega}$-closed subset of (Y,σ). Thus, f is strongly $\hat{\omega}$-closed map.

5. Applications

Theorem 5.1 If (X, τ) is a submaximal space, then every $\hat{\omega}$-open subset of (X, τ) is open in (X, τ).

proof. Suppose that (X, τ) is a submaximal space and V be any $\hat{\omega}$-open subset of (X, τ). By [5] Proposition 3.5, every $\hat{\omega}$-open set is pre-open. By [6] Theorem 4, every pre-open set is open in submaximal space. Therefore, V is $\hat{\omega}$-open subset of (X, τ).

Remark 5.2 Converse of Theorem 5.1 is not always true from the following example.

Example 5.3 Let $X = \{a, b, c\}$ and $\tau = \{ \phi, \{a\}, X\}$. Then the set of all $\hat{\omega}$-open subsets of (X, τ) are $\{ \phi, \{a\}, X\}$. Therefore, every $\hat{\omega}$-open subset of (X, τ) is open in (X, τ) which is not a submaximal space.

Lemma 5.4 In a semi regular space (X, τ), every open subset of (X, τ) is $\hat{\omega}$-open subset of (X, τ).

proof. By [2], every open subset is δ-open subset of (X, τ) and by [5] Proposition 3.2, every δ-open subset is $\hat{\omega}$-open subset of (X, τ). Therefore, every open subset of (X, τ) is $\hat{\omega}$-open subset of (X, τ).

Remark 5.5 Converse of Lemma 5.4, is not always possible from the Example 5.3.

Theorem 5.6 If (X,τ) is submaximal then every contra $\hat{\omega}$ closed map is completely contra $\hat{\omega}$ closed (resp.strongly contra $\hat{\omega}$ closed) map.

Proof. Let F be any $\hat{\omega}$ (resp. a)-closed subset of a submaximal space (X,τ). By Theorem 5.1, F is closed subset of (X,τ) and since f is contra $\hat{\omega}$ closed map, then $f(F)$ is $\hat{\omega}$ open subset of (Y,σ). Therefore, f is completely contra $\hat{\omega}$ closed (resp.strongly contra $\hat{\omega}$ closed) map.

Theorem 5.7 Every strongly contra $\hat{\omega}$-quotient map is contra $\hat{\omega}$-quotient map, provided that both domain and co-domain are semi-regular.

Proof. Let $f:(X,\tau) \to (Y,\sigma)$ be a strongly contra $\hat{\omega}$-quotient map and V be any open subset of (Y,σ). By hypothesis, $f^{-1}(V)$ is $\hat{\omega}$-closed subset of (X,τ). Therefore, f is contra $\hat{\omega}$-continuous map. Suppose that F is any subset of Y such that $f^{-1}(F)$ is closed subset of (X,τ). Since X is semi-regular, by Lemma 5.4, $f^{-1}(F)$ is

$\hat{\omega}$-closed subset of (X,τ). Since f is strongly contra $\hat{\omega}$-quotient map, F is open in the semi-regular space (Y,σ) and by Lemma 5.4, V is $\hat{\omega}$-open subset of (Y,σ).

Theorem 5.8 Every contra $\hat{\omega}$-quotient is strongly contra $\hat{\omega}$-quotient map, provided that both domain and co-domain are submaximal.

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a contra $\hat{\omega}$-quotient map and V be any open subset of (Y,σ). By hypothesis, $f^{-1}(V)$ is $\hat{\omega}$-closed subset of (X,τ). Suppose that F is any subset of Y such that $f^{-1}(F)$ is $\hat{\omega}$-closed subset of (X,τ). Since X is submaximal, by Theorem 5.1, $f^{-1}(F)$ is closed subset of (X,τ). By hypothesis, F is $\hat{\omega}$-open subset of the submaximal space (Y,σ) and by Theorem 5.1, F is open subset of (Y,σ).

Theorem 5.9 Let (Y,σ) be a submaximal space. If $f:(X,\tau)\to(Y,\sigma)$ is surjective,contra $\hat{\omega}$-continuous and completely contra $\hat{\omega}$-closed map, then f is strongly contra $\hat{\omega}$-quotient map.

Proof. Let V be any open subset of (Y,σ). Since f is contra $\hat{\omega}$-continuous map, $f^{-1}(V)$ is $\hat{\omega}$-closed subset of (X,τ). Finally,suppose that F is any subset of Y such that $f^{-1}(F)$ is $\hat{\omega}$-closed subset of (X,τ). Since f is a surjective completely contra $\hat{\omega}$-closed map, $f(f^{-1}(V))=V$ is $\hat{\omega}$-open subset of (Y,σ) and since Y is submaximal, V is open subset of (Y,σ). Thus, f is strongly contra $\hat{\omega}$-quotient map.

Theorem 5.10 (Pasting Lemma for Contra $\hat{\omega}$-irresolute Mappings.)

Let (X,τ) be a topological space such that $X=A\cup B$ where A and B are both pre open and closed subsets of Let f: $(A, \tau|A) \to (Y, \sigma)$ and f: $(B, \tau|B) \to (Y, \sigma)$ be contra ˆω-irresolute functions such that f(x) = g(x) for every $x \in A \cap B$. Then f and g combine to give a contra $\hat{\omega}$-irresolute function $(f\nabla g)(x)=f(x)$ for every $x\in A$ and $(f\nabla g)(y)=g(y)$ for every $y\in B$.

Proof. Let F be $\hat{\omega}$-open subset of (Y,σ). Then $(f\nabla g)^{-1}(F)=f^{-1}(F)\cup g^{-1}(F)$. By hypothesis, $f^{-1}(F)$ and $g^{-1}(F)$ are $\hat{\omega}$-closed subsets of $(A,\tau|_A))$ and $(B,\tau|_B)$ respectively. By [5] Theorem 6.30 (i) (a), $f^{-1}(F)$ and $g^{-1}(F)$ are $\hat{\omega}$-closed subsets of (X,τ). By [5] Theorem 4.11, $f^{-1}(F)\cup g^{-1}(F)$ is $\hat{\omega}$-closed subset of (X,τ). Therefore, $(f\nabla g)$ is contra $\hat{\omega}$-irresolute function.

References

1. Dontchev J. (1996) *Contra-continuous functions and strongly closed spaces*, Int.J.Math.Mat.Sci., 19, 303- 10.
2. Dontchev J. and Ganster M. (1996) *On δ-generalized closed sets and $T_{3/4}$-spaces*, Mem.Fac.Sci.Kochi Univ., (Math.), 17, 15-31.
3. Erdal Ekici. (2008) *On open sets A^*-sets and Decompositions of continuity and super continuity,* Annales Univ. Sci. Budapest,Rolando Eotvos,Sect.Math., 51(1), 39-51.
4. El-Monsef M.E. ; Rose Mary S. ; Lellis Thivagar M. (2007) *On $\alpha\hat{g}$-closed sets in topological spaces",* Assiut University Journal of Mathematics and Computer Science, Vol -36, 43-51.
5. Maki H.; Lellis Thivagar M. ; Santhini S. & Anbuchelvi M. (2013) *New sort of generalized closed sets*, (submitted)
6. Reilly I.L. and Vamanamurthy M.K. (1990) *On Some Questions Concerning Pre-open sets*, Kyungpook Mathematical Journal, 30, No-1, 87-93.
7. Thivagar M.L. (1991) *A note on Quotient Mappings,* Bull.Malaysian Math.Soc.(Second series), 14, 21-30.
8. Thivagar M.L. and Anbuchelvi M. Remarks on non-contra continuity (accepted) International Journal of Advanced Computer and Mathematical Sciences.