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Use of Some Exponential Smoothing Models in Forecasting Some Food Crop Prices in the Upper East Region of Ghana

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Abstract
The study was designed to compare the performance of Holt-Winters multiplicative method with Double exponential smoothing method in forecasting future prices of some selected food crops in the Upper East Region of Ghana and also to examine the trend or direction of movement of the prices.
The conclusion drawn from the study was that the prices have been rising since January 1992, decreasing sometimes but not below the January 1992 prices. This is an indication that all things being equal, the prices of the selected food crops will keep rising (rising trend).
Results from the study revealed that the double exponential smoothing performed better, in four of the five selected food crops in which trend was present, than the Holt-Winters multiplicative method. That is the double exponential model forecasted prices which were much closer to the observed values than the Holt-Winters model. However in the case of the prices of groundnut in which both seasonality and trend were present the Holt-Winters model performed better than the double exponential smoothing. This is a confirmation of the norm that the Holt-Winters model performs better when both trend and seasonality are present whilst the double exponential smoothing performs better when trend is present in a set of data (Minitab User's Guide 2,).
Results from the study also showed that the double exponential smoothing model performs better when given the optimal values. However the optimal values given by the study lie outside the suggested range (*0.70* and *0.95*) for exponential smoothing methods. The study revealed that in practice the discount factors could lie outside the suggested range for exponential smoothing.
From the study it will be recommended that double exponential smoothing models be used for modelling and forecasting the prices of cereals crops in which trend is present whilst the Holt-Winters multiplicative method is used for the leguminous crops in which both trend and seasonality are present in the Upper East Region of Ghana.

1.1 Introduction
The primacy of food production in the sustenance of the livelihood of the majority of the people in Ghana is a truism as about 49.1% of the 24 million live in rural areas where they depend directly or indirectly on agriculture for their livelihood (2010 PHC). However, Ghana like many other African countries seems to be afflicted with persistent food fluctuations over the last two decades.
Food is overwhelmingly the most important item in the household budget in the country. It is over 50% in the consumer price index. Substantial increases in food prices were first noted with great concern in the late 1940's where between 1948 and 1952 the food price index almost doubled. Thus, the food bill in the household expenditure has gone up disproportionately and that much more of the consumer's total income is now spent on food.
It is sometimes alleged that prices of food crops in the Upper East Region are unduly depressed in the post harvest period and that they rise to excessive heights in the period just before harvest. This large increase in prices is attributed to heavy storage losses, exploitative speculation and simple improvidence. To the extent that it is true, it may reduce farmers' incomes and thus their incentive to produce and even provoke actual food shortage.
Severe hunger may result when farmers, who have sold their crops in the glutted post harvest markets, find it necessary to buy them back at two, three, or four times the price in order to feed their families while waiting for the new crops to mature.
The need for a scientific research that will provide evidence to show the direction of price movement cannot be overemphasized. It is alleged that the Holt-Winters method performed consistently in recent forecasting composition when compared with other more sophisticated methods (Makridakis et al, 1982) and that it performs better when both seasonality and trend are present in a given set of data. How far true this statement is compared

to Double Exponential smoothing is what this research is going to look at. It is also said that the Holt-Winters method is suitable for producing short-term forecasting for sales or demand time series. The study seeks to use sales (prices of food crops) data as a case study to compare the performance of these two methods based on the values of their Mean Absolute Percentage Error (MAPE), Mean Squared Deviation (MSD) and Mean Absolute Deviation (MAD) and to confirm or otherwise the validity of the statement that Holt-Winters model performs better when both seasonality and trend are present.

The study seeks to examine the trend of the prices of the food crops under study within the study period (1992-2000) and try to forecast the prices for the next twelve months. It must be emphasized that one needs not expect accurate forecast in this kind of exercise. Even with the original data, there have been serious outliers, which only distort the trend.

Although the researcher has chosen prices of food crops in the Upper East Region as a case study, the prediction of prices of food crops are by no means peculiar to that Region alone but experienced even to a greater extent in many parts of Ghana. It is our hope that the exposition of this problem and perhaps its effects will serve as a basis for further research so that more comprehensive reports and recommendations can be made to the Economic Planners of Ghana to prevent the occurrence of such situations in the future.

Progress in food production is an essential condition that has to be filled to prevent the deterioration of the food situation in the region and the country at large. Accordingly, the study reviews the prices of some of the food crops in the Region. Primarily the study is to afford any worker in the region the opportunity of knowing what to expect of the prices of the food crops and thus determine methods of adjusting to the situation prevailing if the factors remain the same.

The main aim of this paper is to construct a statistical model for analyzing and forecasting prices of food crops in the Upper East of Ghana. In doing this the study will examine the trends in the prices of food crops and their prospects for future, it also seeks to build an exponential smoothing model to analyze the prices of food crops in the Upper East Region of Ghana and to compare the efficiency of Holt-Winters Multiplicative and Double Exponential Smoothing models in forecasting prices of food crops.

1.2 Research Settings

The Upper East Region falls within the guinea savanna zone of Ghana. In general, rainfall is scanty and there is only one main rainy season with a mean of 83.4 mm (Meteorological Service Department rainfall reports) and a long dry season. This sets a limitation on the type and choice of staple food. The Region supports drought-resistant crops. The region has a population of 1,046,545 (2010 population and housing census).

The collection of data on market prices of foodstuffs in the region is mainly the duty of the Policy Planning, Monitoring and Evaluation Division (PPMED) of the Ministry of Food and Agriculture (MoFA) that has its Regional Office in Bolgatanga. PPMED has branches in all the districts of the region, with qualified staff who supervise the collection of data and the general administration of the offices. The division collects data on prices of almost every food crop cultivated within each locality, some of which are millet, maize, sorghum, rice and groundnuts.

Poverty and food insecurity among other things, are chronic in the region and almost all developmental projects aim at boosting the production of food crops. The researcher has decided to look at the prices of food crops because prices affect production, marketing, and consumption as well.

The region consists of thirteen (13) district markets from which data were collected. These are Bawku Municipal, Bawku West, Bolgatanga, Bongo, Kasena/Nankani Municipal, Kasena/Nankani West, Fumbisi, Binduri, Builsa, Pusiga,Garu/Timpani, Talinsi/Nabdam, and Nagodi districts. The provision of sufficient food to meet the biological needs of man is a matter of concern to humanity. The history of man has therefore been replete with the quest for food. While the scarcity of clothing and shelter are likely to cause misery, the inadequacy of food has more severe consequences of under nutrition and malnutrition and in times of acute shortages leads to sickness and premature death.

It was also frequently asserted that the actual prices at which foodstuffs were sold could only be determined by going through the bargaining process thought to determine the individual's terms of each transaction.

Despite all of these alleged deficiencies in the series, the results of the study were not expected to be biased since only average prices were used.

1.3 Materials and Methods

Data for the study were taken from the Policy Planning, Monitoring and Evaluation Division (PPMED) of the Ministry Food and Agriculture (MoFA) monthly reports.

Data on the prices of the food crops in the Upper East Region were available on monthly bases and were also categorized into Districts and so the Regional prices were found by simply finding the averages over the District prices.

After observing that trend and seasonality were present, the Holt-Winters and Double exponential smoothing methods were then employed to analyze and forecast the prices of the various food crops under study.

Both the Double exponential smoothing and Winters multiplicative models were used and based on the values of the Mean Squared Deviation (MSD), the Mean Absolute Percentage Error (MAPE) and Mean Absolute Deviation (MAD), the appropriate model was then chosen.

As a diagnostic check of the model, the sample autocorrelations of the one-step-ahead forecast errors were calculated and compared with their standard errors. Models whose autocorrelations fell within two standard errors were accepted as adequate models for the data. Data for 2001 were held back to check the performances of the chosen models.

A number of smoothing parameters were chosen for the Winter's model whilst the Double exponential smoothing was allowed to use the optimal values given by MINITAB and the set of parameters that gave the smallest sum of squared Errors (SSE) was chosen.

In addition to checking for correlation among the forecast errors, forecast biases were also checked by calculating the mean and standard errors of the one-step-ahead forecasts errors. If the mean fell within ± 2 standard errors, then the forecasts were said to be unbiased. All these were done with the help of the computer software– MINITAB

1.4 Theoretical Reflection of Exponential Smoothing

Forecasting involves making the best possible judgment about some future event. In today's rapidly changing business world such judgments can mean the difference between success and failure. It is no longer reasonable to rely on intuition, or one's feel for the situation, in projecting future sales, inventory needs, personnel requirements, and other important economic or business variables.

The ability to form good forecasts has been highly valued throughout history. Even today various types of fortune-tellers claim to have the power to predict future events. Frequently their predictions turn out to be false. However, occasionally their predictions come out true; apparently often enough to secure a living for these forecasts.

Since future events involve uncertainty, the forecasts are usually not perfect. The basic objective of forecasting is to produce forecasts that are seldom incorrect and that have small forecast errors. In business, industry, economic, and government, policy makers must anticipate the future behavior of many critical variables before they make decisions.

Their decisions depend on forecasts, and they expect these forecasts to be accurate; a forecast system is needed to make such predictions. Each situation that requires a forecast comes with its own unique set of problems and the solutions to one are by no means the solution in another situation. However, certain general principles are common to most forecasting problems and should be incorporated into any forecast system.

1.4.1 Classification of Forecast Methods

Forecast methods may be broadly classified into qualitative and quantitative techniques.

Qualitative or subjective forecast methods are intuitive, largely educated guesses that may not depend on past data. Usually someone else cannot reproduce these forecasts, since the forecaster does not specify explicitly how the available information is incorporated into the forecast. Even though subjective is a non-rigorous approach, it may be quite appropriate and the only reasonable method in certain situation. Forecasts that are based on mathematical or statistical methods are quantitative. Once the underlying model or technique has been chosen corresponding forecasts are determined automatically, they are fully reproducible by any forecaster. Quantitative methods or models can be further classified as deterministic or Probabilistic (stochastic or statistical). In deterministic models the relationship between the variable of interest, Y, and the explanatory or predictor variables $x_1, x_2, \ldots x_n$ is determined exactly: $Y = f(x_1, x_2, \ldots x_n, \alpha_1, \alpha_2, \ldots \alpha_m)$......... 1

The function f and the coefficients $\alpha_1, \alpha_2, \ldots \alpha_m$ are known with certainty. Examples of deterministic relationship are laws in the physical sciences. However in the social sciences, the relationships are usually stochastic.

Measurement errors and variability from other uncontrolled variables introduce random (stochastic) components. This leads to probabilistic or stochastic models of the form,

$Y = f(x_1, x_2, \ldots x_n, \alpha_1, \alpha_2, \ldots \alpha_m) + noise$ 2

where the noise or error component is a realization from a certain probability distribution.

Frequently the functional form f and the coefficients are not known and have to be determined from past data. Usually the data occur in time-ordered sequences referred to as time-series: Statistical models in which the available observations are used to determine the model form are also called empirical which will be used in this write up. In this study we shall look at the single-variable prediction method where we use the past history of the series, z_t where t is the time index and extrapolate it into the future.

1.5 Criteria for choosing a forecast method

The most important criterion for choosing a forecast method is its accuracy or how closely the forecast predicts the actual event. If we denote actual observation at time t with z_t and its forecast which uses the information up to and including time $t-1$, with $z_{t-1}(1)$ then the objective is to find a forecast such that the future forecast error $z_t - z_{t-1}(1)$ is as small as possible. But this is the future forecast error and since z_t has not yet been observed, its value is unknown; we can talk only about its expected value, conditional on the observed history up to and including time t-1. If both negative (over prediction) and positive (under prediction) forecast errors are equally undesirable, it would make sense to choose the forecast such that the mean absolute error, $E|z_t - z_{t-1}(1)|$, or the mean square error $E\{z_t - z_{t-1}(1)\}^2$ is minimized. The forecasts that minimize the mean square error are called minimum mean square error (MMSE) forecasts. When it is used, it leads to simpler mathematical **solutions.**

1.6 Exponential Smoothing

It is a method that is used to estimate the parameters and derive future forecasts for models with stable, uncorrelated error but whose trend components change with time.

The influence of the observations on the parameter estimates diminishes with the age of the observations. Special cases lead to simple, double, and triple exponential smoothing.

This forecasting procedure that was first suggested by C.C.Holt in about 1958, should only be used in its basic form for non-seasonal time series showing no system trend. Of course many time series that arise in practice do contain a trend or seasonal pattern, but these effects can be measured and removed to produce a stationary series. This turns out that, adaptations of exponential smoothing are useful for many types of time series (Bowerman & O'Conell, 1979).

1.7 The Constant mean model

To introduce smoothing methods for the prediction of non-seasonal series, let us consider the general model $z_t = f(t,\alpha) + \varepsilon_t$. The assumption is that the observations are generated from

$z_t = \alpha + \varepsilon_t$ ……..3,

where α is a constant mean level and ε_t is a sequence of uncorrelated errors with constant variance δ^2. Series which follow this model are characterized by random variation around a constant mean, and (3) is referred to as the constant mean model.

If the parameter α is known, the minimum mean square error forecast of a future observation at time n+l, $z_{n+l} = \alpha + \varepsilon_{n+l}$, is given by

$z_n(1) = \alpha$ ………………………………………………………………………………………. 4.

This forecast is unbiased in the sense that the forecast error $z_{n+l} - z_n(l) = \varepsilon_{n+l}$ has expectation $E[z_{n+l} - z_n(l)] = 0$. Its mean square error is given by $E[z_{n+l} - z_n(l)]^2 = E(\varepsilon^2{}_{n+l}) = 0$.

If we again assume that the errors are normally distributed $100(1-\lambda)$ % prediction intervals for a future realization is given by $[\alpha - \mu_{\lambda/2}\sigma, \alpha + \mu_{\lambda/2}\sigma]$, where $\mu_{\lambda/2}$ is the $100(1-\lambda/2)$ % point of the standard normal distribution. λ is the significant level.

If the parameter α is unknown, it is estimated from past data $(z_1, z_2, ... z_n)$ and replace α in (3.4) by its least squares estimate. The least squares estimate is given by the sample mean $\hat{\alpha} = \bar{z} = 1/n\sum_{t=1}^{n} z_t$ and the l–step-ahead forecast of z_{n+l} from time origin is given by

$\hat{z}_n(l) = \bar{z}$ ….. ……………………………………………………………………………..5

These forecasts are the same for all l . They are also unbiased with mean square error $E[z_{n+l} - \hat{z}_n(l)]^2 = \sigma^2(1+1/n)$ ………………………………………………………………6

An unbiased estimate of σ^2 can then be calculated from $\hat{\sigma}^2 = 1/n - \sum_{t=1}^{n}(z_t - \bar{z})^2$ …………...7

A $100(1-\lambda)\%$ prediction interval for a future realization at time $n+1$ is given by $[\bar{z} - t_{\lambda/2}(n-1)\hat{\sigma}(1+1/n)^{1/2}, \bar{z} + t_{\lambda/2}(n-1)\hat{\sigma}(1+1/n)^{1/2}]$, where $t_{\lambda/2}(n-1)$ is the

$100(1-\lambda/2)\%$ point of a t distribution with $n-1$ degrees of freedom.

1.8 Updating forecasts

The forecast at time origin $n+1$ can be written as,

$$\hat{z}_{n+1}(1)=\frac{1}{n+1}(z_1+z_2+...+z_n+z_{n+1})=\frac{1}{n+1}[z_{n+l}+n\hat{z}_n(1)]=\frac{n}{n+1}\hat{z}_n(1)+\frac{1}{n+1}z_{n+l}...... \quad 8$$

Or

$$\hat{z}_{n+l}(1)=\frac{1}{n+1}[z_{n+1}-\hat{z}_n(1)+(n+l)\hat{z}_n(1)]=\hat{z}_n(1)+\frac{1}{n+1}[z_{n+1}-\hat{z}_n(1)] \quad ....\ ....9$$

Equation (8) shows how forecast from time origin $n+1$ can be expressed as a linear combination of the forecast from origin n and the most recent observation. Since the mean α in the model (3) is assumed constant, each observation contributes equally to the forecast. Alternatively, equation (9) expresses the new forecast as the previous forecast, corrected by a fixed fraction of the most recent forecast error. For the computation of the new forecast, only the last observation and the most recent forecast error have to be stored.

1.9 Checking model adequacy

In the constant mean model it is assumed that the observations vary independent around a constant level. To investigate whether this model describes past data adequately, one should always calculate the sample autocorrelations of the residuals. For the constant mean model the residuals are $z_t-\bar{z}$. The sample autocorrelations are given by $r_k=\frac{\sum_{t=k+1}^{n}(z_t-\bar{z})(z_{t-k}-\bar{z})}{\sum_{t=1}^{n}(z_t-\bar{z})^2}$,

k = 1,2,…

To judge their significance, one should compare the estimated autocorrelations with their approximate standard error $n^{-1/2}$. If the sample autocorrelations exceed twice their standard error $(i.e.|r_k|>2n^{-1/2})$, one can conclude that the observations are likely to be correlated and that a constant mean may not be appropriate.

1.10 Locally Constant Mean Model and Simple Exponential Smoothing

The model in equation (3) assumed that the mean is constant over all time periods. As a result, in the forecast computations each observation carries the same weight.

In many instances, however, the assumption of a time constant mean is restrictive, and it would be more reasonable to allow for a mean that moves slowly over time. In such a case it is reasonable to give more weight to the more recent observations and less to the observations in the distant past.

If one chooses weight that decreases geometrically with the age of the observations, the forecast of the future observation at time n+l can be calculated from, $\hat{z}_n(l)=c\sum_{t=0}^{n-1}a^t z_{n-t}=c[z_n+az_{n-1}+...+a^{n-1}z_1]$ 10

The constant $a(|a|<1)$ is a discount coefficient. This coefficient, which should depend on how fast the mean level changes, is usually chosen between 0.7 and 0.95; in many applications value of 0.9 is suggested [Brown (1962)]. The factor $c=\frac{(1-a)}{(1-a^n)}$ is needed to normalize the sum of the weight to 1. Since $\sum_{t=0}^{n-1}a^t=\frac{1-a^n}{1-a}$, it follows that $c\sum_{t=0}^{n-1}a^t=1$

If n is large, then the term a^n in the normalizing constant c goes to zero, and exponentially weighted forecasts can be written as

$$\hat{z}_n(l)=(1-a)\sum_{j\geq 0}a^j z_{n-j}=(1-a)[z_n+az_{n-1}+a^2z_{n-2}+...]... \ ....11$$

The forecast are the same for all lead times l. The coefficient $b=1-a$ is called the smoothing constant and is usually chosen between 0.05 and 0.30. The expression

$S_n = S_n^{[1]} = (1-a)[z_n + az_{n-1} + a^2 z_{n-2} + ...] = b[z_n + (1-b)z_{n-1} + (1-b)^2 z_{n-2} + ...]$ …………12 is called the smoothed statistic or the smoothed value. The last available smoothed statistic S_n serves as forecast for all future observations, $\hat{z}_n(l) = S_n$. Since it is an exponentially weighted average of previous observations, this method is called simple exponential smoothing.

1.11 Updating forecasts

The forecast in (11) or equivalently the smoothed statistic in (12) can be updated in several alternative ways. By simple substitution, it can be shown that

$$S_n = (1-a)z_n + aS_{n-1} = \hat{z}_n(1) = (1-a)z_n + a\hat{z}_{n-1}(1) \quad \text{……………………………………..13}$$

Or

$$S_n = S_{n-1} + (1-a)[z_n - S_{n-1}] = \bar{z}_n(1) = \hat{z}_{n-1}(1) + (1-a)[z_n - \hat{z}_{n-1}(1)] \quad \text{…………………………..14}$$

Equations (13) and (14) show how the forecasts can be updated after a new observation has become available. Equation (13) expresses the new forecast as a combination of the old forecast and the most recent observation. If a is small, more weight is given to the last observation and the information from previous periods is heavily discounted. If a is close to 1, a new observation will change the old forecast only very little. Equation (14) expresses the new forecast as the previous forecast corrected by a fraction ($b = 1-a$) of the last forecast error.

1.12 Implementation of Simple Exponential Smoothing

Equation (13) can be used to update the smoothed statistics at any time period t. In practice, one starts with the first observation z_1 and calculate $S_1 = (1-a)z_1 + aS_0$. This is then substituted into (13) to calculate $S_2 = (1-a)z_2 + aS_1$ and the smoothing is continued until S_n is reached. This is the procedure that is adopted in practice.

To carry out these operations one needs to know

(i) a starting value S_0 and

(ii) a smoothing constant $b = 1-a$.

1.13 Initial Value for S_0

By repeated application of equation (13), it can be shown that $S_n = (1-a)[z_n + az_{n-1} + ... + a^{n-1} z_1] + a^n S_0$. Thus, the influence of S_0 and S_n is negligible, provided n is moderately large and a smaller than 1.

The simple arithmetic average of the historical data $(z_1, z_2, ...z_n)$ is taken as the initial estimate of S_0. Such a choice has also been suggested by Brown (1962) and Montgomery and Johnson (1976). The arithmetic average will perform well, provided that the mean level changes only slowly.

Alternative solutions to choosing the initial value S_0 have been suggested by Makridakis and Wheelwright (1978) where they use the first observation as initial smoothed statistic $S_0 = z_1$, which implies that $S_1 = (1-a)z_1 + aS_0 = z_1$. This choice is preferable if the level changes rapidly (b close to 1 or a close to 0).

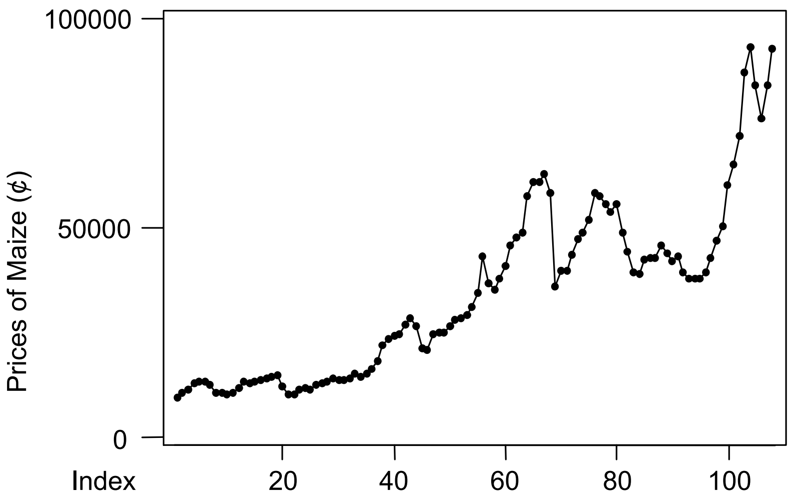
1.14 Results and Discussion

1.14.1 Analysis of Prices of Maize

1.14.1a Exploratory Analysis

Figure 1 below shows a graph of the prices of maize. Data are monthly prices, from January 1992 through December 2000. A visual analysis (of the prices of maize) shows that there is an upward movement of the data, which appears to be accelerating (becoming increasingly steep).

Fig 1: Time Series Plot for Prices of Maize; 1992-2000



A look at the correlation between successive months, shows that the autocorrelation for a k-period lag, r_k, decline towards zero slowly, an indication of the presence of trend and nonstationarity in the data. To determine whether the autocorrelation at lag k is significantly different from zero, the following hypothesis test and rule of thumb were used:

$$H_0 : \rho_k = 0$$

$$H_1 : \rho_k \neq 0$$

For any k, H_0 is rejected if $|r_k| \succ 2/\sqrt{n}$, where n is the number of observations.

In this case 108 observations were used, from January 1992 through December 2000. Thus $2/\sqrt{n} = 2/\sqrt{108} = 0.192 \cong 0.20$. Since all of the autocorrelation coefficients in Table 1.1 are greater than 0.20, we can conclude by our rule of thumb that they are all significantly different from zero. Therefore, we have additional evidence of the presence of trend in the prices of maize data.

From the autocorrelation function of the raw data, the values of r_{12} (0.46) and r_{24} (0.32) are all significantly different from zero. This is an evidence of the presence of seasonal pattern in the data.

Table 1.2 Autocorrelation Function (ACF) of the first difference of prices of MAIZE

Lag	Autocorrelation	Lag	Autocorrelation	Lag	Autocorrelation
1	0.27	10	0.00	19	-0.20
2	-0.07	11	0.03	20	-0.07
3	0.00	12	0.23	21	-0.06
4	0.02	13	0.01	22	-0.10
5	-0.04	14	0.04	23	0.12
6	-0.05	15	-0.02	24	0.22
7	-0.16	16	-0.07	25	0.03
8	-0.04	17	-0.10	26	-0.04
9	0.04	18	-0.12		

From this exploratory analysis we can conclude that there is trend and seasonality, so a possible method of forecasting the future prices of maize is the Winters multiplicative exponential smoothing.

14.1b Exponential smoothing for prices of Maize

The researchers decided to use double exponential smoothing to forecast the prices of maize for the fiscal year 2001 because of the growing trend pattern of the series. The two smoothing parameters are $\alpha = 1.368$ and $\gamma = 0.026$. These smoothing parameters were chosen by MINITAB. However these parameters are outside the suggested range for exponential smoothing this goes to support Brown (1962) that in practice it is possible for the smoothing constants to lie outside the suggested range. The large values of the parameters indicates that the model depends on the last two observations and can change quickly, meaning that the trend changes rapidly with each new observation.

A graph of double exponential smoothing was constructed with smoothing parameters $\alpha = 1.368$ and $\gamma = 0.026$. The graph showed that at the given parameters, the predicted values are much closer to the observed values with MAPE of 7, MAD 2343 and MSD 15758356.

The forecasted values were then calculated as shown below.

The smooth value for February 1993 is

$$F_{14} = \alpha X_{14} + (1-\alpha)(F_{13} + T_{13})$$

= 1.368(13000) + (1-1.368)(13631.47)

= 12,782

The trend estimate is

$$T_{14} = \gamma(F_{14} - F_{13}) + (1-\gamma)T_{13}$$

= 0.026(12,782–13599.6) + 0.974(31.87)

= 9.784

The forecast is

$$H_{14} = F_{14} + 12T_{14}$$

= 12,782+12*9.784

= 12,899.4

Autocorrelation Function of the residuals reveals that all but lag 12 lie within 2σ .

For the Holt-Winters multiplicative model, which uses three smoothing parameters, α' which smoothes the level, γ the trend, and δ the seasonal, the researchers tried several smoothing parameters on the maize data, the parameters that gave the best results were α' =0.42, γ =0.20 and δ =0.10.

These smoothing parameters gave predicted values that were much closer to the actual values. These values gave a MAPE of 9, MAD of 3157 and MSD of 21184243.

Let us now forecast the prices of maize using the Winters multiplicative model with parameters α' =0.42, γ =0.20 and δ =0.10.

The method used in calculating the forecast price of maize for February 1993 is as shown.

The smoothed value for February 1993 is

$$F_{14} = 0.42 * \frac{13000}{0.89042} + 0.58(9152.3 + 328.19)$$

= 11601.35

The seasonal estimate is

$$S_{14} = 0.1 * \frac{13000}{11601.35} + 0.9 * 0.89409$$

= 0.91727

The trend estimate is

T_{14} = 0.2(11601.35–9152.3) + 0.8*328.19

= 489.81 + 262.552

= 752.362.

The forecast for February 1993 is

H_{14} = (11601.35+12*752.362) 0.91727

= 18,923.0

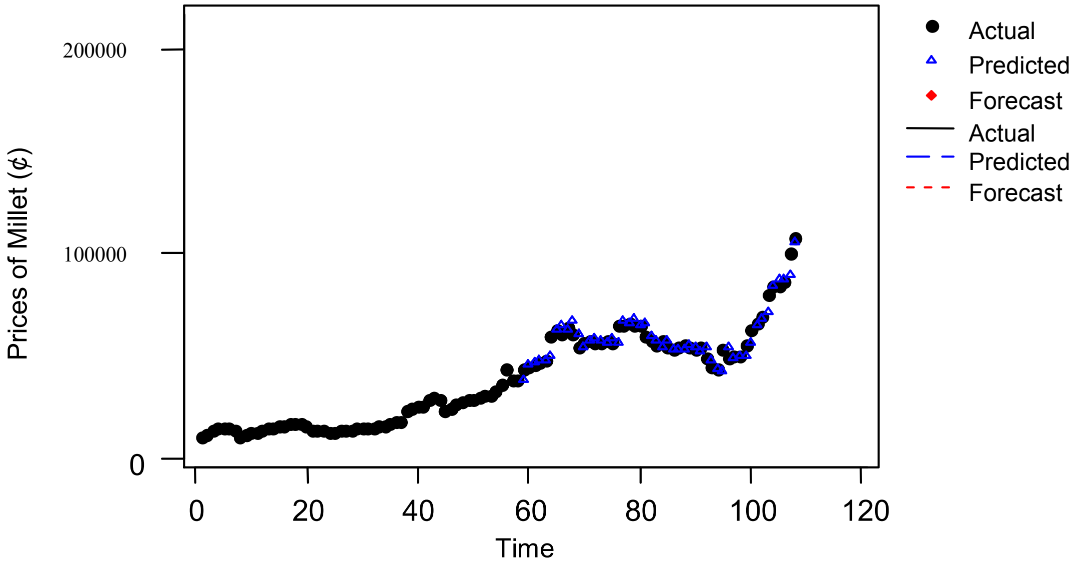
The three accuracy measures, MAPE, MAD, and MSD were 7.0, 2343, and 15758356 respectively for double exponential smoothing fit, compared to 9, 3157, and 21184243, respectively for Winters multiplicative model. Because these values are smaller for double exponential smoothing, we can judge that the double exponential smoothing provides a slightly better fit to the prices of maize data when optimal weights are used.

Also comparing the forecast values to the observed values, forecasts from the double exponential smoothing are much closer to the observed values than those from the winter's model. Even though prices of foodstuff are said to be seasonal, it is seen from the analysis that prices of maize are better forecasted with double exponential smoothing model than the Holt-Winters multiplicative model.

1.14.2a Analysis of the Prices of Millet

Like the maize prices, data for the prices of millet are monthly prices from January 1992 through December 2000. Similarly, a look at the data shows an upward trend in the prices, a trend that is not so much different from that of the maize. Correlation between successive months indicates that the autocorrelation for r_k does not decline towards zero quickly. This shows that the prices of millet within the study period are not stationary and also indicates the presence of trend in the data.

The figure below is a graphical representation of double exponential smoothing with smoothing parameters $\alpha = 1.084$ and $\gamma = 0.150$.



From the graph it is seen that at the given parameters the predicted values are much closer to the observed values with MAPE of 5, MAD 1951 and MSD 9638595.

The procedure for calculating the forecasted values is as shown below.

The smoothed value for February 1993 is:

$F_{14}= \alpha X_{14} + (1- \alpha)(F_{13}+T_{13})$

= 1.083(14694) + (1-1.083)(14460+410.42)

= 14679.35

For the trend estimate, we have

$T_{14}= \gamma(F_{14}-F_{13}) + (1- \gamma) T_{13}$

= 0.150(1479.35-14460) + 0.850*410.42

= 381.757

The forecast for February 2010 is

$H_{14}= F_{14}+12T_{14}$

= 14679.35+12*381.75

= 19,260.43

Autocorrelation functions of the residuals reveal that all but r_{12} are not significantly different from zero. Thus the errors lie within 2σ indicating that the model is a good one. Hence the forecast errors at one-step-ahead are uncorrelated.

From the descriptive statistics, the trend has a mean of 665 with standard deviation of 1109 indicating is a positive trend, which means that all things being equal the prices of millet will keep rising over the years.

Let us analyze the millet data using the Winters exponential model. After trying so many smoothing parameters, the parameters that gave the best results are $\alpha = 0.45$, $\gamma = 0.20$ and $\delta = 0.10$.

The technique in calculating the forecasted values is as shown below.

The smoothing value for February 1993 is

$$F_{14} = 0.45\left(\frac{14694}{0.87882}\right) + (1-0.45)(9502+323.54)$$
$= 7524.06 + 5404.04$
$= 12928.10$

The seasonal estimate is

$$S_{14} = 0.1 * \frac{14694}{12928.10} + 0.9 * 0.87882$$
$= 0.90459$

The trend value is

$T_{14} = 0.2*(12928.10-9502) + 0.8*323.54$
$= 994.052$

The forecast for February 1994 is

$H_{26} = (12928.10+12*994.052)0.90459$
$= 22371.05$

Analysis of the errors or residuals reveals that there is a high significant spike at lags 1, 2, 12, and 24. This shows that at one-step-ahead, the forecast errors are still correlated.
The accuracy measures MAPE, MAD, and MSD were 8, 2741, and 13293433, respectively for the winters compared to 5, 1951 and 9638595 respectively for double exponential smoothing.
On the basis of the values of MAPE, MAD, and MSD, the double exponential smoothing model has smaller values compared to the Winters model. An indication that the double exponential smoothing model provides a slightly better fit to the data than the Winters model.
Again, comparing the forecasted values to the actual values, the values given by the double exponential smoothing model are much closer to the actual values than those given by the Holt-Winters method.

1.14.3 Analysis of the Prices of Groundnut

1.14.3a Exploratory Analysis

Like the other two data sets, prices of groundnut against time also show an upward movement of the prices, indicating a positive trend. The correlation between successive months shows that, the autocorrelation coefficients do not decline towards zero rapidly. This is an evidence of the presence of trend and nonstationarity in the data.
The autocorrelations function of the de-trended data confirms that seasonality is present in the data which is seen in lag 12 and lag 24 where the correlations coefficients are 0.25 and 0.30 respectively.
From the exploratory analysis we can say that the data contain both trend and seasonality.

1.14.3b Exponential Analysis

1.14.3b(i) Double Exponential Smoothing

The two parameters chosen for the double exponential smoothing by Minitab are $\alpha = 1.216$ and $\gamma = 0.026$. Again, this value of α is outside the suggested range for exponential smoothing. Like the other crops, the prices of groundnuts is stochastic as such can change quickly with each new observation. With these parameters the predicted values are much closer to the actual or observed values with we have a MAPE of 9, MAD 6009 and MSD 96346738.
Like the other crops, the procedure used in calculating the forecast for the prices of groundnut is as follows.
The smoothing value for February 1993 is

$F_{14} = 1.216(24321)–0.216(24962–21715)$
$= 24229.52$

The trend estimate is

$T_{14} = 0.26(24229.52–24962)–0.974*217.51.$
$= -230.90$

The forecast for February 1994 is

$H_{26} = (24229.57–12*230.90)$
$= 21458.73$

The autocorrelation coefficients of the residuals show that the residuals all except r_3 and r_{23} lie within 2σ, which means that the prices of groundnut in the Region within the study period are random.
Descriptive statistics also indicate that the trend has a mean of 587.8 and standard deviation of 810.7. That means that all things being equal the prices of groundnut will keep rising.

1.14.3b(ii) Winters Multiplicative model

Analysis of the data using Winters method gave $\alpha = 0.55$, $\gamma = 0.20$ and $\delta = 0.10$ with MAPE of 9, MAD

5800, and MSD 73695273. With these parameters the predicted values are much closer to the observed values.

The forecasted prices for groundnut were calculated with the following methods.
The smoothing value for February 1993 is

$$F_{14} = 0.55(\frac{24321}{0.89048}) + 0.45(16989 + 1227.82)$$
$$= 15021.72 + 819756$$
$$= 23219.29$$

The trend estimate is

$$T_{14} = 0.20(23219.29–16989+(0.8*1227.82)$$
$$= 2228.3$$

The seasonality estimate is

$$S_{14} = 0.1(\frac{24321}{23219.29}) + 0.9 * 0.89048$$
$$= 0.90617$$

The forecast for February 1994 is

$$H_{26} =(23219.29+12*2228.3)*0.90617$$
$$= 45271.24$$

Analysis of the residuals shows that there is high positive spike at lag 1 which is typical of an autoregressive process. This also tells us that the prices of groundnut are random and does not follow any systematic procedure. The accuracy measures, MAPE, MAD and MSD were 9, 6009 and 96346738 respectively for double exponential smoothing fit compared to 9, 5800 and 73695273 for winters method. Because two of the values MAD and MSD are smaller for the Winters method, we can judge that the Winters method is a better fit for the prices of groundnut data. This goes to support the view that if trend and seasonality are both present in a given set of data, the best method is the Holt-Winters method.
Descriptive statistics also reveals that the trend of the prices of groundnut is positive (1346) indicating a growing trend. This means the prices of groundnut will keep rising like the prices of the other crops.

1.15 Conclusion and Recommendation

The study was designed to compare the performance of Holt-Winters multiplicative method with Double exponential smoothing method in forecasting future prices of some selected food crops in the Upper East Region of Ghana. It was also to analyze the prices of the selected food crops and to examine the trend in which the prices of these crops moved over the period.
The study indicates that, there has been a rising trend in each case. Observations from the analysis show that the prices have been rising since January 1992, decreasing sometimes but not below the January 1992 prices.
The study was also to forecast the prices for the next twelve months. The Holt-Winters multiplicative and double exponential smoothing methods were used. In cases where trend was present, the double exponential smoothing gave a much closer approximation to the observed values than the Holt-Winters method as seen in the prices of Maize and Millet. However in the case of groundnut where both trend and seasonality were present, the Holt-Winters multiplicative model gave a better fit to the data than the double exponential smoothing. This is a confirmation of the assertion that double exponential smoothing performs better when trend is present as seen in the prices of Maize and Millet.
On the basis of the MAPE, MAD and MSD values, it was seen that double exponential smoothing performed better than the Holt-winters multiplicative model in the maize and millet price data. However the Holt-winters multiplicative model performed better than the double exponential smoothing in the groundnut price data. This is a confirmation of the norm that Holt-winters multiplicative model performs better when seasonality and trend are present as was seen in the prices of groundnut.
The analysis from this study further indicates that most discount factors that gave the best results lie outside the range suggested for double exponential smoothing. This finding falls in line with Brown's (1962) suggestion after setting a range between 0.84 and 0.97 for discount factors, that in practice it is possible for the smoothing parameters to lie outside the range suggested for exponential smoothing.
The study also showed that prices of food crops of successive months during the study period were not independent as successful forecasting models were built for all the prices of the food crops under study.
Since visual inspection of the data alone cannot lead to correct conclusions about the order of exponential smoothing, it is therefore recommended that reliable identification tools such as the sample autocorrelation and sample partial autocorrelation functions should be considered. Furthermore, a smoothing constant of 0.1 or discount coefficient of 0.9 will not always lead to good forecast.

It will be recommended that the Holt-Winters multiplicative procedure be used in modeling and forecasting prices of food crops in which both seasonality and trend are present with these two components (trend and seasonality) being multiplicative in a given set of data. The Double exponential smoothing procedure be used when trend or seasonality is present

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