# Minimization of Losses on Electric Power Transmission Lines

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**Abstract**

Availability of electric power has been the most powerful vehicle for facilitating economic, industrial and social developments of any nation. Electric power is transmitted by means of transmission lines which deliver bulk power from generating stations to load centres and consumers. For electric power to get to the final consumers in proper form and quality, losses along the lines must be reduced to the barest minimum. In this paper, a mathematical model of losses along electric power transmission lines was developed using a combination of ohmic and corona losses. The resulting model, which is a nonlinear multivariable unconstrained optimization problem, was minimized using the classical optimization technique. From the results, we were able to see that power losses on transmission lines will be minimized if we transmit electric power at a very low current and at an operating voltage that is very close to the critical disruptive voltage. Also the spacing between the conductors should be large in comparison to their diameters. These results, gotten by the use of an analytical method, conform to the existing results for power transmission.

**Keywords**: Power Losses, Minimization, Transmission, Classical Optimization, Mathematical Model.

## 1. Introduction

Electrical energy is generated at power stations which are usually located far away from load centres. Thus, a network of conductors between the power stations and the consumers is required in order to harness the power generated. This network of conductors may be divided into two main components, namely, the transmission system and the distribution system. Accurate knowledge of power losses on transmission lines and their minimization is a critical component for efficient flow of power in an electrical network. Power losses result in lower power availability to final consumers. Hence, adequate measures need to be taken to reduce power losses to the barest minimum.

Power plants' planning in a way to meet the power network load demand is one of the most important and essential issues in power systems. Since transmission lines connect generating plants and substations in power network, the analysis, computation and reduction of transmission losses in these networks are of great concern to scientists and engineers.

A lot of research works have been carried out on the analysis, computation and reduction of transmission losses. Zakariya (2010) made a comparison between the corona power loss associated with HVDC transmission lines and the ohmic power loss. The corona power loss and ohmic power loss were measured and computed for different transmission line configurations and under fair weather and rainy conditions. Numphetch et al. (2011) worked on loss minimization using optimal power flow based on swarm intelligences. Thabendra et al. (2009) considered multi-objective optimization methods for power loss minimization and voltage stability while Abdullah et al. (2010) looked at transmission loss minimization and power installation cost using evolutionary computation for improvement of voltage stability. Bagriyanik et al. (2003) used a fuzzy multi-objective optimization and genetic algorithm-based method to find optimum power system operating conditions. In addition to active power losses, series reactive power losses of transmission system were also considered as one of the multiple objectives. Onohaebi and Odiase (2010) considered the relationship between distance and loadings on power losses using the existing 330 KV Nigerian transmission network as a case study in his empirical modelling of power losses as a function of line loadings and lengths while Moghadam and Berahmandpour (2010) developed a new method for calculating transmission power losses based on exact modelling of ohmic loss, to mention a few. In all these research work, much emphasis has been on reduction of losses using design and construction technique, optimal power flow based on swarm intelligences and evolutionary methods. A better approach would have been the one that utilizes the concept of classical optimization for minimizing the losses by employing mathematical modelling and differential calculus as working tools.

Therefore, in this paper, we formulated a mathematical model for power losses, on transmission lines, by making use of the formulae for ohmic and corona losses. The resulting model was minimized using the classical optimization technique and the results are in agreement with the existing rules of power transmission.

## 2. Mathematical Model for Power Losses

The main reason for losses in transmission and sub-transmission lines is the resistance of conductors against the flow of current. The production of heat in the conductor as a result of the flow of current increases its temperature. This rise in the conductor's temperature further increases the resistance of the conductor and this will consequently increase the losses. This implies that ohmic power loss is the main component of losses in transmission and sub-transmission lines, Mehta and Mehta (2008) and Gupta (2008).

The value of the ohmic power loss, Wadhwa (2009), is given as

$$L_{Ohmic} = I^2 R KW/Km/Phase \qquad (1)$$

where

I denotes current along the conductor and

R represents resistance of the conductor.

The formation of corona on transmission line is associated with a loss of power, which will have some effect on the efficiency of the transmission line. The corona power loss for a fair weather condition, Mehta and Mehta (2008), Wadhwa (2009), Gupta (2008) and James (2005), has the value

$$L_{Corona} = 242\frac{(f+25)}{\delta}.\sqrt{(\frac{r}{d})}.(V-V_0)^2.10^{-5} KW/Km/Phase \qquad (2)$$

where

$f$ represents the frequency of transmission,

$\delta$ denotes the air density factor,

$r$ is radius of the conductor,

$d$ represents the space between the transmission lines,

$V$ is the operating voltage and

$V_0$ denotes the distruptive voltage.

Taking the total power loss on transmission lines to be the summation of ohmic and corona loss, we have

$$T_{Loss} = L_{Ohmic} + L_{Corona} \qquad (3)$$

i.e

$$T_{Loss} = I^2 R + 242\frac{(f+25)}{\delta}.\sqrt{(\frac{r}{d})}.(V-V_0)^2.10^{-5} KW/Km/Phase \qquad (4)$$

The general form of equation (4) is given by

$$T_{Loss} = I^2\frac{\rho L}{A} + 242\frac{(f+25)}{\delta}\sqrt[4]{(\frac{A}{\pi d^2})}.(V-V_0)^2.10^{-5} KW/Km/Phase \qquad (5)$$

where

$\rho$ is the resistivity of the conductor,

$L$ denotes the length of the conductor and

$A$ is the cross-sectional area of the conductor.

## 3. Minimization of Power Losses

The problem of finding the optimum electric power loss during transmission can therefore be posed as

$$\underset{(I,V,d)}{Minimize}\, T_{Loss} = I^2\frac{\rho L}{A} + 242\frac{(f+25)}{\delta}\sqrt[4]{(\frac{A}{\pi d^2})}.(V-V_0)^2.10^{-5} KW/Km/Phase \qquad (6)$$

This is a nonlinear multivariable unconstrained optimization problem. Assuming that the transmission related factors are continuous, then (6) can be solved using the classical method of optimization.

To determine the stationary points of (6), Rao (1998), we differentiate with respect to the selected variables to get

$$\frac{\partial T_{Loss}}{\partial I} = \frac{2I\rho L}{A} \qquad (7)$$

$$\frac{\partial T_{Loss}}{\partial V} = 484\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi d^2})}.(V-V_0)10^{-5} \qquad (8)$$

$$\frac{\partial T_{Loss}}{\partial d} = -121\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi})}.(V-V_0)^2 d^{-\frac{3}{2}}10^{-5} \qquad (9)$$

Equations (7), (8) and (9) give the extremum points as $I = 0,\ V = V_0\ and\ d \to \infty$.

The second derivatives with respect to the variables are

$$\frac{\partial^2 T_{Loss}}{\partial I^2} = \frac{2\rho L}{A} \qquad (10)$$

$$\frac{\partial^2 T_{Loss}}{\partial I \partial V} = 0 \qquad (11)$$

$$\frac{\partial^2 T_{Loss}}{\partial I \partial d} = 0 \qquad (12)$$

$$\frac{\partial^2 T_{Loss}}{\partial V \partial I} = 0 \quad (13)$$

$$\frac{\partial^2 T_{Loss}}{\partial V^2} = 484\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi d^2})}.10^{-5} \quad (14)$$

$$\frac{\partial^2 T_{Loss}}{\partial V \partial d} = -242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5} \quad (15)$$

$$\frac{\partial^2 T_{Loss}}{\partial d \partial I} = 0 \quad (16)$$

$$\frac{\partial^2 T_{Loss}}{\partial d \partial V} = -242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5} \quad (17)$$

$$\frac{\partial^2 T_{Loss}}{\partial d^2} = \frac{363}{2}.\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi})}.(V-V_0)^2 d^{-\frac{5}{2}}10^{-5} \quad (18)$$

The Hessian matrix, Rao (1998), is therefore given by

$$H = \begin{pmatrix} \frac{\partial^2 T_{Loss}}{\partial I^2} & \frac{\partial^2 T_{Loss}}{\partial I \partial V} & \frac{\partial^2 T_{Loss}}{\partial I \partial d} \\ \frac{\partial^2 T_{Loss}}{\partial V \partial I} & \frac{\partial^2 T_{Loss}}{\partial V^2} & \frac{\partial^2 T_{Loss}}{\partial V \partial d} \\ \frac{\partial^2 T_{Loss}}{\partial d \partial I} & \frac{\partial^2 T_{Loss}}{\partial d \partial V} & \frac{\partial^2 T_{Loss}}{\partial d^2} \end{pmatrix}$$

i.e.,

$$H = \begin{pmatrix} \frac{2\rho L}{A} & 0 & 0 \\ 0 & 484\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi d^2})}.10^{-5} & -242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5} \\ 0 & -242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5} & \frac{363}{2}.\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi})}.(V-V_0)^2 d^{-\frac{5}{2}}10^{-5} \end{pmatrix}$$

for which,

$$H_1 = \left|\frac{2\rho L}{A}\right| > 0$$

$$H_2 = \begin{vmatrix} \frac{2\rho L}{A} & 0 \\ 0 & 484\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi d^2})}.10^{-5} \end{vmatrix} = \frac{2\rho L}{A}.484\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi d^2})}.10^{-5} > 0$$

$$H_3 = \begin{vmatrix} \frac{2\rho L}{A} & 0 & 0 \\ 0 & 484\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi d^2})}.10^{-5} & -242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5} \\ 0 & -242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5} & \frac{363}{2}.\frac{(f+25)}{\delta}.\sqrt[4]{(\frac{A}{\pi})}.(V-V_0)^2 d^{-\frac{5}{2}}10^{-5} \end{vmatrix}$$

$$= \frac{2\rho L}{A}\left[\left(484\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi d^2}\right)}.10^{-5}\right)\left(\frac{363}{2}.\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)^2 d^{-\frac{5}{2}}10^{-5}\right) - (242\frac{(f+25)}{\delta}.\sqrt[4]{\left(\frac{A}{\pi}\right)}.(V-V_0)d^{-\frac{3}{2}}10^{-5})^2\right] > 0$$

## 4. Discussion on Results

Since $H_1, H_2$ and $H_3$ are all greater than zero, then it shows that the Hessian matrix of power losses over transmission lines is positive definite at the extremum values. Hence the power loss is minimum at $I = 0,\ V = V_0\ and\ d \to \infty$.

Technically, this implies that the total power losses on transmission lines will only be minimum if

(i) power is transmitted at a very low current along transmission lines. This will reduce the ohmic or line loss on the conductors to the barest minimum. This conforms to the principle of electric power transmission.

(ii) the operating voltage is equal to the critical disruptive voltage. When this happens, there is no ionisation of air around the conductor and hence no corona is formed. Therefore, there will be no corona loss and

(iii) the spacing between the conductors on the transmission line should be large. This is because; an increase in the spacing between conductors reduces the electro-static stresses. This therefore reduces the corona effect. If the spacing between the conductors is made very large as compared to their diameter, there may not be any corona effect or losses on the line.

**Conclusion**

The classical optimization technique has been used to minimize power losses on transmission lines thereby getting the same results for optimum power transmission by making use of an analytical method. The application of the classical optimization technique to the mathematical model of power losses on transmission lines has therefore provide a better understanding of the problem of power losses on high voltage transmission lines.

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