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On Nano Continuity

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Abstract
The purpose of this paper to propose a new class of functions called nano continuous functions and derive their characterizations in terms of nano closed sets, nano closure and nano interior. There is also an attempt to define nano-open maps, nano closed maps and nano homeomorphism
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1. Introduction

Continuity of functions is one of the core concepts of topology. In general, a continuous function is one, for which small changes in the input result in small changes in the output. The notion of Nano topology was introduced by Lellis Thivagar [3], which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined nano closed sets, nano-interior and nano closure. In this paper we have introduced a new class of functions on nanotopological spaces called nano continuous functions and derived their charaterizations in terms of nano closed sets, nano closure and nano interior. We have also established nano-open maps, nanoclosed maps and nano homeomorphisms and their representations in terms of nano closure and nano interior.

2. Preliminaries

Definition 2.1 [5]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation.Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U , R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [5]: If (U, R) is an approximation space and $X, Y \subseteq U$, then

i) $L_R(X) \subseteq X \subseteq U_R(X)$
ii) $L_R(\varphi) = U_R(\varphi) = \varphi$
iii) $L_R(U) = U_R(U) = U$
iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
ix) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
x) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
xi) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3 [3]: Let U be a non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U , called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and (U, $\tau_R(X)$) is called the nano topological space. $[\tau_R(X)]^C$ is called as the dual nano topology of $\tau_R(X)$.

Elements of $[\tau_R(X)]^C$ are called as nano closed sets.

Remark 2.4 [3]: The basis for the nano topology $\tau_R(X)$ with respect to X is given by $\beta_R(X) = \{$ U, L_R (X) ,B_R(X) $\}$.

Definition 2.5 [3] If (U, $\tau_R(X)$) is a nano topological space with respect to X where X ⊆ U and if A ⊆ U, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest nano closed set containing A.

Remark 2.6 :Throughout this paper, U and V are non-empty, finite universes; X ⊆ U and Y ⊆ V ; U/R and V/R' denote the families of equivalence classes by equivalence relations R and R' respetively on U and V. (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) are the nano topological spaces with respect to X and Y respectively.

3.Nano continuity

Definition 3.1 : Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be nano topological spaces. Then a mapping f : (U, $\tau_R(X)$) →(V, $\tau_{R'}(Y)$) is nano continuous on U if the inverse image of every nano-open set in V is nano-open in U.

Example 3.2 Let U = $\{a,b,c,d\}$ with U/R= $\{\{a,c\},\{b\},\{d\}\}$. Let $X = \{a,d\} \subseteq$ U. Then $\tau_R(X) = \{U, \phi, \{d\}, \{a,c,d\}, \{a,c\}\}$. Let $V = \{x,y,z,w\}$ with $V/R' = \{\{x\},\{y,z\},\{w\}\}$ and $Y = \{x,z\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x,y,z\}, \{y,z\}\}$. Define f: U → V f as f(a) = y, f(b) = w, f(c) = z, f(d) = x. Then $f^{-1}(\{x\}) = \{d\}, f^{-1}(\{x,y,z\}) = \{a,c,d\}$ and $f^{-1}(\{y,z\}) = \{a,c\}$. That is, the inverse image of every nano-open set in V is nano -open in U. Therefore, f is nano continuous.

The following theorem characterizes nano continuous functions in terms of nano closed sets.

Theorem 3.3 : A function f : (U, $\tau_R(X)$) →(V, $\tau_{R'}(Y)$) is nano continuous if and only if the inverse image of every nano closed set in V is nano closed in U.

Proof: Let f be nano continuous and F be nano closed in V. That is, V - F is nano-open in V Since f is nano continuous, $f^{-1}(V - F)$ is nano-open in U. That is, U - $f^{-1}(F)$ is nano-open in U. Therefore, $f^{-1}(F)$ is nano closed in U. Thus, the inverse image of every nano closed set in V is nano closed in U , if f is nano continuous on U. Conversely, let the inverse image of every nano closed set be nano closed. Let G be nano-open in V. Then V - G is nano closed in V. Then, $f^{-1}(V - G)$ is nano closed in U. That is,
U - $f^{-1}(G)$ is nano closed in U. Therefore, $f^{-1}(G)$ is nano-open in U . Thus,the inverse image of every nano-open set in V is nano-open in U. That is, f is nano continuous on U.

In the following theorem, we establish a characterization of nano continuous functions in terms of nano closure.

Theorem 3.4 : A function f : (U, $\tau_R(X)$) →(V, $\tau_{R'}(Y)$) is nano continuous if and only if $f(NCl(A)) \subseteq NCl(f(A))$ for every subset A of U.

Proof: Let f be nano continuous and $A \subseteq U$. Then $f(A) \subseteq V$. NCl(f(A)) is nano closed in V. Since f is nano continuous, $f^{-1}(NCl(f(A)))$ is nano closed in U. Since $f(A) \subseteq NCl(f(A))$, $A \subseteq f^{-1}(NCl(f(A)))$. Thus $f^{-1}(NCl(f(A))$ is a nano closed set containing A. But, NCl(A) is the smallest nano closed set containing A. Therefore $NCl(A) \subseteq f^{-1}(NCl(A))$. That is , $f(NCl(A)) \subseteq NCl(f(A))$. Conversely, let f(NCl(A)) ⊆ NCl(f(A)) for every subset A of U. If F is nano closed in V, since $f^{-1}(F) \subseteq U$, $f(NCl(f^{-1}(F))) \subseteq NCl(f(f^{-1}(F))) \subseteq NCl(F)$.
That is, $NCl(f^{-1}(F)) \subseteq f^{-1}(NCl(F)) = f^{-1}(F))$, since F is nano closed. Thus $NCl(f^{-1}(F) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq NCl(f^{-1}(F))$. Therefore, $NCl(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is nano closed in U for every nano closed set F in V . That is, f is nano continuous.

Remark 3.5 : If f : (U, $\tau_R(X)$) →(V, $\tau_{R'}(Y)$) is nano continuous, then $f(NCl(A))$ is not necessarily equal to $\mathsf{N}Cl(f(A))$ where A ⊆ U. For example, let $\mathsf{U}=\{a,b,c,d\}$; $\mathsf{U}/R=\{\{a\},\{b,d\},\{c\}\}$. Let $X=\{a,c,d\}$. Then $\tau_R(X)=\{\mathsf{U},\phi,\{a,c\},\{b,d\}\}$. Let $\mathsf{V}=\{x,y,z,w\}$ with $\mathsf{V}/R'=\{\{x,z\},\{y\},\{w\}\}$. Let $Y=\{x,y\}$. Then $\tau_{R'}(Y)=\{\mathsf{V},\phi,\{y\},\{x,y,z\},\{x,z\}\}$. Let $f:(\mathsf{U},\tau_R(X))\to(\mathsf{V},\tau_{R'}(Y))$ be given by f(a) = y, f(b) = x, f(c) = y, f(d) = x. Then $f^{-1}(\mathsf{V})=\mathsf{U}, f^{-1}(\phi)=\phi$, $f^{-1}(\{y\})=\{a,c\}, f^{-1}(\{x,y,z\})=\mathsf{U}$ and $f^{-1}(\{x,z\})=\{b,d\}$. That is, the inverse image of every nano-open set in V is nano-open in U. Therefore, f is nano continuous on U. Let $A=\{a,c\}\subseteq\mathsf{V}$. Then $f(\mathsf{N}Cl(A))=f(\{a,c\})=\{y\}$. But, $\mathsf{N}Cl(f(A))=\mathsf{N}Cl(\{y\})=\{y,w\}$. Thus, $f(\mathsf{N}Cl(A)\neq\mathsf{N}Cl(f(A))$, even though f is nano continuous. That is, equality does not hold in the previous theorem when f is nano continuous.

Theorem 3.6 *:* Let $(\mathsf{U},\tau_R(X))$ and $(\mathsf{V},\tau_{R'}(Y))$ be two nanotopological spaces where $X\subseteq\mathsf{U}$ and $Y\subseteq\mathsf{V}$. Then $\tau_{R'}(Y)=\{\mathsf{V},\phi,L_{R'}(Y),U_{R'}(Y),\ B_{R'}(Y)\}$ and its basis is given by $\mathsf{B}_{R'}=\{\mathsf{V},L_{R'}(Y),B_{R'}(Y)\}$. A function $f:(\mathsf{U},\tau_R(X))\to(\mathsf{V},\tau_{R'}(Y))$ is nano continuous if and only if the inverse image of every member of $\mathsf{B}_{R'}$ is nano-open in U.

Proof: Let f be nano continuous on U. Let $B\in\mathsf{B}_{R'}$. Then B is nano-open in V. That is, $B\in\tau_{R'}(Y)$. Since f is nano continuous, $f^{-1}(B)\in\tau_R(X)$. That is,the invese image of every member of $\mathsf{B}_{R'}$ is nano-open in U. Conversely, let the inverse image of every member of $\mathsf{B}_{R'}$ be nano-open in U. Let G be a nano-open in V. Then $G=\bigcup\{B:B\in\mathsf{B}_1\}$, where $\mathsf{B}_1\subset\mathsf{B}_{R'}$. Then $f^{-1}(G)=f^{-1}(\bigcup\{B:B\in\mathsf{B}_1\})=\bigcup\{f^{-1}(B):B\in\mathsf{B}_1\}$, where each $f^{-1}(B)$ is nano-open in U and hence their union, which is $f^{-1}(G)$ is nano-open in U. Thus f is nano continuous on U.

The above theorem characterizes nano continuous functions in terms of basis elements. In the following theorem, we characterize nano continuous functions in terms of inverse image of nano closure.

Theorem 3.7 *:* A function $f:(\mathsf{U},\tau_R(X))\to(\mathsf{V},\tau_{R'}(Y))$ is nano continuous if and only if $\mathsf{N}Cl(f^{-1}(B))\subseteq f^{-1}(\mathsf{N}Cl(B))$ for every subset B of V.

Proof: If f is nano continuous and $B\subseteq\mathsf{V}$, $\mathsf{N}Cl(B)$ is nano closed in V and hence $f^{-1}(\mathsf{N}Cl(B))$ is nano closed in U. Therefore, $\mathsf{N}Cl[f^{-1}(\mathsf{N}Cl(B))]=f^{-1}(\mathsf{N}Cl(B))$. Since $B\subseteq\mathsf{N}Cl(B)$, $f^{-1}(B)\subseteq f^{-1}(\mathsf{N}Cl(B))$. Therefore, $\mathsf{N}Cl(f^{-1}(B))\subseteq\mathsf{N}Cl(f^{-1}(\mathsf{N}Cl(B)))=f^{-1}(\mathsf{N}Cl(B))$. That is, $\mathsf{N}Cl(f^{-1}(B))\subseteq f^{-1}(\mathsf{N}Cl(B))$. Conversely, let $\mathsf{N}Cl(f^{-1}(B))\subseteq f^{-1}(\mathsf{N}Cl(B))$ for every $B\subseteq\mathsf{V}$. Let B be nanoclosed in V. Then $\mathsf{N}Cl(B)=B$. By assumption, $\mathsf{N}Clf^{-1}(B)\subseteq f^{-1}(\mathsf{N}Cl(B))=f^{-1}(B)$. Thus, $\mathsf{N}Clf^{-1}(B)\subseteq f^{-1}(B)$. But $f^{-1}(B)\subseteq\mathsf{N}Cl(f^{-1}(B))$. Therefore, $\mathsf{N}Cl(f^{-1}(B))=f^{-1}(B)$. That is, $f^{-1}(B)$ is nano closed in U for every nano closed set B in V. Therefore, f is nano continuous on U.

The following thereom establishes a criteria for nano continuous functions in terms of inverse image of nano interior of a subset of V.

Theorem 3.8 *:* A function $f:(\mathsf{U},\tau_R(X))\to(\mathsf{V},\tau_{R'}(Y))$ is nano continuous on U if and only if $f^{-1}(\mathsf{N}Int(B))\subseteq\mathsf{N}Int(f^{-1}(B))$ for every subset B of V.

Proof: Let f be nano continuous and $B\subseteq\mathsf{V}$. Then $\mathsf{N}Int(B)$ is nano-open in $(\mathsf{V},\tau_{R'}(Y))$. Therefore $f^{-1}(\mathsf{N}Int(B))$ is nano-open in $(\mathsf{U},\tau_R(X))$. That is, $f^{-1}(\mathsf{N}Int(B))=\mathsf{N}Int[f^{-1}(\mathsf{N}Int(B))]$. Also, $\mathsf{N}Int(B)\subseteq B$ implies that $f^{-1}(\mathsf{N}Int(B))\subseteq f^{-1}(B)$. Therefore $\mathsf{N}Int[f^{-1}(\mathsf{N}Int(B))]\subseteq\mathsf{N}Int(f^{-1}(B))$. That is, $f^{-1}(\mathsf{N}Int(B))\subseteq\mathsf{N}Int(f^{-1}(B))$. Conversely, let $f^{-1}(\mathsf{N}Int(B))\subseteq\mathsf{N}Int(f^{-1}(B))$ for every subset B of V. If B is nano-open in V, $\mathsf{N}Int(B)=B$. Also,

$f^{-1}(\mathrm{N}Int(B)) \subseteq \mathrm{N}Int(f^{-1}(B))$. That is, $f^{-1}(B) \subseteq \mathrm{N}Int(f^{-1}(B))$. But $\mathrm{N}Int(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore, $f^{-1}(B) = \mathrm{N}Int(f^{-1}(B))$. Thus, $f^{-1}(B)$ is nano-open in U for every nano-open set B in V. Therefore, f is nano continuous.

Example 3.9 *:* Let $\mathrm{U} = \{a,b,c,d\}$ with $\mathrm{U}/R = \{\{a,d\},\{b\},\{c\}\}$. Let $X = \{a,c\} \subseteq \mathrm{U}$. Then the nanotopology, $\tau_R(X)$ with respect to X is given by $\{\mathrm{U},\phi,\{c\},\{a,c,d\},\{a,d\}\}$ and hence the nanoclosed sets in U are $\mathrm{U},\phi,\{a,b,d\},\{b\}$ and $\{b,c\}$. Let $\mathrm{V} = \{x,y,z,w\}$ with $\mathrm{V}/R' = \{\{x\},\{y\},\{z\},\{w\}\}$. Let $Y = \{x,w\} \subseteq \mathrm{V}$. Then the nanotopology on V with respect to Y is given by $\tau_{R'}(Y) = \{\mathrm{V},\phi,\{x,w\}\}$, and the nanoclosed sets in V are V,ϕ and $\{y,z\}$. Define $f:\mathrm{U}\to\mathrm{V}$ as $f(a)=x$, $f(b)=y$, $f(c)=z$ and $f(d)=w$. Then f is nano continuous on U, since inverse image of every nano-open set in V is nano-open in U. Let $B=\{y\}\subset\mathrm{V}$. Then $f^{-1}(\mathrm{N}Cl(B)) = f^{-1}(\{y,z\}) = \{b,c\}$ and $\mathrm{N}Cl(f^{-1}(B)) = \{b\}$. Thus, $\mathrm{N}Cl(f^{-1}(B)) \neq f^{-1}(\mathrm{N}Cl(B))$. Also when $A = \{x,z,w\} \subseteq \mathrm{V}$, $f^{-1}(\mathrm{N}Int(A)) = f^{-1}(\{x,w\}) = \{a,d\}$ but $\mathrm{N}Int(f^{-1}(A)) = \mathrm{N}Int(\{a,c,d\}) = \{a,c,d\}$. That is, $f^{-1}(\mathrm{N}Int(A)) \neq \mathrm{N}Int(f^{-1}(A))$. Thus, equality does not hold in theorems 5.6 and 5.7 when f is nano continuous.

Theorem 3.10 *:* If $(\mathrm{U},\tau_R(X))$ and $(\mathrm{V},\tau_{R'}(Y))$ are nano topological spaces with respect to $X\subseteq\mathrm{U}$ and $Y\subseteq\mathrm{V}$ respectively, then for any function $f:\mathrm{U}\to\mathrm{V}$, the following are equivalent:

1. f is nano continuous.
2. The inverse image of every nano closed set in V is nano closed in U.
3. $f(\mathrm{N}Cl(A)) \subset \mathrm{N}Cl(f(A))$ for every subset A of V.
4. The inverse image of every member of the basis $B_{R'}$ of $\tau_{R'}(Y)$ is nano-open in U.
5. $\mathrm{N}Cl(f^{-1}(B)) \subseteq f^{-1}(\mathrm{N}Cl(B))$ for every subset B of V.
6. $f^{-1}(\mathrm{N}Int(B)) \subset \mathrm{N}Int(f^{-1}(B))$ for every subset B of V.

Proof of the theorem follows from theorems 3.3 to 3.8.

Definition 3.11 *:* A subset A of a nanotopological space $(\mathrm{U},\tau_R(X))$ is said to be nano dense if $\mathrm{N}Cl(A)=\mathrm{U}$.

Remark 3.12 *:* Since $\mathrm{N}Cl(X)=\mathrm{U}$ is rough topological space $(\mathrm{U},\tau_R(X))$ with respect to X where $X\subset\mathrm{U}$, X is nano dense in U.

Example 3.13 *:* Let $\mathrm{U}=\{a,b,c,d\}$ with $\mathrm{U}/R=\{\{a\},\{b,c\},\{d\}\}$. Let $X=\{a,c\}$. Then $\tau_R(X)=\{\mathrm{U},\phi,\{a\},\{a,b,c\},\{b,c\}\}$ and the rough closed sets in U are $\mathrm{U},\phi,\{b,c,d\},\{d\}$ and $\{a,d\}$. If $\mathrm{V}=\{x,y,z,w\}$ with $\mathrm{V}/R'=\{\{x\},\{y\},\{z,w\}\}$ and $Y=\{x,z\}$, then $\tau_{R'}(Y)=\{\mathrm{V},\phi,\{x\},\{x,z,w\},\{z,w\}\}$ and the nanoclosed sets in V are $\mathrm{V},\phi,\{y,z,w\},\{y\},\{x,y\}$. Define a function $f:\mathrm{U}\to\mathrm{V}$ as $f(a)=z, f(b)=z, f(c)=w$ and $f(d)=y$. Then f is nano continuous since the inverse image of every nano-open set in V is nano-open in U. Let $A=\{a,b,d\}\subseteq\mathrm{U}$. $\mathrm{N}Cl(A)=\mathrm{U}$ and hence A is nano dense in U. But $\mathrm{N}Clf(A)=\mathrm{N}Cl(\{y,z\})=\{y,z,w\}\neq\mathrm{V}$. Therefore, f(A) is not nano dene even though A is nano dense and f is nano continuous. Thus, a nano continuous function does not map nano dense sets into nano dense sets.

In the following theorem, we establish that a nano continous function maps nano dense sets into nano dense sets, provided it is onto.

Theorem 3.14 *:* Let $f:(\mathrm{U},\tau_R(X))\to(\mathrm{V},\tau_{R'}(Y))$ be an onto, nano continuous function. If A is nano dense in U, then $f(A)$ is nano dense in V.

Proof: Since A is nano dense in $\mathrm{U},\mathrm{N}Cl(A)=\mathrm{U}$. Then $f(\mathrm{N}Cl(A))=f(\mathrm{U})=\mathrm{V}$, since f is onto. Since f is nano continuous on $\mathrm{U}, f(\mathrm{N}Cl(A))\subseteq\mathrm{N}Cl(f(A))$. Therefore, $\mathrm{V}\subseteq\mathrm{N}Cl(f(A))$. But $\mathrm{N}Cl(f(A))\subseteq\mathrm{V}$. Therefore, $\mathrm{N}Cl(f(A))=\mathrm{V}$. That is, f(A) is nano dense in V. Thus, a nano continuous function maps nano dense sets into nano dense sets, provided it is onto.

4. Nano-open maps, Nano closed maps and Nano homeomorphism

Definition 4.1 : A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is a nano-open map if the image of every nano-open set in U is nano-open in V. The mapping f is said to be a nanoclosed map if the image of every nanoclosed set in U is nanoclosed in V.

Theorem 4.2 : A mapping $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nanoclosed map if and only if $NCl(f(A))\subseteq f(NCl(A))$, for every subset A of U.

Proof: If f is nanoclosed, $f(NCl(A))$ is nanoclosed in V, since $NCl(A)$ is nano closed in U. Since $A\subseteq NCl(A)$, $f(A)\subseteq f(NCl(A))$ .Thus $f(NCl(A))$ is a nano closed set containing f(A). Therefore, $NCl(f(A))\subseteq f(NCl(A))$. Conversely, if $NCl(f(A))\subseteq f(NCl(A))$ for every subset A of U and if F is nanoclosed in U, then $NCl(F)=F$ and hence $f(F)\subseteq NCl(f(F))\subseteq f(NCl(F))=f(F)$. Thus, $f(F)=NCl(f(F))$. That is, $f(F)$ is nanoclosed in V. Therefore, f is a nanoclosed map.

Theorem 4.3 : A mapping $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano-open map if and only if $f(NInt(A))\subseteq NInt(f(A))$, for every subset $A\subseteq U$.

Proof is similar to that of theorem 4.2

Definition 4.4 : A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be a nano homeomorphism if

1. f is 1-1 and onto
2. f is nano continuous and
3. f is nano-open

Theorem 4.5 : Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a one-one onto mapping. Then f is a nano homeomorphism if and only if f is nano closed and nanocontinuous.

Proof: Let f be a nano homeomorphism. Then f is nano continuous. Let F be an arbitrary nano closed set in $(U,\tau_R(X))$. Then U-F is nano-open. Since f is nano- open, $f(U-F)$ is nano- open in V. That is, $V-f(F)$ is nano-open in V. Therefore, $f(F)$ is nano-closed in V. Thus, the image of every nano closed set in U is nano closed in V. That is, f is nano closed. Conversely, let f be nano closed and nano continuous. Let G be nano-open in $(U,\tau_R(X))$. Then U-G is nano closed in U. Since f is nano closed, $f(U-G)=V-f(G)$ is nano closed in V. Therefore, $f(G)$ is nano-open in V. Thus, f is nano-open and hence f is a nano homeomorphism.

The following theorem provides a condition on a nano continuous function under which equality holds in theorem 3.3

Theorem 4.6 : A one-one map f of $(U,\tau_R(X))$ onto $(V,\tau_{R'}(Y))$ is a nano homeomorphism iff $f(NCl(A))=NCl[f(A)]$ for every subset A of U.

Proof: If f is a nano homeomorphism, f is nano continuous and nano closed. If $A\subseteq U$, $f(NCl(A))\subset NCl(f(A))$, since f is nano continuous. Since $NCl(A)$ is nano closed in U and f is nano closed, $f(NCl(A))$ is nano closed in V. $NCl(f(NCl(A)))=f(NCl(A))$. Since $A\subseteq NCl(A), f(A)\subseteq f(NCl(A))$ and hence $NCl(f(A))\subseteq NCl[f(NCl(A))]=f(NCl(A))$. Therefore, $NCl(f(A))\subseteq f(NCl(A))$. Thus, $f(NCl(A))=NCl(f(A))$ if f is a nano homeomorphism. Conversely, if $f(NCl(A))=NCl(f(A))$ for every subset A of U, then f is nano continuous . If A is nano closed in $U, NCl(A)=A$ which implies $f(NCl(A))=f(A)$. Therefore, $NCl(f(A))=f(A)$. Thus, f(A) is nano closed in V, for every nano closed set A in U. That is f is nano closed. Also f is nano continuous. Thus, f is a nano homeomorphism.

5. Application

In this section we apply the concept of nano continuous functions in a day-to-day problem.

Consider the cost of a cab ride as a function of distance travelled. Let $U=\{x_1,x_2,x_3,x_4,x_5,x_6\}$ be the universe of distances of six different places from a railway junction and let $V=\{a,b,c,d,e,f\}$ be the universe of cab fares to reach the six destinations in U from the railway junction . We know that the fares depend on the distance of the places.Let $\mathsf{U}/R = \{\{x_1\},\{x_2,x_4\},\{x_3,x_5\},\{x_6\}\}$ and let $X = \{x_1,x_2,x_3\}$, a subset of U .Then the nano topology on U is given by $\tau_R(X) = \{\mathsf{U},\phi,\{x_1\},\{x_1,x_2,x_3,x_4,x_5\},\{x_2,x_3,x_4,x_5\}\}$. Let $\mathsf{V}/R' = \{\{a\},\{b,d\},\{c,e\},\{f\}\}$ and let Y = $\{a,b,c\}$. Then the nano topology $\tau_{R'}(Y)$ on V with respect to Y is given by $\{\mathsf{V},\phi,\{a\},\{a,b,c,d,e\},\{b,c,d,e\}\}$. Define $f:\mathsf{U}\to\mathsf{V}$ as $f(x_1)=a, f(x_2)=b, f(x_3)=c,$ $, f(x_4)=d, f(x_5)=e, f(x_6)=f.$ Then $f^{-1}(\mathsf{U})=\mathsf{V}, f^{-1}(\phi)=\phi, f^{-1}(\{a\})=\{x_1\},$ $f^{-1}(\{a,b,c,d,e\}) = \{x_1,x_2,x_3,x_4,x_5\}$ and $f^{-1}(\{b,c,d,e\}) = \{x_2,x_3,x_4,x_5\}$. That is, the inverse image of every nano-open set in V is nano-open in U. Therefore f is nano continuous.Also we note that the image of every nano-open set in U is nano-open in V and f is a bijection . Thus, f is a nano homeomorphism. Therefore, the cost of a cab ride, as a function of distance travelled, is a nano homeomorphism.

6. Conclusion

The theory of nano continuous functions has a wide variety of applications in real life. In this paper, we have shown that the cost of cab rides, as a function of distance travelled, is not only a nano continuous function but also a nano homeophormism.Similarly, nano continuous functions have a wide range of applications such as growth of a plant over time, depreciaton of machine and temperature at various times of the day.Thus, nano continuous functions, in near future, can be applied to more day-to-day situations.

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