

Portfolio Optimization of Pension Fund Contribution in Nigeria

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Abstract

In this research work, we investigate the optimality of pension portfolio of 2004 pension reform guidelines in Nigeria using the Net Asset Values (NSA) from the First Alliance and Benefits Ltd as our scarce resources.

The resources are distributed as different portfolios of asset classes for comparative analysis. Linear programming technique is used to carry out the analysis of the data using TORA Operations Research software to select the optimal mix of opportunities that would maximize the return while meeting the investment conditions set by each portfolio class and other constraints by Pen Com.

We conclude that the Pen Com guided portfolio is not optimum as revealed by the hypothesis test.

Keywords: Portfolio Optimization, Contributory Pension Scheme, Defined Benefit Scheme Linear Programming.

1. Introduction

Generally, pension is the amount of money set aside by an employer or employee or both to ensure that at retirement there is something to fall back on as income.

Pension scheme is a system in which an employer pays certain amount of money regularly into a pension fund while the employee also pays some money into the same pension fund which forms the aggregate of what the employee gets at the time of retirement (Egbe 2009).

Before 2004, Nigeria had practiced a Pay-As-You-Go (unfunded) defined benefit pension scheme in which an employee's pension benefits are paid from the employer's current revenue or by government out of current taxes. This scheme was burdened with a lot of problems and increasingly became unsustainable. The huge deficit, arbitrary increases in salaries and pension along with poor administrative structure brought about the need for pension reform. The general problem in defined benefit scheme led to a private system where employees find their own retirements through compulsory savings. This is a fully funded, defined contributory scheme that necessitates the employer paying into the pension account a minimum of the employee's 7.5% of his pensionable salary to the same account to make a total of 15% minimum of the pensionable salary. The contributions are deducted and transferred into the relevant retirement savings account managed by pension Fund Administrators (PFAs) and Pension Fund Custodians (PFCs). While PFAs invest the contributions in Government, and Corporate bonds, Shares, Bank Deposits and Certificates as well as Open Ended Investment Funds, among others, the PFCs hold the assets and funds of the pension funds (PENCOM 2004).

The base line argument in this study is that the Defined Benefits (DB) lacked the care ingredients of proper investment of the accumulated funds. In the light of the short comings, this research work draws its most important essence "portfolio optimization for pension contribution".

A portfolio cannot be optimized if it is not diversified as a first step in the right direction. The basic objective of diversification is to reduce the variability of portfolio returns without jeopardizing the expected rate of return.

Thus, the primary issue in attaining optimal diversification is not the component securities in the portfolio, but the nature of relationships among them.

We therefore investigate in this study if the pension portfolio for pension contributions based on 2004 pension reform guideline is optimum.

There is the expectation that portfolio optimization entails correct diversification and proper timing. No single fund may time the market well, but investors who own more than one fund may switch between optimum market segments. The basic idea is that a pension plan would maintain a constant mix of stocks and bonds if a market training strategy is not followed.

In working to achieve our objective, we employ appropriate investment return maximization and risk minimization techniques that will give optimum growth to pension contributors.

1.2 Significance and Scope of the Study

This work is expected to contribute to existing literatures on contributory pension scheme globally based on the understanding of the Nigeria Pension Reform Act of 2004.

It will also assist in providing empirical result of comparison of different portfolio classes based on pension

contribution and approved investment classes. This will in no doubt help Pension Fund Administrators on the optimal mix of investment.

1.3 Pension Reform Act (PRA) 2004

The Pension Reform Act (PRA) 2004 provides for the establishment of a contributory pension scheme for any employment in the Federal Republic of Nigeria. It stipulates payment of retirement benefits to employees to whom the scheme applies: (a) public sector employees (b) private sector employees in a firm with staff strength in excess of 5 employees.

Objective of the scheme are to:

- (i) ensure seamless funding of the retirement scheme by assisting improvident individuals save in order to cater for their likelihood at old age.
- (ii) ensure that private and public sector employee receives his retirement benefits as at when due.
- (iii) establish a uniform set of rules, regulations and standards for the administration and payments of retirement benefits.

1.4 Establishment of National Pension Commission (PENCOM)

The duties of PENCOM as reflected in PENCOM (2004) include:

- (i) Regulation and supervision of effective administration of pension matters in Nigeria.
- (ii) Approval, licensing and supervision of all pension fund administrators.
- (iii) Establishment of standards, rules and issuance of guidelines for the management and investment of pension funds under the act.

1.5 Pension Fund Administrators (PFAs)

The PRA (2004) Act provides that from the commencement of the plan or scheme, pension funds shall only be managed by pension Fund Administrators (PFAs) licensed by PENCOM under the act to perform the following functions:

- (a) Open Retirement Saving Account (RSA)
- (b) Invest and manage pension funds and assets in accordance with the provision of the Act.
- (c) Maintain books of account relating to pension funds managed by it.
- (d) Provide regular information on investment strategy, returns and other performance indicators to PENCOM and employees
- (e) Payment of retirement benefits in accordance with the provision of the Act.

1.6 Pension Fund Custodians (PFCs)

PEN COM Act 2004 stipulates that pension funds and assets are to be held solely in custody for the Pension Fund Administrators (PFAs) by an independent Pension Fund custodian (PFC). The PFC has the responsibilities of:

- (i) receiving the total contributions remitted by the employer within 24 hours and notify the PFA of same and hold the pension assets in safe custody on trust for the employee and beneficiaries of the RSA.
- (ii) Settling of transactions and undertaking activities relating to the administration of pension fund investments including the collection of dividends.
- (iii) executing relevant proxy in favour of the PFAs for the purpose of voting in relation to the investments.

The PFCs provides some control over the activities of the PFAs and provides a hedge against unauthorized access or trading. The custodian is in no way empowered to utilize any pension fund assets in its custody to meet its own financial obligations or that of the third party.

1.7 Definitions of Terminologies

- (1) **Pension Fund:** This is the accumulation of assets created from contributions and the investments. In other words, it is any plan, fund or scheme which provides retirement income (Bodie et al. 2009).
- (2) **Asset Allocation:** This is the decision of how a fund should be invested across each of several asset classes, assuming that neutral capital market conditions exist.
- (3) **Return on Investment:** This is the evaluation of the overall success of an investment by comparing the investment returns with the amount of investment made initially.
- (4) **Return on Assets:** This compares the net income with the invested capital as measured by average total assets. The return on assets ratio measures how effectively those assets generate profits.
- (5) **Bond:** This is a security issued by a borrower that obligates the issuer to make specified payments to the holder over a specified period.
- (6) **Hybrid Fund:** This is a mutual fund that invests in both stock and bonds. It offers investors the opportunity to diversify their portfolios with a single investment vehicle.
- (7) **Hybrid Securities:** These are complex capital instruments issued by companies to diversify their fund base and manage their cost of capital.
- (8) **Money Market Instruments:** These are debt securities that generally give the owner the unconditional right to receive a stated, fixed sum of money on a specified date. Included are such instruments as

- treasury bills, negotiable certificates of deposit, etc. issued under note issuance facilities (IMF 2003).
- (9) **Fixed Income Securities:** These are investments that provide a return in the form of fixed periodic payments and the eventual return of principal at maturity.
 - (10) **Variable Income Securities:** These are investments in which the rate of return is determined by market and hence, the returns continuously changes with the market dynamics.
 - (11) **Portfolio Return:** This is the proportion weighted combination of the constituent assets' returns.
 - (12) **Nominal Yield:** This is the rate of interest on a bond which often serves as the descriptive term for the bond.
 - (13) **Current Yield:** This is the one period rate of return on a bond. It is the ratio of the money value of one period interest on a bond to the current market price of the bond.
 - (14) **Yield to Maturity:** This is the rate of return on a bond over its entire life span (the holding period rate of return).

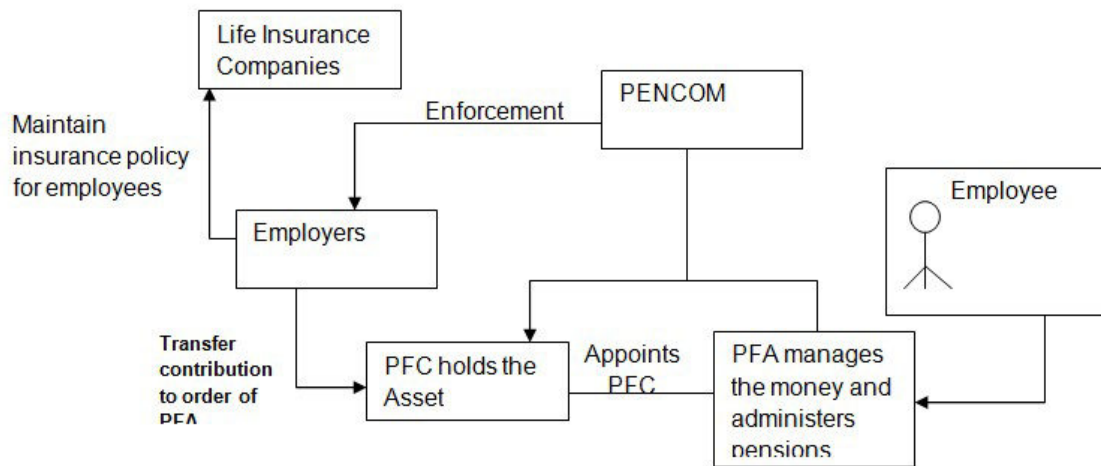


Figure 1. Structure of Contributory Pension Scheme (CPS)

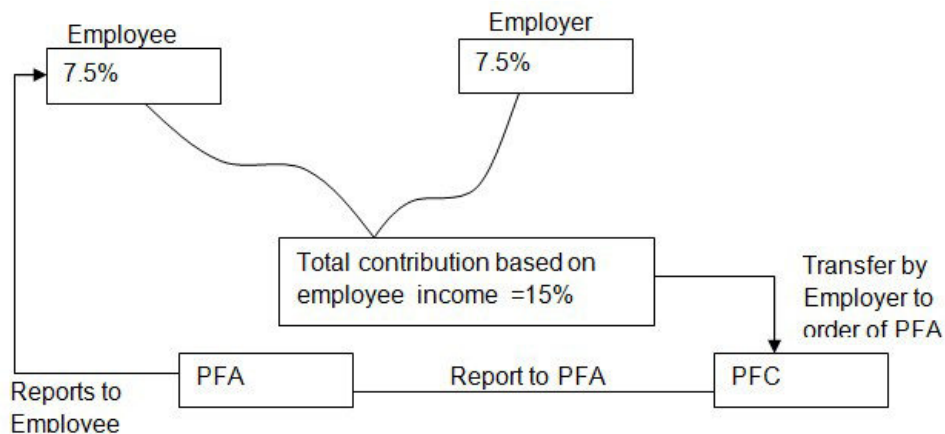


Figure 2. Contribution Fund Flow

1.8 Investment Guidelines for the PFAs

The Nigerian Pension Fund Administrators respect and obey the investment guidelines of PEN COM which include asset classifications. The major asset classes consist of equities of rated companies in Nigeria, bonds of Federal Government and approved corporations, money market instruments from rated financial intermediaries and hybrid funds (Demola 2011)

Specifically, pension fund assets are permitted by PEN COM Act 2004 to invest in the following asset classes: equities, money market, infrastructure funds, private equity funds, open/close-end and hybrid funds, global depository receipts/notes and euro-bonds, bankers' acceptances and commercial paper and bonds.

Table1. Investment Limits and Performance Bench Mark

S/N	Asset Class	Global Portfolio Limit	Per Issuer Limit	Per Issue Limit
1.	Government Securities	FGN 80% State and Local Governments: (i) 20% for issues with guarantee (ii) 3% for issue without guarantee	Maximum of 80% to issues Maximum of 5% of pension fund asset in my state or Local Government	80% of the issue Based on credit rating of bond/debt instrument (i) BBB Rating = 16% of issue (ii) A Rating = 18% of issue (iii) AA Rating = 20% of issue
2	Corporate bonds debt including REITs, mortgages and asset backed securities	35% budget of maximum of 15% in infrastructure bonds	Maximum of 5% of pension assets under management in total issues of any one corporate entity	Based on credit rating of bond/debt instrument. (i) BBB Rating = 16% of issue (ii) A Rating = 18% of issue (iii) AA Rating = 20% of issue
3	Money Market Instruments (certificate of deposits, bankers acceptances and commercial papers)	35%	(a) Maximum investment of PF-assets in all instruments issued by one bank shall be subject to its risk rating: (i) BBB Rating = 3% of assets (ii) A Rating = 4% of assets (iii) AA Rating = 5% of assets (b) Maximum of 3% assets under management in money market instruments issued by any one discount house with a minimum rating of A (c) Maximum of 5% of assets under management in corporate papers any corporate entity	Applicable to commercial paper issues only. Based on credit rating of corporate entity issuing the commercial paper. (i) A rating = 18% of issue (ii) AA rating and above = 20% of issue
4.	Ordinary shares	25%	Maximum of 5% of assets in any one corporate entity	Maximum of 4.5% of the issued capital of any corporate entity
5.	Open, closed-end and Hybrid funds (including REITs)	20%	Maximum of 5% of assets in any one issuer	Maximum of 10% of the value of any one fund

2. Portfolio Optimizations

Portfolio Optimizations refers to mean variance optimization. This means the expected returns of the investment and measure of the risk associated with the portfolio.

Mathematically, optimization problems are stated as follows

Minimize the Risk

Minimize the variance $x^t V_x$ with a specified expected return $r = b^t x$ subject to linear or non linear constraints.

(i) **(Maximize the Expected Return)**

Maximize the expected return $r = b^t x$ with a specified variance $v = x^t V$

(ii) **(Maximize Expected Return with Risk Aversion)**

Maximize $\lambda b^t x - x^t V_x$ subject to a linear or non linear constraints. The risk aversion parameter is denoted by λ . The maximization problem is also equivalent to minimize $x^t V_x - \lambda b^t x$

(iii) **(Minimize Risk)**

Minimize the variance $x^2 V_x$ subject to linear or non-linear constraints.

(iv) **(Maximize the Expected Return)**

Maximize the expected return $r = b^t x$ subject to linear or non linear constraints. The maximization problem is equivalent to minimize $-b^t x$.

(v) **(Minimize the Expected Return)**

Minimize the expected return $r = b^t x$ subject to linear or non linear constraints.

(vi) **(Minimise Risk with respect to a Benchmark)**

Minimize $(x - x_b)^t V(x - x_a)$ with a specified expected relative return r defined by $r = b^t (x - x_a)$ subject to linear or non linear constraints where x_b denotes a benchmark portfolio (Elton & Gruber 1995; Chang et al. 1996; Markowitz 1991; Hensel et al. 1999)

2.1 Optimal Solutions to Portfolio

2.1.1 Numerical Algorithm Group Method (NAGM).

The mathematical problem of portfolio optimization can be formulated in many ways terminating as linear or non-linear constraints, equality and inequality constraints. To use the NAG method, we employ

Where y_i is the transaction cost for equity i

$$p_i \leq y_i, q_i \leq y_i, \text{ for } i = 1, 2, \dots, n$$

2.1.4 Portfolio Return Maximization using Linear Programming Technique

A portfolio return optimization using linear programming technique produces these components;

- (i) Decision variables to be determined as components of asset classes:
 - (a) equity weight (b) bond weight (c) money market instruments weight (d) hybrid instrument weight
- (ii) Objective to optimized: The portfolio mix needs a maximal return. Thus, we need to maximize the return of the portfolio as a function of the rate of returns of the components of the portfolio.
 If an asset class has a fixed return rate, then we can set an objective function on the basis of its individual return and weight it contributes to the portfolio mix
- (iii) Constraints that the solution must satisfy: These include the weight each class in the portfolio occupies as related to the scarce resource and the minimum tolerable risk.
- (iv) Data which quantify the relationships represented in the objective function and the constraints.

3. Research Methodology and Data

In this work, we employ linear programming technique to formulate our analytical frame work. The technique is used to select the optimal mix of opportunities that will maximize return while meeting the investment conditions set by each portfolio class and other constraints by PEN COM.

Time Series Data (Quarterly) sourced from First Alliance Pension and Benefits Limited are in use for this study. Net Asset Values are used as scarce resources and distributed as different portfolios of different asset classes for comparative analysis using TORA Operations Research software.

The rates of return of the selected portfolio classes will be tested against the rates of return of PEN COM guide portfolio of PFAs.

3.1 Research Hypotheses and Test Statistic

The rates of return of the selected portfolio classes are tested against the rates of return of PENCOR guided portfolios of PFAs.

$$H_0: \mu_{PRA} \leq \mu_{OPT}$$

- PRA = Pension Reform Act Portfolio
- OPT = Optimized Portfolio

Using $\alpha = 0.05$ significance level and knowing fully well that a large positive value of $d = \bar{X}_{PRA} - \bar{X}_{OPT}$ will refute the null hypothesis, the test is upper-tailed.

We also define our t-satistic test as follows:

$$t = \frac{\bar{X}_{PRA} - \bar{X}_{OPT}}{\sqrt{\frac{(n_{PRA} - 1)S_{PRA}^2 + (n_{OPT} - 1)S_{OPT}^2}{n_{PRA} + n_{OPT} - 2} \left(\frac{1}{n_{PRA}} + \frac{1}{n_{OPT}} \right)}}$$

3.2 Specification of the Model

In defining the decision variables, we itemize the security classes in the portfolio which shares the net asset values that are available.

Let X_1 , X_2 , and X_3 represent the values of the allocations to the security classes in the portfolio which are ordinary shares amount, government securities amount and money market amount respectively.

The total net asset values typify the scarce resources which must be optimized to form the following classes:

Table 2. Ratio of Asset Classes

	Equity (W_1)	Govt. securities (W_2)	Money market (W_3)
Conservative	25%	40%	35%
Moderately Aggressive	55%	20%	25%
Aggressive	70%	10%	25%

Table 3. Loss Rate

Security	Equity	Govt. securities	Money market
Maximum Expected Loss Rate (S_i)	15%	0%	5%

Source: Two Year Transaction Records of Pension Industry

The maximum portfolio loss is $\frac{1}{n} \sum_{i=1}^n S_i W_i$

where S_i = loss rate of security class
 W_i = weight of security class in portfolio
 n = number of security classes in the portfolio

Any value of X_i^s that satisfies all constraints constitutes a feasible solution, otherwise the solution is infeasible.

The goal of the problem is to find the best feasible solution (the optimum that maximizes the total return)
 Thus, the objective function is:

$$\text{Maximize } P_i = \sum R_i(1 - S_i)X_i - \sum S_i X_i \dots \dots \dots (9)$$

where P_i = portfolio of the asset classes
 R_i = rate of return of securities
 X_i = value of securities in the portfolio

3.3 Formulation of the Constraints

(a) The Constraint on the Total Portfolio Asset (Net Asset Value) is given as

$$X_1 + X_2 + X_3 \leq A \dots \dots \dots (10)$$

where A is the constraint of the Net Asset Value

(b) The restrictions on ratio of allocating funds to the security classes considered are specified as follows:

$$X_1 \leq W_1 (X_1 + X_2 + X_3) \dots \dots \dots (11)$$

$$X_2 \geq W_2 (X_1 + X_2 + X_3) \dots \dots \dots (12)$$

$$X_3 \leq W_3 (X_1 + X_2 + X_3) \dots \dots \dots (13)$$

(c) The Limitation on Loss of Expected Return is given as

$$S_1 X_1 + S_2 X_3 \leq \sum_{i=1}^n S_i W_i X_i \dots \dots \dots (14)$$

(d) The non-negativity constraint is given as $X_1, X_2, X_3 \geq 0$

3.4 Equation and Model Expectation

The maximum value of P in equation (9) is expected to increase with increased volume of equity weight from the total net asset value. This implies that as the portfolio moves from conservative to aggressive portfolio, the value of P increases in that direction. This is from the expectation that investment return is more from a higher risky security than from lower risky investments.

It is also expected that P increases as the portfolio changes from conservation to aggressive, even when the values of R_i in equation (9) changes proportionally. The bond limit constraint inequality (12) is expected to pose the risk of infeasible solution. This implies that there will be the use of artificial variable and in this case, we expect that the model will have a feasible space.

4. Discussion of Data and Results

From the data table in appendix A, equity has a weight of 23.36% of the total asset value of ₦4,124,547,984 as its opening balance. Its closing balance stands at ₦1,541,711,872 with a total return of ₦578,141,952.01 which provides a return rate of 60%. The closing balance is an aggregate of dividend, bonuses and the equity value in total market capitalization.

From the same table, government security opening balance is ₦1,966,113,403.68 carrying asset weight of 47.66% in a total net asset value of ₦4,124,547,984. This security with a closing balance of ₦2,182,385,877 contributes a return of ₦216,272,474.30 which is a return rate of 11%.

Money market instrument opened with a balance of ₦1,194,864,661.94 which is a weight of 28.96% of the total net asset value. With a closing balance of ₦1,672,810,525, money market instrument made a return of ₦477,945.864.40 within the period under review. This return contributes 40% return rate.

The aggregate return from the three security classes, is ₦1,272,360,291 which translates to 30.8% return rate.

4.1 Optimization Results

From the data used for optimizing return of different portfolio classes, the net asset value used is ₦11.32bn, rate of return from equity investment is 60%, rate of return from government securities is 11% and rate of return from money market instrument is 40%.

The input data satisfying equations (9) and (14) for conservative portfolio class is presented below

Table 4. Conservative Portfolio Class

	Equity (X ₁)	Govt. securities (X ₂)	Money market (X ₃)	
Maximize	0.36	0.11	0.31	
Subject to				
Net Asset value	1.0	1.0	1.0	≤ 11.32
Equity	0.75	- 0.25	- 0.25	≤ 0
Govt. securities	- 0.40	- 0.60	- 0.40	≥ 0
Money market	- 0.35	- 0.35	0.65	≤ 0
Prob of loss	0.1425	- 1.00	0.0425	

Results

- (a) Objective value = ₦2.751bn
- (b) Objective value contributions:
 - (i) Equity = ₦1.0188bn
 - (ii) Govt. Securities = ₦0.4982bn
 - (iii) Money Market ₦1.2282bn

From the objective value contributions, money market instrument contributed the highest value of ₦1.2282bn. The ratio of the money market instrument is lower than the government securities, but higher than equity in the conservative portfolio class.

We therefore establish that the money market returned higher than equity because it has higher ratio in the portfolio class and returned higher than government securities because it has higher rate of return in the objective function.

The result of the objective value of ₦2.7451bn also reveals a portfolio return rate of 24.25%. This is lower than the return rate of the portfolio class in appendix A. The result of conservative portfolio class also shows that a unit increase in the Net Asset value increases the objective value by ₦0.2425b. A one percent increase in equity value increases the objective value by ₦0.05bn. A one percent increase in government securities increases the objective value by ₦0.2bn. There is also no economic advantage in increasing the ratio of money market instrument in the conservative portfolio since the dual price of money market is zero.

Suppose d₁, d₂ and d₃ denote changes (positive or negative) in the security class ratio of the portfolios, then these set of inequalities must be satisfied simultaneously:

$$2.83 + 0.25d_2 + d_3 \geq 0$$

$$4.5380 - d_1 + 0.4d_2 \geq 0$$

$$3.9620 + d_1 + 0.35d_2 - d_3 \geq 0$$

To maintain the optimality of the conservative portfolio,
 Maximize P = (0.36 + d₁)X₁ + (0.11 + d₂)X₂ + (0.31 + d₃)X₃

For d₁, d₂ and d₃ to be positive or negative, then these inequalities derived from the result must be satisfied simultaneously:

$$0.2 + d_3 - d_2 > 0$$

$$0.2425 + 0.25d_1 + 0.4d_2 + 0.35d_3 > 0$$

$$0.05 + d_1 - d_3 > 0$$

Solution matrix to the inequalities above satisfying the conditions to maintain the optimal value of the objective function is

$$\left\{ (d_1 d_2 d_3) \begin{bmatrix} 0 & 0.25 & 1 \\ -1 & 0.4 & 0 \\ 1 & 0.35 & -1 \end{bmatrix} \right\}^T \geq \begin{bmatrix} 0.2 \\ 0.2425 \\ 0.05 \end{bmatrix}$$

The input data satisfying the set of inequalities (9) and (14) for moderately aggressive portfolio is presented below:

Table 5. Moderately Aggressive Portfolio

	Equity (X ₁)	Govt. Securities (X ₂)	Money Market (X ₃)	
Maximize	0.36	0.11	0.31	
Subject to				
Net Asset Value	1.0	1.0	1.0	≤ 11.32
Equity	0.45	- 0.55	- 0.55	≤ 0
Govt. securities	- 0.20	0.80	- 0.20	≥ 0
Money market	- 0.25	- 0.25	0.75	≤ 0
Total loss	0.1425	- 1.0	0.0425	≤ 0

Results

- (a) Objective value = ~~N~~3.3677bn
- (b) Objective value contributions:
 - (i) Equity = ~~N~~2.2414bn
 - (iii) Govt. Securities = ~~N~~0.2490bn
 - (iv) Money Market ~~N~~0.8773bn

From the aggregate contributions for the different securities, equity contributes the highest value among other security classes.

The result of the objective value of 3.3677 also shows portfolio return rate of 29.75% which is still lower than the return rate generated from the data on appendix 1.0 and its associated asset ratio in the portfolio.

The d_i s may also be positive or negative and the following set of inequalities must be satisfied simultaneously;

$$0.2 - d_4 + d_5 \geq 0$$

$$0.2975 + 0.55d_3 + 0.2d_4 + 0.25d_5 \geq 0$$

$$0.05 + d_3 - d_5 \geq 0$$

The optimality of the objective value of moderately conservative portfolio remains optimal over a range of value if

$$\text{maximize } P = (0.36 + d_3)X_1 + (0.11 + d_4)X_2 + (0.31 + d_5)X_3$$

The following sets of inequalities derived from the result are satisfied simultaneously if d_i s are positive or negative:

- (i) $6.2260 + 0.55d_4 + d_5 \geq 0$

- (ii) $2.2640 - d_3 + 0.2d_4 \geq 0$

- (iii) $2.83 + d_3 + 0.25d_4 - d_5 \geq 0$

The solution matrix to the inequalities above is given as:

$$\left\{ (d_3 d_4 d_5) \begin{bmatrix} 0 & 0.55 & 1 \\ -1 & 0.2 & 0 \\ 1 & 0.25 & -1 \end{bmatrix} \right\}^T \geq \begin{bmatrix} 0.2 \\ 0.2975 \\ 0.05 \end{bmatrix}$$

The input data satisfying the objective function (9) and set of inequalities (10) through (14) for aggressive portfolio class is presented below:

Table 6. Aggressive Portfolio Class

	Equity (X_1)	Govt. Securities (X_2)	Money Market (X_3)	
Maximize	0.36	0.11	0.31	
Subject to				
Net Asset Value	1.0	1.0	1.0	≤ 11.32
Equity	0.3	- 0.7	- 0.7	≤ 0
Govt. Securities	- 0.1	0.9	- 0.1	≥ 0
Money Market	- 0.2	- 0.2	0.8	≤ 0
Total Loss	0.1425	- 1.0	0.0425	≤ 0

Results

- (a) Objective value = ~~N~~3.36611bn
- (b) Objective value contributions
 - (i) Equity = ~~N~~2.8528bn
 - (ii) Govt. Security = ~~N~~0.1344bn
 - (iii) Money Market ~~N~~0.6741bn

The result above shows that equity which had the highest percent in the portfolio also returned the highest value of N2.8528bn. The consistent increase in return over an increase in the weight of the asset classes in the portfolio suggests a direct relationship between size of asset in a portfolio and its return.

As in moderately aggressive portfolio, we set d_6 , d_7 and d_8 to changes (positive or negative) in the values of the right hand side of the constraint inequalities for equity, government securities and money market respectively.

For the feasibility condition of the dual price claim to hold, the following sets of inequalities are satisfied simultaneously:

$$0.323 + 0.7d_6 + 0.1d_7 + 0.1921d_8 \geq 0$$

$$0.308 + d_6 + 0.0959d_7 - 1.0959d_8 \geq 0$$

$$0.1918 - 0.9592d_7 + 0.9592d_8 \geq 0$$

For the objective value of the aggressive portfolio to remain feasible over any change,

$$\text{maximize } P = (0.36 + d_6)X_1 + (0.11 + d_7)X_2 + (0.31 + d_8)X_3$$

The following set of inequalities must be satisfied simultaneously:

- (i) $7.9240 + 0.7d_6 + d_7 > 0$
- (ii) $1.2216 + 0.1079d_6 + 0.0959 d_7 - 0.9592d_8 \geq 0$
- (iii) $2.1744 + 0.1921d_6 - 1.095d_7 + 0.9592 \geq 0$

The solution matrix to the above inequalities satisfying the conditions for optimal values of the objective function is given by;

$$\left\{ (d_6 \ d_7 \ d_8) \begin{bmatrix} 0.7 & 1 & 0 \\ 0.1079 & 1 & 0 \\ 0.1921 - 0.0959 & 0.9592 & \end{bmatrix} \right\}^T \geq \begin{bmatrix} 0.3234 \\ 0.0308 \\ 0.1918 \end{bmatrix}$$

Table 7. Dual Prices and Investment Expectation

	Equity (X ₁)	Govt. Securities (X ₂)	Money Market (X ₃)
Conservative	0.05	- 0.2	0.0
Moderate	0.05	- 0.2	0.0
Aggressive	0.05	- 0.2	0.0

Table 8. Investment Expectation

Net Asset Value	Pen Com guided Portfolio	Conservative Portfolio	Moderately Aggressive	Aggressive
₦11.32bn	27.72%	24.24%	2.976	32.32%

Table 9. t – Statistic Comparison of Return Rate

Period	PPENCOM Guided Return (%)	Return from optimized Portfolio (%)
First Quarter (Q ₁)	9.12	6.06
Second Quarter (Q ₂)	6.0	7.44
Third Quarter (Q ₃)	5.0	8.08
Fourth Quarter (Q ₄)	7.6	7.19

The computed value of t = -0.12 falls within the acceptance region. This implies that the null hypothesis stating that the rate of return from PENCOM guided portfolio class is not optimal should not be rejected.

5. Conclusion

The objective of this research work has been achieved because the PENCOM guided portfolio is not the optimum as revealed by the hypothesis test. However, the performance is not a deviation from the performance of the best estimated portfolio.

It is observed that some investment areas such as REIT markets, mortgage backed up securities and hybrid open and close fund investments in which pension fund would have benefited were completely absent in this research work. This implies that more work still needs to be done in this area of study.

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Appendices

Appendix A

Asset Class	Opening Balance (₦)	Closing Balance (₦)	Return (₦)	Rate of Return (%)
Equity	963,569,920.28	1,541,711,872.00	578,141,952.01	60
FGN Bond	1,966,113,403.68	2,182,385,877.0	216,272,474.30	11
Money Market	1,194,864,661.94	1,672,810,525.00	477,945,864.40	40
Totals	4,124,547,984.00	5,396,908,274.00	1,272,360,291.00	30.84

Source: PFAs Unpublished Report

Appendix B

	March-June	June-September	September-December	December-March
Opening	4,124,547,984.00	4,503,034,906.00	4,775,223,373.00	5,011,979,022.00
Closing	4,503,034,906.00	4,775,223,373.00	5,011,979,022.00	5,393,687,109.00
Difference	378,486,922.00	272,188,467.30	236,755,649.10	381,708,087.30
RR (%)	0.09 \equiv 9.12%	0.06 \equiv 6.0%	0.05 \equiv 5.0%	0.08 \equiv 7.6%

Source: PFAs PENCOM Guided Portfolio Quarterly 2007 – 2008 Unpublished

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