

Dynamic Relationship between Production Growth Rates of Three Major Cereals in Ghana

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Abstract
Cereals play a major role in contributing to agricultural Gross Domestic Product and the economy, and are also used for the preparation of several local dishes and drinks in Ghana. Several linkages have been hypothesised of the relationship among the production growth rates of cereals. This study employed a Vector Autoregressive (VAR) model to investigate the relationship between the production growth rates of three major cereals in Ghana. The VAR model favoured VAR at lag 1 which indicated that, in addition to the bivariate unidirectional production growth rate causalities; there is also a bilateral causality between production growth rate in Millet and production growth rate in Milled Rice and a Rice to Corn unidirectional production growth rate causality. A diagnostic test revealed that the VAR (1) model was stable as it satisfies the stability condition. Also, the univariate ARCH-LM test and Ljung-Box test revealed that the model is free from conditional heteroscedasticity and serial correlation respectively. The Impulse Response Function and the Forecast Error Variance Decomposition were further employed to interpret the VAR (1) model. The Forecast Error Variance Decomposition revealed that growth rate in Millet production explains an appreciable amount of the forecast uncertainty in Rice and Corn.
Keywords: Production, Growth rates, Corn, Millet, Milled Rice, Granger-causality.

1. Introduction

Umpteen of people over the world depend on cereals as their sources of livelihood. Rice, Corn and Millet are among the most important cereals grown all over the world and feed several millions of the world's population. In Ghana, these cereals are grown all over the country especially in the Northern regions and also have great socio-economic importance: They play a major role in contributing to agricultural Gross Domestic Product (GDP) and the economy of Ghana, and are also used for the preparation of several local dishes and drinks both for commercial and household consumption.

A number of researches have been done on cereals using univariate time series analysis. These include the work done by Badmus and Ariyo (2011) on forecasting area of cultivation and production of maize in Nigeria. Najeeb *et al*., (2005) employed Box-Jenkins model to forecast wheat area and production in Pakistan. In Ghana, Suleman and Sarpong (2012a) modeled milled rice production using the Box-Jenkins approach. In another study, Suleman and Sarpong (2012b) modeled production and consumption of corn in Ghana using ARIMA models.

Several linkages have been hypothesised of the relationship among the production of cereals. This study therefore explored the dynamic relationship between the production growth rates of Milled Rice, Corn and Millet in Ghana.

2. Materials and Methods

The study was carried out using the production data of Corn, Millet and Milled Rice from 1960 to 2012 collected from a secondary source (Index Mundi, 2013). The data for the cereals were transformed to obtain the growth rates in the production of each of these cereals. The growth rate for each cereal is given by

$$\text{growth rate} = 100 \times \ln\left(\frac{C_t}{C_{t-1}}\right)$$

where C_t and C_{t-1} are the production of the cereal at time t and $t-1$ respectively.

2.1 Augmented Dickey-Fuller Test

The order of integration of data was investigated using the Augmented Dickey-Fuller (ADF) test. The regression model employed by Dickey and Fuller (1979) is given by;

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta Y_{t-i} + \varepsilon_t$$

where α is a constant, β the coefficient on time trend series, $\sum_{i=1}^{p} \gamma_i \Delta Y_{t-i}$ is the sum of the lagged values of the dependent variable ΔY_t and p is the lag order of the autoregressive process. The parameter of interest in the ADF test is δ. For $\delta = 0$, the series contains unit root and hence non-stationary. The choice of the starting augmentation order depends on; data periodicity, significance of γ_i estimates and white noise residuals. The test statistic for the ADF test is given by

$$ADF = \frac{\hat{\delta}}{\mathrm{SE}(\hat{\delta})}$$

where $\mathrm{SE}(\hat{\delta})$ is the standard error of the least square estimate of $\hat{\delta}$. The null hypothesis is rejected if the test statistic is greater than the critical value.

2.2 Vector Autoregressive (VAR) Model

A VAR process consists of a set of K endogenous variables $\boldsymbol{Y_t} = (y_{1t}, y_{2t}, \dots, y_{kt})$ for $k = 1, 2, \dots, K$. A VAR process of order p is given by

$$\boldsymbol{Y_t} = \boldsymbol{A_1 Y_{t-1}} + \boldsymbol{A_2 Y_{t-2}} + \dots + \boldsymbol{A_p Y_{t-p}} + \boldsymbol{u_t}$$

where $\boldsymbol{A_i}$ are $(K \times K)$ coefficient matrices for $i = 1, 2, \dots, p$ and $\boldsymbol{u_t}$ is a K-dimensional white noise process with time invariant positive definite covariance matrix. An important characteristic of a VAR (*p*) process is its stability. This implies that given sufficient starting values, the VAR (*p*) process generates stationary time series with time invariant means, variances and covariance structure. The stability is determined by evaluating the reverse characteristic polynomial

$\det(I_k - A_1 z - \dots - A_p z^p) \neq 0$ for $|z| \leq 1$. If the solution of the reverse characteristic polynomial has a root $z = 1$, then either some or all the variables in the VAR (*p*) process are integrated of order one. In practice, the stability of an empirical VAR (*p*) process can be analysed by calculating the eigenvalues of the coefficient matrix. If the moduli of the eigenvalues of $\boldsymbol{A_i}$ are less than one, then the VAR (*p*) process is stable.

2.3 VAR Lag Order Selection

An essential step in fitting a VAR (*p*) process is determining the optimum lag for the process. In this study, the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC) were employed to determine the optimum lag length for VAR (*p*) process. The criteria are given by

$$AIC = \ln\left|\widehat{\sum\nolimits_u (p)}\right| + \frac{2}{T} pK^2$$

$$HQIC = \ln\left|\widehat{\sum\nolimits_u (p)}\right| + \frac{2\ln\ln(T)}{T} pK^2$$

$$SIC = \ln\left|\widehat{\sum\nolimits_u (p)}\right| + \frac{\ln(T)}{T} pK^2$$

where *T* is the number of observations, *p* assigns the lag order and $\widehat{\sum_u (p)} = T^{-1} \sum_{t=1}^{T} \widehat{u_t} \widehat{u_t'}$.

2.4 Impulse Response Function

The impulse response function was used to investigate the dynamic interaction between the endogenous variables and is based upon the Wold representation of the VAR (*p*) process. The Wold representation is based on the orthogonal errors $\boldsymbol{\eta}_t$ and is given by

$$\mathbf{Y_t} = \boldsymbol{\mu} + \boldsymbol{\Theta_0 \eta_t} + \boldsymbol{\Theta_1 \eta_{t-1}} + \boldsymbol{\Theta_2 \eta_{t-2}} + \dots$$

where $\boldsymbol{\Theta_0}$ is a lower triangular matrix. The impulse response to the orthogonal shocks $\boldsymbol{\eta_{jt}}$ are

$$\frac{\partial y_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial y_{i,t}}{\partial \eta_{j,t-s}} = \Theta_{ij}^{s} \quad i, j = 1,2, \dots, n, s > 0$$

where Θ_{ij}^{s} is the $(i, j)th$ element of $\boldsymbol{\Theta_0}$. For *n* variables there are n^2 possible impulse response functions.

2.5 Forecast Error Variance Decomposition (FEVD)

The FEVD was used to determine the contribution of the j^{th} variable to the h-step forecast error variance of the i^{th} variable. The FEVD is given by

$$FEVD_{i,j}(h) = \frac{\sigma_{\eta_j}^2 \sum_{s=0}^{h-1} (\Theta_{ij}^{s})^2}{\sigma_{\eta_1}^2 \sum_{s=0}^{h-1} (\Theta_{i1}^{s})^2 + \dots + \sigma_{\eta_n}^2 \sum_{s=0}^{h-1} (\Theta_{in}^{s})^2} \quad i, j = 1, 2, \dots, n$$

where $\sigma_{\eta_j}^2$ is the variance of η_{jt}. A VAR (*p*) process with *n* variables will have n^2 $FEVD_{i,j}(h)$ values.

2.6 Causality Test

A variable y_t is said to Granger-cause a variable z_t if the past values of y_t has additional power in forecasting z_t after controlling for the past of z_t (Gelper and Croux, 2007). Causality may be classified as unidirectional, bilateral or independent (Gujurati, 2003).

3.0 Results and Discussion

Table 1 shows the ADF test performed on the growth rate in production of the cereals. The test performed with constant only and with constant and trend revealed that the data was stationary. The stationarity in the production growth rate of the cereals is affirmed by the time series plot of the data. As shown in Figure 1, the data for the three cereals fluctuates about a fixed point indicating that the growth rate in production of the cereals is stationary. This property of the data is a good justification for fitting the Vector Autoregressive model. The appropriate lag order for the model was selected using the information criterion: From Table 2, the AIC, HQIC, and SBIC selected lag 1 as the optimum lag order for the model as it had the least value for all the information criteria.

Thus, VAR (1) was estimated for the production growth rate as shown in Table 3. The lag1 value for Millet is useful in predicting the growth rate in Rice production while the lag 1 values for Rice and Corn are not. The lag 1 value for Millet is useful in predicting the growth rate in Rice production while that of Corn and Rice itself are not. The lag 1 values for Rice and Corn are useful in predicting the growth rate in Corn production while that of Millet is not. In addition, the lag 1 values for Rice and Millet are useful in predicting Millet production while that of corn is not.

The stability of the VAR (1) model was investigated. The results revealed the model was stable as all the eigenvalues have modulus less than one as shown in Table 4. This affirms that all the series used are stationary as revealed by the ADF test. Also, the CUSUM plot in Figure 2 affirms that the model is stable as the recursive residuals for the individual equations are within the confidence limit.

The univariate Ljung-Box test and ARCH-LM test were used to diagnose the model and as shown in Table 5 and Table 6, the model residuals are free from serial correlation and conditional heteroscedasticity respectively; this indicates that the fitted model is adequate. The model was then used to investigate Granger causality among the cereals. Table 7 revealed that Millet Granger-cause Rice and Rice Granger-cause Millet, thus there is a bilateral causality between Millet and Rice. Also, Corn does not Granger-cause Millet and Rice but Corn and Millet Granger-cause Rice and Corn and Rice Granger-cause Millet: These results imply that, the growth rate in Corn production alone cannot be used to predict the growth rate in the production of the other cereals unless combined with that of another cereal. In addition, growth rate in Rice production Granger-cause growth rate in Corn production.

The Impulse Response analysis in Figure 3 depicts the way the cereals in the model interact following a shock in the VAR model. When the impulse variable is Rice, in the first period Rice reacts positively to a shock in its own values followed by a negative response in the second and third period. The fourth period shows a positive response followed by a stable response for the rest of the periods. Millet reacted negatively to a shock in Rice in the second period followed by a positive reaction in the third period and then a stable response for the rest of the periods. Corn reacted positively in the second period, negatively in the third period, positively in the fourth period and the followed by a stable response for the rest of the periods. When the impulse variable is Millet, Rice reacted positively in the second period, negatively in the third period and then positively in the fourth period followed by a stable response for the rest of the period. Millet reacts positively to a shock in itself in the first period, negatively in the second period, positively in the third period and then followed by a stable response for the rest of the periods. Corn reacted positively in the first three periods followed by negative response in the fourth period and then a positive response in the fifth period. When the impulse variable is Corn, Rice reacted positively for the first two periods, negatively in the third period and then a positive response in the fourth period followed by a stable response for the rest of the periods. Millet reacted positively in the first period, negatively in the second period, positively in the third period and the followed by a stable response for the rest of the periods. Corn reacted positively in the first, second and fifth period for a shock in itself. The third and fourth period exhibited negative response followed by stable response for the other periods.

The Impulse Response analysis does not clearly show the magnitude of the relationship among the variables. The Variance Decomposition for the variables was therefore examined. Table 8 displays the Variance Decomposition for Rice. Aside Rice itself, the influence of Millet contributes most in forecasting the uncertainty of Rice. For instance at period ten, about 82.03% of the variance in Rice appears to have been explained by innovations in Rice, while 12.56% and 5.41% was explained by innovations in Millet and Corn respectively. Also, apart from Millet itself, the influence of Rice contributes most in forecasting the uncertainty in Millet as shown in Table 9. At period ten about 71.39% of the variance in Millet appears to have been explained by innovations in Millet, while 26.84% and 1.77% was explained by innovations in Rice and Corn respectively. Finally, apart from Corn itself, the influence of Millet contributes most in forecasting the uncertainty of Corn. At period ten about 63.07% of the variance in Corn appears to have been explained by innovations in Corn, while 22.54% and 14.40% was explained by innovations in Millet and Rice respectively as shown in Tale 10.

4.0 Conclusion

In this study, the relationship between the production growth rates of three major cereals in Ghana was investigated. The results revealed that there was bilateral causality between Rice and Millet. Also, the growth rate in Rice production Granger-cause the growth rate in Corn production. The growth rate in Corn production cannot be used in predicting growth rate in the production of the other cereals. The Forecast Error Variance

Decomposition revealed that growth rate in Millet production explains an appreciable amount of the forecast uncertainty in Rice and Corn

5.0 References

Adenomon, M. O., Ojehomon, V. E. T., and Oyejola, B. A. (2013). Modeling the Dynamic Relationship Between Rainfall and Temperature Time Series Data In Niger State, Nigeria. *Mathematical Theory and Modeling*, 3 (4): 53-70.

Badmus, M. A, and Ariyo, O. S. (2011). Forecasting Cultivated Areas and Production of Maize in Nigeria Using ARIMA model. *Asian Journal of Agricultural Sciences*, 3 (3): 171-176.

Dickey, D. A., and Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit-root. *Journal of American Statistical Association*, *74*: 427-431.

Gelper, S., and Croux, C. (2007). Multivariate Out-of-Sample Tests for Granger Causality. *Computational Statistics and Data Analysis*, 51: 3319-3329.

Gujurati, D. N. (2003). Basic Econometrics. Fourth Edition. New Delhi. The McGraw-Hill Co.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. *Econometrica*, 37 (3): 424-438.

Index Mundi (2013). Production and Consumption of Cereals by Country. www.indexmundi.com, Accessed on 15th May, 2013.

Lütkepohl, H., (2005). New Introduction to Multiple Time Series Analysis. New York: Springer Berlin Heidelberg.

Najeeb, I., Khuda, B., Asif, M., and Abid, S. A. (2005). Use of ARIMA Model for Forecasting Wheat Area and Production in Pakistan. *Journal of Agricultural and Social Sciences*, 1 (2): 120-122.

Suleman, N. and Sarpong, S. (2012). Forecasting Milled Rice Production in Ghana Using Box-Jenkins Approach. *International Journal of Agricultural Management and Development*, 2 (2): 79-84.

Suleman, N., and Sarpong, S. (2012). Production and Consumption of Corn in Ghana: Forcasting Using ARIMA Models. *Asian Journal of Agricultural Sciences*, 4 (4): 249-253.

Table 1: Augmented Dickey-Fuller test of series

	Constant		Constant+ Trend	
Cereal	Test Statistic	*P*-value	Test Statistic	*P*-value
Rice	-4.5725	0.0001	-4.5140	0.0014
Millet	-4.5058	0.0001	-4.6730	0.0007
Corn	-11.7446	0.0000	-11.6300	0.0000

Table 2: Lag selection criteria

Information Criteria	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
AIC	27.9933*	28.0921	28.3491	28.3745	28.3800
HQIC	28.1266*	28.3587	28.7491	28.9078	29.0466
SBIC	28.3476*	28.8007	29.4120	29.7917	30.1514

*: Means best based on the information criteria

Table 3: VAR (1) model for the three cereals

Equations	Variables	Coefficient	Std. Error	Z- Statistic	P> |Z|
Millet	Millet.L1	-0.6383	0.1290	-4.9500	0.0000
	Rice.L1	0.5957	0.1190	5.0100	0.0000
	Corn.L1	0.0817	0.1502	0.5400	0.5870
Rice	Millet.L1	-0.4987	0.1396	-3.5700	0.0000
	Rice.L1	-0.0844	0.1287	-0.6600	0.5120
	Corn.L1	0.2945	0.1626	1.8100	0.0700
Corn	Millet.L1	-0.1605	0.1214	-1.3200	0.1860
	Rice.L1	0.3639	0.1120	3.2500	0.0010
	Corn.L1	-0.4281	0.1414	-3.0300	0.0020

Table 4: VAR (1) stability condition

Eigenvalue	Modulus
-0.5309661	0.530966
-0.3099085+0.3756596*i*	0.486994
-0.3099085-0.3756596*i*	0.486994

Table 5: Univariate Ljung-Box test

Equations	Lag	Test statistic	*P*-value
Millet	12	12.2469	0.4260
	24	27.4470	0.2840
Rice	12	12.9585	0.3720
	24	27.9099	0.2640
Corn	12	11.1312	0.1580
	24	33.8319	0.0877

Table 6: Univariate ARCH-LM test

Equations	Lag	Test statistic	*P*-value
Millet	12	6.9559	0.4601
	24	24.7011	0.4317
Rice	12	11.8218	0.8605
	24	24.5295	0.4221
Corn	12	5.2239	0.9501
	24	26.9352	0.3075

Table 7: Granger causality test

Equations	Excluded	Chi2	df	Prob> Chi2
Millet	Rice	25.0710	1	0.0000
	Corn	0.2957	1	0.5870
	ALL	25.984	2	0.0000
Rice	Millet	12.768	1	0.0000
	Corn	3.2812	1	0.0700
	ALL	12.833	2	0.0020
Corn	Millet	1.7472	1	0.1860
	Rice	10.5600	1	0.0010
	ALL	11.0930	2	0.0040

Table 8: Forecast Error Variance Decomposition for Rice

Period	Std. Error	Rice	Millet	Corn
1	27.0575	100.0000	0.0000	0.0000
2	29.2964	86.1454	9.7777	4.0769
3	30.4590	82.1066	12.5240	5.3694
4	30.7152	82.0869	12.5090	5.4040
5	30.7391	82.0833	12.5171	5.3995
6	30.7481	82.0384	12.5525	5.4091
7	30.7519	82.0324	12.5573	5.4103
8	30.7525	82.0330	12.5569	5.4101
9	30.7526	82.0326	12.5571	5.4103
10	30.7526	82.0325	12.5572	5.4103

Table 9: Forecast Error Variance Decomposition for Millet

Period	Std. Error	Rice	Millet	Corn
1	25.0058	3.2778	96.7222	0.0000
2	32.2002	20.1832	79.5571	0.2597
3	33.8826	26.6777	72.8138	0.5085
4	34.1752	26.9849	71.7600	1.2552
5	34.2920	26.8160	71.5231	1.6609
6	34.3290	26.8307	71.4128	1.7565
7	34.3341	26.8404	71.3920	1.7675
8	34.3345	26.8400	71.3919	1.7682
9	34.3348	26.8401	71.3917	1.7682
10	34.3348	26.8404	71.3914	1.7683

Table 10: Forecast Error Variance Decomposition for Corn

Period	Std. Error	Rice	Millet	Corn
1	23.5362	7.0991	20.0551	72.8458
2	27.2189	10.8954	24.6536	64.4510
3	28.5361	14.2453	23.3049	62.4498
4	28.9276	14.5520	22.6968	62.7512
5	29.0599	14.4250	22.5857	62.9893
6	29.1000	14.4015	22.5500	63.0485
7	29.1083	14.4001	22.5380	63.0619
8	29.1098	14.3989	22.5362	63.0649
9	29.1101	14.3987	22.5359	63.0654
10	29.1103	14.3988	22.5357	63.0654

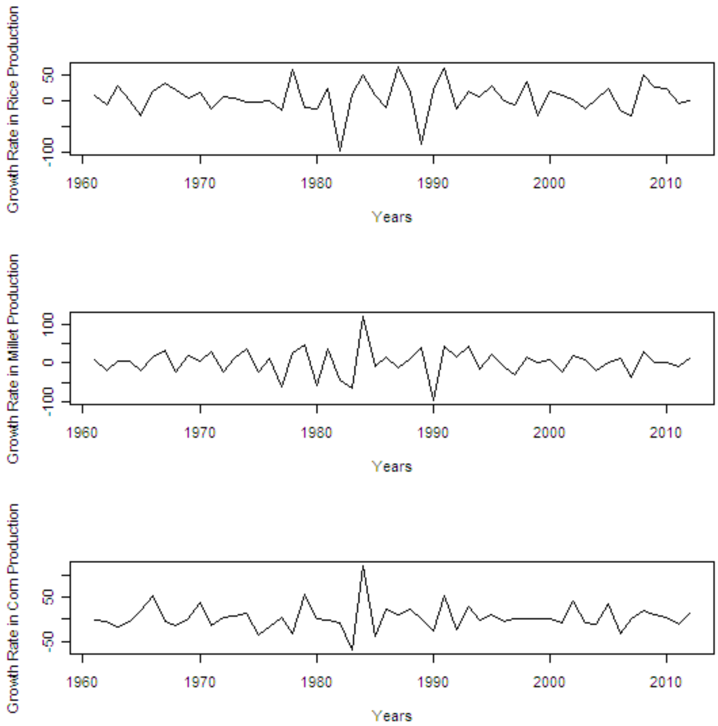

Figure 1: Time Series plot of production growth rate of cereals

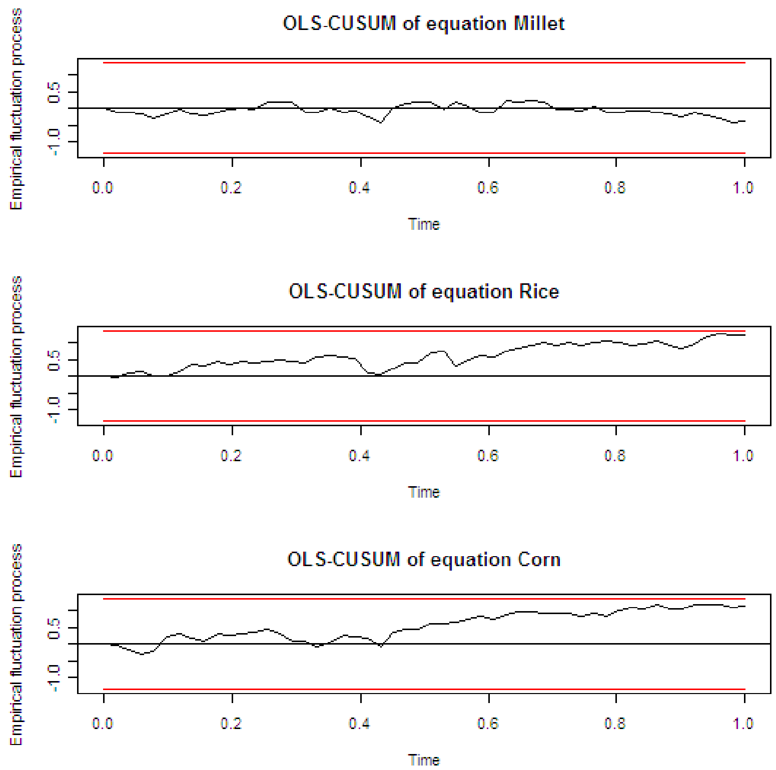

Figure 2: OLS-CUSUM plot of VAR (1) equations

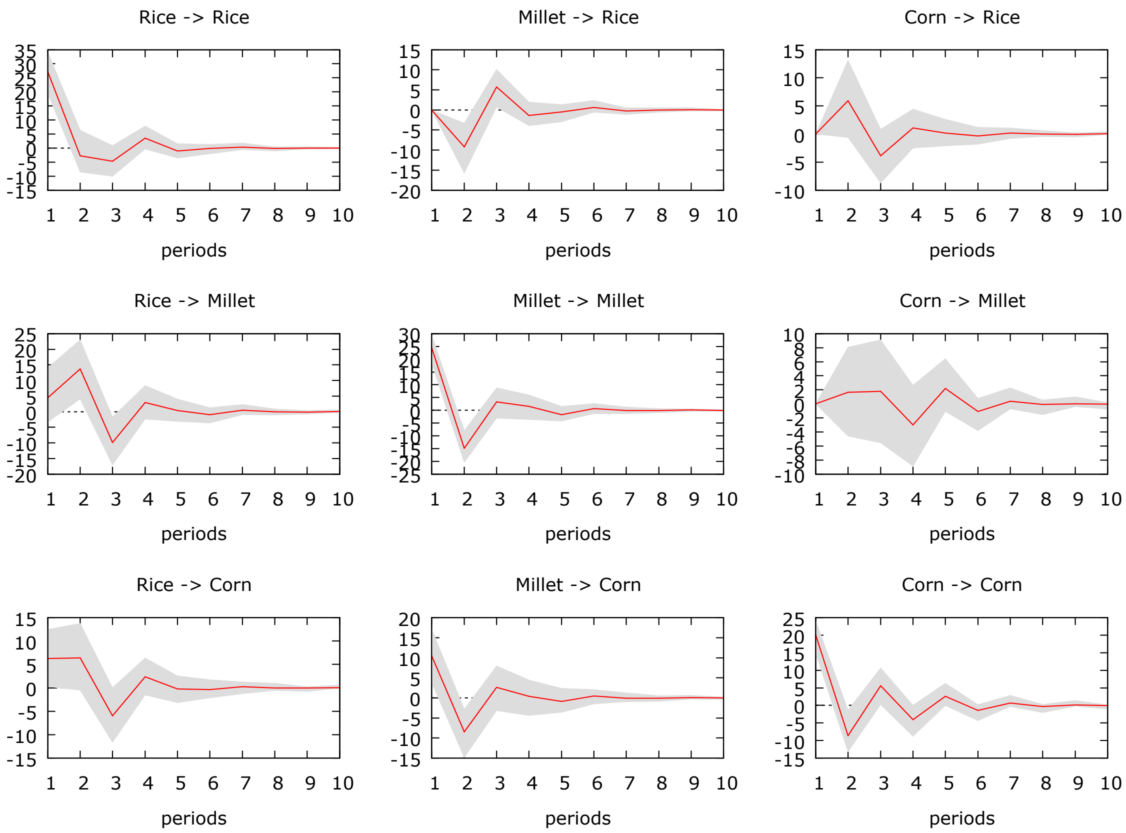

Figure 3: Impulse Response analysis of cereals