

Gamma Function for Different Negative Numbers and Its Applications

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Abstract

The purpose of this paper is to develop and obtain a formula for the gamma function (according to science researchers) for different negative numbers using mathematical inference and Stirling's Asymptotic Formula. Comparison between the previous formulae and developed formula for negative numbers such as $(-0.5, -1.5)$ have been conducted, so the results were identical.

Introduction

In mathematics, the gamma function (represented by the capital Greek letter Γ) is an extension of the factorial function, with its argument shifted down by 1, to real and complex numbers.

The gamma function is a component in various probability-distribution functions, and as such it is applicable in the fields of probability and statistics, as well as combinatorics.

The behavior for negative n is more intricate. Euler's integral does not converge for $n \leq 0$, but the function it defines in the positive complex half-plane has a unique analytic continuation to the negative half-plane.

The gamma function is nonzero everywhere along the real line, although it comes arbitrarily close to zero as $n \rightarrow \infty$.

In fact, researchers need to deal with the gamma function for different numbers (integers or fractional) in resolving some of the issues facing them in their studies associated with this function, specially statistical distributions, for example (Weibull distribution and Beta distribution). Also the gamma function for positive numbers can be obtained using well-known mathematical formulae, but for negative numbers are unknown except some numbers, such as -0.5 , -1.5 , -2.5 and so on. In this paper, the development of gamma function for different negative numbers have been performed using mathematical inference and Stirling's Asymptotic Formula.

The literature review related with the gamma function for different numbers (integers or fractional). (Spiegel, 1968, 1986) dealt with the gamma function for numbers such as, -0.5 , -1.5 , -2.5 and so on. Also the gamma function for different numbers is shown in figure 1 so that for helping to develop and obtain the gamma function for different negative numbers. Also (Abramowitz, 1965) and (Tuma, 1970) viewed and dealt with calculation of the gamma function (Γn) for positive numbers and some negative numbers. (Buck, 1978, 1986, 2013) dealt with the gamma function (Γn), $n < 0$, for example $\Gamma -0.5$, $\Gamma -1.5$.

From this Literature Review, it can be seen that there is no dealt with negative numbers which differ from -0.5 , -1.5 , -2.5 and so on. In this paper it has been dealt and calculated different negative numbers such as, -0.2 , -0.4 , -0.6 , -1.1 , -1.9 and so on. Some particular values of gamma function are given, (www.google, 2013).

The purpose of this paper was to develop and obtain a formula for the gamma function for different negative numbers, develop and create a table of gamma function for different negative numbers.

Materials And Methods

In this section a brief discussion of methodology and Gamma function issues. The gamma function, if n is a positive integer can be written as follows :

$$\Gamma n = (n-1)! \quad (1)$$

The gamma function is defined for all complex numbers except the negative integers and zero. For complex numbers with a positive real part, it is defined via an improper integral that converges is given by formula (4).

The behavior of $\Gamma(n)$ for an increasing positive variable is simple : it grows quickly — faster than an exponential function. Asymptotically as $n \rightarrow \infty$, the magnitude of the gamma function is given by Stirling's formula

$$\Gamma n+1 \sim \text{square root}(2 \pi n(n/e)), \quad (2)$$

here the symbol \sim means that the quotient of both sides converges to 1.

The gamma function for different numbers is shown in figure 1 in order to explain some points about it. Also previous Gamma function formulae and Gamma function developed are presented as follows:

From mathematical references, figure 1 shows that there are different negative numbers for Gamma function without determination of mathematical formula for calculating it. This figure produced starting point and important issue of the development Gamma function values.

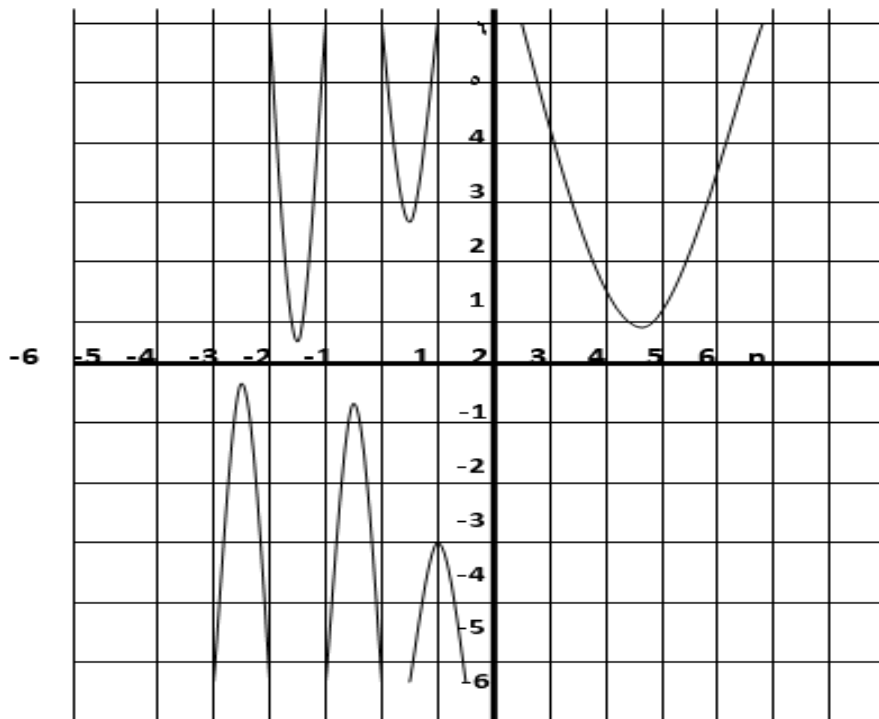


Figure 1 Gamma function for different values (Spiegel,1968, p 101)

It can be seen that from figure1:

Firstly: Positive Numbers:

As (n) approaches from zero, result that Γn becomes large and large, then it becomes indefinite for numbers near from zero. Mathematically, it can be declared, see (Tuma,1970).

From the formula 1, it can be seen that:

$$\Gamma n = \lim_{r \rightarrow \infty} \frac{r! r^n}{n(n+1)\dots(n+r)} \quad (3)$$

if $n = 0 \rightarrow \Gamma n = (r! / 0)$, it becomes indefinite.

Secondly: Negative Numbers

1- For the numbers $-1 < n < 0$, Γn becomes negative or As it approaches from (-1) it becomes indefinite. Note that for case (-0.5), Γn are between (-3) and (-4).

2- For the numbers $-2 < n < -1$, Γn becomes positive or As it approaches from (-2) it becomes indefinite too. Note that for case (-1.5), Γn are between (2) and (3).

3- And so on for others numbers, Γn values are between negative and positive and it approaches more and more to zero.

With regard to the formulae of the gamma function account shall be as follows

Mathematically and see (Abramowitz,1965 p156), the definition of the gamma function as follows:

$$\Gamma n = \int_0^{\infty} t^{n-1} e^{-t} dt, \quad n > 0 \quad (4)$$

$$\Gamma_{n+1} = n \Gamma_n = n!, \quad n = 0, 1, \dots \quad (5)$$

As $n < 0$, (Buck, 1978, p299) stated:

$$\Gamma_n = \Gamma_{n+1} / n, \quad n < 0 \tag{6}$$

According to (6) as an application, (Buck, 1978, p299) stated:

$$\Gamma_{-0.5} = \Gamma_{0.5} / -0.5 = \sqrt{\pi} / -0.5 = -2\sqrt{\pi}$$

$$\Gamma_{-1.5} = \Gamma_{-0.5} / -1.5 = 4/3 \sqrt{\pi}$$

So let $-n = -m + 1/2$

The general formula as follows:

$$\Gamma_{-n} = \Gamma_{-m+1/2} = \{(-1)^m 2^m \sqrt{\pi}\} / \{1, 3, 5, \dots, (2m-1)\}, \tag{7}$$

$$m = 1, 2, 3, \dots, n = -0.5, -1.5, \dots$$

From(7), it can be seen that the value of the gamma function for the numbers- 0.5, -1.5, -2.5 and so on based on the formation of a series of repeated substitutions for factorial that number to be up to the first term of the extent $\Gamma_{0.5}$. So what about negative numbers that differ from the -0.5, -1.5,-2,5 and so on. Developed formula will be as follows:

Through mathematical inference, can be obtained on the gamma function for any negative number to form a series of repeated substitutions for factorial to a negative number refer to formula(6) to be up to a positive first term is greater than zero and can be expressed by developed formula as follows:

$$\Gamma_{-n} = \Gamma_{-n+m} / \{(-n)(-n+1), \dots, (-n+m-1)\} \tag{8}$$

$$m = \text{Integer} | -n+1 |, m > n$$

$-n+m$ calculate as a positive number of log series Stirling,s Asymptotic Formula, see(Spiegel,1968):

$$\text{Log } \Gamma_h = (h-0.5) \log h - 1/2 \text{Log } 2\pi + 1 / (12h) - 1 / (360h^3) + 1 / (1260h^5) - \dots \tag{9}$$

$$\Gamma_h = \text{Exp}\{\log \Gamma_h\}, h = -n+m$$

Where Γ_{-n+m} calculate as a positive number of log series Stirling's Asymptotic Formula, see(Spiegel,1968):

Results AND DISCUSSION

In this section, we have dealt with some particular values , developed and create a table of gamma function for different negative numbers.

Firstly, some particular values to verify that developed formula (8) and compare it with formula (7) on two numbers (- 0.5, - 1.5) when $m=0$ as follows:

- 1- The number (- 0.5), developed gamma formula(8)

$$\Gamma_{-0.5} = \Gamma_{0.5} / -0.5 = -3.545$$

But, according to the formula (7) as follows:

$$\Gamma_{-0.5} = -2\sqrt{\pi} = -3.545$$

- 2- The number (- 1.5), developed gamma formula (8)

$$\Gamma_{-1.5} = \Gamma_{-0.5} / -1.5 = 2.363 = 2.36$$

But, according to the formula (7) as follows:

$$\Gamma_{-1.5} = 4/3 \sqrt{\pi} = 2.363 = 2.36$$

- 3- Thus, with others some particular values based on formula (8) only, such as (-0.3, -0.6,-1.0,-1.2,-1.9) the results are :

$$\Gamma_{-0.3} = -4.38, \Gamma_{-0.6} = -3.7, \Gamma_{-1.0} = \infty, \Gamma_{-1.2} = 4.85, \Gamma_{-1.9} = 5.56$$

- 4- Some particular values of gamma function are given, (www.google,2013).

$$\Gamma(-\frac{3}{2}) = \frac{4}{3}\sqrt{\pi} \approx 2.3632718012017$$

$$\Gamma(-1) = (-2)! = \infty$$

$$\Gamma(-\frac{1}{2}) = -2\sqrt{\pi} \approx -3.544907701811$$

$$\Gamma(0) = (-1)! = \infty$$

$$\Gamma(\frac{1}{2}) = \sqrt{\pi} \approx 1.772453850905$$

$$\Gamma(1) = 0! = 1$$

$$\Gamma(\frac{3}{2}) = \frac{1}{2}\sqrt{\pi} \approx 0.88622692545$$

$$\Gamma(2) = 1! = 1$$

$$\Gamma(\frac{5}{2}) = \frac{3}{4}\sqrt{\pi} \approx 1.32934038818$$

$$\Gamma(3) = 2! = 2$$

$$\Gamma(\frac{7}{2}) = \frac{15}{8}\sqrt{\pi} \approx 3.32335097045$$

$$\Gamma(4) = 3! = 6$$

From these values, it can be seen that there is no dealt with negative numbers which differ from $-0.5, -1.5, -2.5$ and so on and which are identical with negative numbers obtained by formula(8).

Secondly, developed and create a table of gamma function for different negative numbers is shown on table 1.

Table 1 the gamma function for different negative numbers ($n < 0$) according to the formula(8)

m n	0	1	2	3	4	5	6	7	8	9
0.0	∞	-100.59	-50.60	-33.94	-25.62	-20.63	-17.31	-14.94	-13.16	-11.79
- 0.1	-10.69	-9.79	-9.04	-8.42	-7.88	-7.42	-7.01	-6.66	-6.35	-6.07
- 0.2	-5.82	-5.60	-5.40	-5.22	-5.05	-4.90	-4.77	-4.64	-4.53	-4.42
- 0.3	-4.38	-4.24	-4.16	-4.09	-4.02	-3.96	-3.90	-3.85	-3.80	-3.76
- 0.4	-3.78	-3.69	-2.66	-3.63	-3.61	-3.59	-3.58	-3.56	-3.55	-3.55
- 0.5	-3.54	-3.60	-3.54	-3.56	-3.57	-3.58	-3.60	-3.62	-3.64	-3.67
- 0.6	-3.70	-3.73	-3.77	-3.82	-3.86	-3.92	-3.98	-4.04	-4.11	-4.19
- 0.7	-4.28	-4.37	-4.46	-4.58	-4.70	-4.83	-4.98	-5.14	-5.32	-5.52
- 0.8	-5.74	-5.98	-6.26	-6.57	-6.91	-7.32	-7.78	-8.31	-8.94	-9.68
- 0.9	-12.57	-11.67	-13.10	-14.81	17.18-	-20.50	-25.48	-33.80	-50.45	-100.4
- 1.0	∞	99.59	49.61	32.95	24.63	19.65	16.33	13.96	12.19	10.81
- 1.1	9.72	8.82	8.08	7.95	6.91	6.45	6.05	5.69	5.38	5.10
- 1.2	4.85	4.63	4.42	4.24	4.07	3.92	3.78	3.65	3.54	3.43
- 1.3	3.33	3.24	3.15	3.07	2.10	2.93	2.87	2.86	2.76	2.71
- 1.4	2.66	2.62	2.58	2.54	2.51	2.48	2.45	2.42	2.40	2.38
- 1.5	2.36	2.35	2.34	2.32	2.32	2.31	2.31	2.30	2.30	2.37
- 1.6	2.31	2.32	2.33	2.34	2.36	2.37	2.40	2.42	2.45	2.48
- 1.7	2.51	2.55	2.60	2.65	2.70	2.76	2.83	2.91	2.99	3.09
- 1.8	3.19	3.31	3.44	3.59	3.76	3.96	4.18	4.44	4.75	5.12
- 1.9	5.56	6.11	6.79	7.67	8.85	10.51	13.00	17.16	25.48	50.47

Conclusion

In this paper, from literature review, there are no studies dealt with the gamma function for negative numbers vary, for example, -0.5, -1.5, and so on. In this paper, a formula was developed for gamma function for different negative numbers, for example, -0.2 -0.4 , -0.6, - 1.1, -1.9, and so on. Developed and create a table of gamma function for different negative numbers, which can be used by researchers interested in. Also compression between the previous formulae and developed formula of gamma function for negative numbers, for example (- 0.5,-1.5) have been conducted, so the results were identical.

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