

Mathematical Model for Estimating the Standard of Living of Nigerians and Achieving the First Agenda of the Vision 20; 2020

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Abstract

The research is concerned with the development of a mathematical model for Nigerians' standard of living and the achievement of the first item on the vision 20; 2020 agenda. The model was optimized to know whether it is possible for humans to have maximum or minimum standard of living. The optimization result showed that there is no specific standard of living value that can momentarily stop human existence but the snag indicated based on the work is that, those whose standard of living is critical (very close to zero percent) can take to negative ways of getting income in order to survive and thus, pose a challenge to the achievement of the overall vision 2020. Equally, the model was validated and observation about the model's results and the questionnaire data was made, indicating that the results from our model and the questionnaire data have a higher degree of correlation of approximately 70% or +0.7 ranking correlation coefficient and thus recommending the model as a standard measure for estimating the standard of living.

Key words: MDGs, Government Parameter, Vision 20-2020, Social status, HDR, MPI, constraint, Saddle-point, extremum.

1.0 Introduction

Notable of remark here is that when standard of living is improved, poverty is tackled and the first agenda on the Vision 20; 2020 which is to eradicate extreme poverty and hunger is achieved in that regard. In recent times, the application of mathematics to economic issues like poverty, investment and growth rate has gained dominance. Meanwhile, in as much as poverty has become a global issue today through Millennium Development Goals (MDGs) and a national matter domesticated in vision 20; 2020, we wish to remark that the empirical examination of the impact of econometric social policy on the objective of poverty alleviation especially at the micro level, requires appropriate instrument such as mathematical modelling.

Mathematical modelling of course accentuates the beauty of mathematics and can help produce urgent result for handling poverty and thereby making it possible to achieve the Vision 20; 2020 at large. According to Benyah (2008), mathematical models are useful experimental tools for building and testing theories, accessing quantitative conjectures, answering specific questions, determining sensitivities to changes in parameter values, and estimating key parameters from data. But, Brauer (1990) advised that the closer mathematical model assumptions are to reality of dynamics, the more difficult the mathematical analysis, hence the need to simplify assumptions without losing track of the situation or dynamics at hand.

Thus, the need for this research and the choice of using mathematical modelling approach cannot be over emphasized.

2.0 Methodology

2.1 Formulation of Model Equations

To develop the research model equation, all possible parameters associated with the measurement of the standard of living will be outlined. However, while some are very important others can be ignored. Thus, the ones in the frontline are mainly considered before the negligible ones are done away with accordingly. These parameters are as discussed in the next section.

2.1.1 Definition of Parameters/Notations

1. **Monthly income (I):** this is the total earning of a family per month expressed as,

$$I = \frac{v(1 + ar)(1 + b)}{f} \quad (2.0)$$

- Where: v = the amount of earnings of the principal (basic) worker of the family
 a = the ratio between the amount of earnings of the principal and secondary workers in the family.
 r = the amount earned by other working members of the family
 b = the ratio of other incomes of the family to the total earnings of the family
 f = Family size. [As by *Aganbegian and Rimashevskaja (2006)*]
2. **Total Expenditures (E):** total money spent per month by a family.
 3. **Social status/Position of the man q, (in percent):** is the position a man occupies in the society which he works to maintain and working harder increases his standard of living as a result.
 4. **The man's societal expectation p,(in percent):** is the expectation of a society from a man which increases as his standard of living rises and therefore makes him to work harder to keep the standard and not let the society down.
 5. **Family size (F):** this is the total number of persons that make up a household.
 6. **Standard of living parameter (S):** it refers to the level of comfort, wealth, happiness, material goods and necessities available to a household in a certain socioeconomic class in a certain geographic area. The standard of living is closely linked to quality of life Raymond (2011). It can be very high above 100% and very low closer to zero percent.

2.1.2 Some Basic Assumptions

- Let $T = \frac{(\text{social status or position of the man} + \text{societal Expectations from the man})}{2}$

$$T = \frac{(q + p)}{2} \quad (2.1)$$

The equation (2.1) is formed by finding the average of the two variables p and q this way because; the two variables are closely related.

- Also as p is the expectation of a society from a man which increases as his standard of living rises and therefore makes him to work harder to keep the standard and not let the society down, also q is the position a man occupies in the society which he works to maintain (his position) and working harder increases his standard of living; then, the man's standard of living is directly proportional to T. Which means,

$$S \propto T, S = K_1 T \quad (2.2)$$

Where K_1 = constant of proportionality

- A man's standard of living increases as his level of income I, increases. Hence,
 $S = K_2 I$, (Where K_2 = constant) (2.3)
- When one's level of expenditure, E increases, his standard of living decreases. Hence,

$$S \propto \frac{1}{E}; \quad S = \frac{K_3}{E} \quad (2.4)$$

- Family size, F at the end of expenditure is a natural factor that affects one's standard of living indirectly as it increases as the man's standard of living is reducing. This is because the family size definition could include extended family also, which depends on an individual. Hence, this factor does not depend on other factors directly neither does any of them affect it. Thus, it is seen as an additive factor.

$$S \propto \frac{1}{F}, \quad S = \frac{K_4}{F} \quad (2.5)$$

2.1.3 Establishment of Model Parameter Relationships

From equation (2.5) in the section above,

$$\text{Let } S = \frac{K}{F} \quad (\text{where } K \text{ is a constant}) \quad (2.6)$$

Also, we would postulate as follows,

- Let the man's standard of living S; vary jointly as his social status T and inversely as his spending or expenditure level E, per time. Then Mathematically,

$$S = \frac{B_1 T}{E} \quad (\text{Where } B_1 \text{ is a constant}) \quad (2.7)$$

From above, since equation (2.5) is an additive factor, then combining (2.6) and (2.7) gives

$$2S = \frac{K}{F} + \frac{B_1 T}{E}$$

$$S = \frac{A}{F} + \frac{BT}{E} \quad (\text{Where: } A = \frac{K}{2}, B = \frac{B_1}{2}) \quad (2.8)$$

2.1.4 Relationship of Other Model Parameters With Income

By general consideration, since when income I is low with other factors high, the individual's economic standard of living will be low. Implying that, income rate affects all other factors. This means,

$$S = K_I \left(\frac{A}{F} + \frac{BT}{E} \right) I \quad (2.9)$$

$$\therefore S = \frac{\alpha I}{F} + \frac{\beta TI}{E}; \alpha = AK_I, \beta = BK_I \quad (2.10)$$

Where: E = Expenditure level

F = family size

T = social status/societal expectations, I = Income

S = standard of living and α, β are constants.

2.1.5 Analysing the Model to Evaluate its Equation Constants

To evaluate the constants in the model above, equation (2.10) is going to be differentiated partially with respect to α and β respectively. To do this, we have to minimize the model using *least squares method* as follows: From (2.10) we let,

$$Z_{\min} = \min \sum \left(S_i - \frac{\alpha I_i}{F_i} - \frac{\beta T_i I_i}{E_i} \right)^2 \quad i=1, 2, 3, \dots, n \quad (2.11)$$

$$\frac{\partial Z}{\partial \alpha} = -2 \sum \left(S_i - \frac{\alpha I_i}{F_i} - \frac{\beta T_i I_i}{E_i} \right) \frac{I_i}{F_i} = 0 \quad (2.12)$$

$$\frac{\partial Z}{\partial \beta} = -2 \sum \left(S_i - \frac{\alpha I_i}{F_i} - \frac{\beta T_i I_i}{E_i} \right) \frac{T_i I_i}{E_i} = 0 \quad (2.13)$$

$$\therefore \alpha \sum \frac{I_i^2}{F_i^2} + \beta \sum \frac{T_i I_i^2}{E_i F_i} = \sum \frac{S_i I_i}{F_i} \quad (2.14)$$

$$\alpha \sum \frac{T_i I_i^2}{F_i E_i} + \beta \sum \frac{T_i^2 I_i^2}{E_i^2} = \sum \frac{S_i T_i I_i}{E_i} \quad (2.15)$$

$i = 1, 2, 3, \dots, n$. But, for this problem $n = 20$.

Meanwhile Equations (2.14) and (2.15) are to be solved simultaneously for α and β .

2.1.5.1 Research Instrument Used

The research instrument used is known as the questionnaire. This is because our research is a kind of opinion poll research. However in distributing the questionnaire, only the set of income earners that usually have a clear cut record of their income were considered. This is because; some craftsmen and business persons find it difficult to keep record of the flow of income and such type of data or record may affect the authenticity of our research model. Also, in the questionnaire there are questions designed to checkmate fake responses. Wherever any element of fake response is discovered, the questionnaire is destroyed accordingly. 200 copies of questionnaire were distributed. But only 20 copies which satisfied our acceptability test were considered. We attached a fake response (F.R.) test/acceptability test to each questionnaire such which any one that became successful was considered for the research.

2.1.5.2 Evaluation of the Equation Constants Using the Questionnaire Data in Table 3

Solving equation (2.14) and (2.15) in the section above simultaneously, where from the table above,

$$\sum \frac{T_i I_i^2}{E_i F_i} = 1104306482, \quad \sum \frac{T_i^2 I_i^2}{E_i^2} = 2685124583,$$

$$\sum \frac{S_i T_i I_i}{E_i} = 170099.1667, \quad \sum \frac{S_i I_i}{F_i} = 92305.47619, \quad \sum \frac{I_i^2}{F_i^2} = 934161637.5$$

Using the data collected: $\alpha = 4.6561047 \times 10^{-5}, \beta = 4.4199625 \times 10^{-5}$

$$S = \frac{4.6561047 \times 10^{-5} I}{F} + \frac{4.4199625 \times 10^{-5} TI}{E} \quad (2.16)$$

The equation (2.16) above is our model equation for the people's standard of living before the introduction of government parameter and other natural factors and it is in a way similar to that of Negedu (2010). Although he

did not consider some of our issues and neither formulated his model as explicitly as this nor optimize and supply information about the model's extreme values.

3.0 Results And Discussion

In the concluding part of the previous chapter, data were collected in order to be able to evaluate our emerging model equation constants. Thus, our new model equation now is

$$S = \frac{4.6561047 \times 10^{-5} I}{F} + \frac{4.4199625 \times 10^{-5} TI}{E} \quad (3.1)$$

3.1 Optimization of the Model

This approach is a technique for programming/optimizing an objective function, subject to an inequality or equation constraints. Hence, we wish to optimize our model *subject to a constraint* as below.

Optimizing the Model Using Lagrange's Multiplier Subject to E = 184 + 0.7064 * I as Constraint [from analysed Keynesian Expenditure Theory by Javitz *et al.* (2010)]

Since from our equation (3.1) above,

$$\Rightarrow S = \frac{\alpha I}{F} + \frac{\beta TI}{E}$$

Then setting the constraint equal to zero and multiplying it by our Lagrange multiplier (where λ is our Lagrange parameter), to get our Lagrange function will give,

$$S = \frac{\alpha I}{F} + \frac{\beta TI}{E} + \lambda(E - 184 - 0.7064I) \quad (3.2)$$

Let by supposition the social status of the man T, (at optimal level) = maximum = 100% = $\frac{100}{100} = 1$.

Then, differentiating equation (3.2) partially with respect to F, E, I and λ gives

$$\frac{\partial S}{\partial F} = \frac{-\alpha I}{F^2} = 0 \quad (3.3)$$

$$\frac{\partial S}{\partial E} = \frac{-\beta I}{E^2} + \lambda = 0 \quad (3.4)$$

$$\frac{\partial S}{\partial I} = \frac{\alpha}{F} + \frac{\beta}{E} - 0.7064 \lambda = 0 \quad (3.5)$$

$$\frac{\partial S}{\partial \lambda} = E - 184 - 0.7064I = 0 \quad (3.6)$$

From equation (3.6),

$$I = \frac{E - 184}{0.7064} \quad (3.7)$$

Putting (3.7) into (3.3)

$$\Rightarrow E = 184\% \text{ (Meaning, at optimal the man's expenditure level is at the extreme, above 100\%)}$$

Thus, by back substitution into (3.7),

$$I = 0 \text{ (this means the man's income is of no effect, at optimal level)}$$

To eliminate λ in (3.4) and (3.5) above, we perform (3.5) + (3.4) x 0.7064 to give

$$\frac{-0.7064 \beta I}{E^2} + \frac{\alpha}{F} + \frac{\beta}{E} = 0$$

$$\frac{\alpha}{F} = -\frac{\beta}{1.84} \text{ (Since } I=0 \text{ and } E=1.84)$$

$$F = -\frac{184(4.6561047 \times 10^{-5})}{4.4199625 \times 10^{-5}} < 1$$

$\therefore F < 1$ person (it means he cannot even take care of just 1 person at optimal level.)

Also, by substituting our values of I and E into equation (3.4),

$$\lambda = 0 \text{ and } S = 0.$$

Thus, our critical values are: E = 184%, T = 100%, $\lambda = 0$, F < 1 person and I = 0, S = 0 as E tends to maximum.

This implies that, Standard of living of the man S, $\rightarrow 0$ as his expenditure level E $\rightarrow \infty$

And this in real life means that the man's Standard of living tends to 0% as his expenditure level E, is geometrically or infinitely rising.

3.2 Observation on the Nature of the Models' Extreme Values

This stage is necessary in order to know whether the model's extreme point S_0 is a minimum or a maximum. Generally, given a function $f(x, y)$ that obeys the continuity of the partial derivatives and we

Let: $A = \frac{\partial^2 f}{\partial x^2}$; $B = \frac{\partial^2 f}{\partial x \partial y}$ and $C = \frac{\partial^2 f}{\partial y^2}$ then,

- 1) If $B^2 - AC < 0$, then $f(x, y)$ has extreme value at (x_0, y_0) and minimum if $A > 0$ and it is maximum if $A < 0$
- 2) If $B^2 - AC > 0$, or $AC < B^2$ then $f(x, y)$ has no extreme value. That is, it has a saddle point at (x_0, y_0) .
- 3) If $B^2 - AC = 0$, then no information is derivable about its extreme values.

Similarly, importing the same idea in our reduced model equation variables,

$$S = \left(\frac{\alpha}{F} + \frac{\beta}{E} - 0.00001\right)I; \text{ where, } I = \frac{E}{0.7064} - \frac{184}{0.7064}, \text{ from (3.7) \& our constraint equ.}$$

$$\therefore S = \left(\frac{\alpha}{F} + \frac{\beta}{E}\right)\left(\frac{E}{0.7064} - \frac{184}{0.7064}\right)$$

$$\begin{aligned} \text{And } \frac{\partial S}{\partial F} &= \frac{-\alpha}{F^2} \left(\frac{E}{0.7064} - \frac{184}{0.7064}\right) \\ \frac{\partial S}{\partial E} &= \frac{-\beta}{E^2} \left(\frac{E}{0.7064} - \frac{184}{0.7064}\right) + \frac{1}{0.7064} \left(\frac{\alpha}{F} + \frac{\beta}{E}\right) \\ &= \left(\frac{-\beta}{0.7064 E} + \frac{184 \beta}{0.7064 E^2}\right) + \left(\frac{1.4156 \alpha}{F} + \frac{1.4156 \beta}{E}\right) \end{aligned}$$

$$\therefore \frac{\partial^2 S}{\partial F^2} = \frac{2\alpha}{F^3} \left(\frac{E}{0.7064} - \frac{184}{0.7064}\right), \quad \frac{\partial^2 S}{\partial F^2} \Big|_{E=184} = 0, \text{ (by substituting our critical values)}$$

$$\frac{\partial^2 S}{\partial E^2} = \left(\frac{\beta}{0.7064 E^2} - \frac{520.98 \beta}{E^3} - \frac{1.4156 \beta}{E^2}\right)$$

$$\therefore \frac{\partial^2 S}{\partial E^2} = \frac{0.00002016 \beta}{0.7064 E^2} - \frac{520.98 \beta}{E^3}$$

$$\frac{\partial^2 S}{\partial F \partial E} = \frac{\partial}{\partial F} \left(\frac{\partial S}{\partial E}\right) = \frac{-\alpha}{F^2} \left(\frac{1}{0.7064}\right)$$

$$\text{If, } \frac{\partial^2 S}{\partial E^2} = A, \quad \frac{\partial^2 S}{\partial F \partial E} = B \text{ and } \frac{\partial^2 S}{\partial F^2} = C,$$

Then, by substituting our critical values,

$$B^2 - AC = \left(\frac{-4.6561047 \times 10^{-5}}{1}\right)^2 \left(\frac{1}{0.7064}\right)^2 - 0 > 0 \quad (3.8)$$

Hence, $B^2 - AC > 0$

(It means S has a Saddle-point and has no local *extremum* or it has no specific optimal value)

3.3 Physical Application of the Model/ Validation of our Model Data

Table 4 below shows the validation of the first ten rows of our questionnaire data and the corresponding absolute error between the model results and our raw questionnaire data using our model equation (2.16) and a value approximately +0.7 ranking correlation coefficient.

3.4 Discussion of Results

From our optimization result in equation (3.8) above, it was noticed at various levels of our model analysis that there is no particular level of standard of living of a man/household that can be called the maximum or minimum standard of living. This is consequent upon obtaining "Saddle-point" as the model's extreme/optimal value. Thus, no value of a person's economic standard is critical enough to momentarily stop his existence. This then implies that human beings can exist even at critical level of standard of living that is close to zero. Hence, the proportion of the population that has critical standard of living who will not die until help comes their way through poverty alleviation programmes like NAPEP and CGS or they seek for help themselves either through positive or negative means, forms the main factor for much security threats in our country. As it is logical to state that, if there is no food and Biology says a man needs food to survive, then for these people, the next option for survival is stealing as deduced from the view of a particular scholar, Nagarjuna (2010) in Osho Library. The effect of

this neglected population again can affect the achievement of the vision 20; 2020 agendas if not taken care of.

4.0 Conclusion

Since, the model has spotted that insecurity and other social vices in this country is caused by gross disregard of the critically poor people, then it is time to learn to use all the country's poverty alleviation programmes and supports judiciously in order to achieve the first agenda of the Vision 20; 2020. Any decision or activity contrary to this is anti- Vision 20; 2020 and another way of saying the vision cannot be achieved at the time speculated. More so, as the country is in a hurry to obtain economic stability, this research is timely and the utilization of all its specifications alike.

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Tables

Table 1: Nigerians poverty profile as analysed by National Bureau of Statistics

Absolute Poverty 0%	Food Poverty 38.7%	Relative Poverty 30.3%	Non-poor 31%
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Source: National bureau of Statistics, 2011. Yemi (2012)

Table 2: Comparison of Nigerian poverty profile with some selected countries by UNESCO

	MPI value	Head count (%)	Intensity of deprivation (%)	Population vulnerable to Poverty (%)	Population in severe Poverty (%)	Population below income poverty Line (%)
Nigeria	0.310	54.1	57.3	17.8	33.9	64.4
Ethiopia	0.562	0.562	63.5	6.1	72.3	39.0
Dem. Rep. of Congo	0.393	73.2	53.7			

Source: Nigeria's MPI for 2011 relative to selected countries (HDR, 2011)

Table 3: Questionnaire data for determining our emerging equation constants

Monthly Income (I)	Government parameter g_p as % of income	Family size (F)	Expenditure Level (Ex 0.01) Per Month in (%)	Social status/ Societal Expectation (T x0.01) in%	Standard of Living (S) in % before g_p
7000	560	8	0.6	0.7	40
17000	2040	1	0.9	0.6	129
4500	450	8	0.2	0.5	52
8000	560	3	0.6	0.6	48
4000	360	1	0.6	0.6	36
17500	2275	9	0.4	0.6	125
4000	300	4	0.2	0.4	40
4700	376	5	0.6	0.6	25
8000	480	2	0.4	0.6	72
4300	387	7	0.2	0.4	41
12000	1320	5	0.8	0.7	58
7000	490	7	0.8	0.7	32
10500	1155	3	0.6	0.6	62
8500	765	8	0.6	0.8	55
1500	90	1	0.6	0.6	14
2000	140	1	0.4	0.5	20
22000	1980	1	0.8	0.75	94
4000	280	3	0.4	0.4	24
18000	1800	2	0.6	0.7	135
4800	480	2	0.8	0.6	27

Source: Data from the distributed questionnaire (2011)

Table4: Comparison between the Questionnaire & Model values of the standard of living

PERSONS	STANDARD OF LIVING (%) (calculated by our model)	STANDARD OF LIVING (%) (questionnaire data)	Error (Absolute)
Man A	40.17	40	0.17%
Man B	129.25	129	0.25%
Man C	52.34	52	0.34%
Man D	47.78	48	0.22%
Man E	36.30	36	0.30%
Man F	125.08	125	0.08%
Man G	40.06	40	0.06%
Man H	25.15	25	0.15%
Man I	71.66	72	0.34%
Man J	40.87	41	0.13%

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