On Continuity of Complex Fuzzy Functions

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Abstract
In this paper, Some important theorems on fuzzy type I-continuous and II-continuous of complex fuzzy functions mapping generalized rectangular valued bounded closed complex complement normalized fuzzy numbers into itself are proved.
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1. Introduction

It is well known that fuzzy complex numbers and fuzzy complex analysis were first introduced by (Buckley, 1989; Buckley and Qu, 1991, 1992). Scholars did series research about the properies of fuzzy complex number from various aspects (Quan, 1996; Ma *et al.*, 2009; Zheng and Ha, 2009). But these achievements were very abstract, and it did not consummate until today. In view of (Buckley, 1989), Guangquan (1992) discussed the limit theory of the sequence of fuzzy complex numbers in detail, giving a series of results about limit theory, which are the counterparts of well-known results valid for real numbers in classical mathematics analysis. Buckley (1989) suggested that introducing a metric on the space of fuzzy complex numbers provide to study convergence, continuity and differentiation of fuzzy complex function (Chun and Ma, 1998; Qiu *et al.*, 2000, 2001; Ousmane and Congxin, 2003; Shengquan, 2006; Cai, 2009; Sabir, 2012). On the basis of Buckley's work, some authors continued research and have extensively studied the theory of fuzzy complex numbers and fuzzy complex analysis (Wu and Qiu, 1999; Zengtai and Shengquan, 2006; Qiu and Shu, 2008; Sun and Guo, 2010; Sabir *et al.*, 2012b). Sabir *et al.* (2012a) giving the definitions of the complement normalized fuzzy numbers (CNFNs), bounded closed complex CNFNs (BCCCNFNs), generalized rectangular valued BCCCNFNs (GRVBCCCNFNs) and discussed some of their basic properties. In section two, we first review the definitions and characterizations related to fuzzy complex sets. We will also present the notations needed in the rest of the paper. In the last section, some theorems on the continuity of complex fuzzy functions are proved.

2. Priliminaries

A fuzzy set $\tilde{A}$ defined on the universal set X is a function $\mu(\tilde{A},x) : X \to [0,1]$. Frequently, we will write $\mu_{\tilde{A}}(x)$ instead of $\mu(\tilde{A},x)$. The family of all fuzzy sets in X is denoted by $\mathcal{F}(X)$. The α–level of a fuzzy set $\tilde{A}$, denoted by ${}^{\alpha+}\tilde{A}$, is the non-fuzzy set of all elements of the universal set that belongs to the fuzzy set $\tilde{A}$ at least to the degree $\alpha \in [0,1]$. The weak α–level ${}^{\alpha-}\tilde{A}$ of a fuzzy set $\tilde{A} \in \mathcal{F}(X)$ is the crisp set that contains all elements of the universal set whose membership grades in the given set are greater than but do not include the specified value of α. The largest value of α for which the α-level is not empty is called the height of a fuzzy set $\tilde{A}$ denoted $\alpha_{\tilde{A}}^{max}$. The core of a fuzzy set $\tilde{A}$ is the non-fuzzy set of all points in the universal set X at which $sup_x\, \mu_{\tilde{A}}(x)$ is essentially attained.

Let $\tilde{A}_i \in \mathcal{F}(X)$. Then the union of fuzzy sets $\tilde{A}_i$, denoted $\bigcup_i \tilde{A}_i$, is defined by $\mu_{\bigcup_i \tilde{A}_i}(x) = sup_x\, \mu_{\tilde{A}_i}(x) = \bigvee_x \mu_{\tilde{A}_i}(x)$, the intersection of fuzzy sets $\tilde{A}_i$, denoted $\bigcap_i \tilde{A}_i$, is defined by $\mu_{\bigcap_i \tilde{A}_i}(x) = inf_x\, \mu_{\tilde{A}_i}(x) = \bigwedge_x \mu_{\tilde{A}_i}(x)$, and the complement of $\tilde{A}_i$, denoted $\neg\tilde{A}_i$, is defined by $\mu_{\tilde{A}_i}(x) + \mu_{\neg\tilde{A}_i}(x) = 1$, for all x in the universal set X.

A fuzzy number ã is a fuzzy set defined on the set of real numbers R^1 characterized by means of a membership function $\mu_{\tilde{a}}(x): R^1 \to [0,1]$, which satisfies: (1) $\tilde{a}$ is upper semicontinuous, (2) $\mu_{\tilde{a}}(x) = 0$ outside some interval $[c,d]$, (3) There are real numbers a, b such that $c \le a \le b \le d$ and $\mu_{\tilde{a}}(x)$ is increasing on $[c,a]$, $\mu_{\tilde{a}}(x)$ is decreasing on $[b,d]$, $\mu_{\tilde{a}}(x) = 1, a \le x \le b$. We denote the set of all fuzzy numbers by $\mathcal{F}^\star$. A fuzzy complex number $\hat{Z}$ is defined by its membership functions $\mu_{\hat{Z}}(z)$ which is a mapping from the set of ordinary complex numbers into [0,1] if and only if $\mu_{\hat{Z}}(z)$ is continuous; ${}^{\alpha-}\hat{Z}$ is open, bounded, and connected; and ${}^{1+}\hat{Z}$ is non-empty, compact, and arcwise connected. We use $\mathcal{F}^{\star\star}$ to the set of all fuzzy complex numbers.

Let $f(z',z'') = w$ be any mapping from $\mathbb{C} \times \mathbb{C}$ into $\mathbb{C}$. Buckley (1989) extend f to $\mathcal{F}^{\star\star} \times \mathcal{F}^{\star\star}$ into $\mathcal{F}^{\star\star}$ and write $f(\widehat{Z'},\widehat{Z''}) = \widehat{W}$ if $\mu_{\widehat{W}}(w) = \bigvee_{f(z',z'')=w} (\mu_{\widehat{Z'}}(z') \wedge \mu_{\widehat{Z''}}(z''))$. One obtains $\widehat{W} = \widehat{Z'} \oplus \widehat{Z''}$ or $\widehat{W} = \widehat{Z'} \odot \widehat{Z''}$ by using $f(\widehat{Z'},\widehat{Z''}) = \widehat{Z'} \oplus \widehat{Z''}$ or $f(\widehat{Z'},\widehat{Z''}) = \widehat{Z'} \odot \widehat{Z''}$, respectively.

3. Properties of Continuous Complex Fuzzy Functions

In this section, we give the continuity of complex fuzzy function mapping GRVBCCCNFNs into itself. Most results, definitions and standard notations on fuzzy complex analysis which are used in this section can be found in Sabir *et al.* (2012a). Some of the results in this section are without proofs owing to the simplicities.

Definition 3.1. Let $\tilde{\mathcal{G}}^* \subseteq [\tilde{\mathcal{F}}_{\neg N}^{**}]$, and $\tilde{F}$ be a mapping from $\tilde{\mathcal{G}}^*$ to the set of all GRVBCCCNFNs. If for arbitrary $[\tilde{Z}] \in \tilde{\mathcal{G}}^*$, there exists unique $[\tilde{W}] \in [\tilde{\mathcal{F}}_{\neg N}^{**}]$, make $\tilde{F}([\tilde{Z}]) = [\tilde{W}]$, we call $\tilde{F}$ a complex fuzzy function defined on $\tilde{\mathcal{G}}^*$. Let $[\tilde{W}] \in [\tilde{\mathcal{F}}_{\neg N}^{**}]$, we say $\tilde{F}([\tilde{Z}])$ is fuzzy continuous at $[\tilde{W}]$ if for all $\bar{\varepsilon} > \bar{0}$ there exists $\bar{\delta} > \bar{0}$ such that $\tilde{\Gamma}\left(\tilde{F}([\tilde{Z}]), \tilde{F}([\tilde{W}])\right) \prec \bar{\varepsilon}$ as $\tilde{\Gamma}([\tilde{Z}], [\tilde{W}]) \prec \bar{\delta}$.

Definition 3.2. We say $\tilde{F}([\tilde{Z}])$ is fuzzy type I-continuous (resp. II-continuous) at $[\tilde{W}] \in [\tilde{\mathcal{F}}_{\neg N}^{**}]$ if for each $\bar{\varepsilon} > \bar{0}$ there is $\bar{\delta} > \bar{0}$ such that $\tilde{F}([\tilde{Z}]) \prec \tilde{F}([\tilde{W}])[+]\bar{\varepsilon}$ (resp. $\tilde{F}([\tilde{W}]) \prec \tilde{F}([\tilde{Z}])[+]\bar{\varepsilon}$) whenever $[\tilde{Z}][-][\tilde{W}] \prec \bar{\delta}$.

Theorem 3.3. Let $\tilde{F}$ and $\tilde{G}$ both are fuzzy continuous at GRVBCCCNFN $[\tilde{W}]$ then so is $\tilde{F}[*]\tilde{G}$ for $[*] \in \{[+], [-], [\cdot], [/], [\vee], [\wedge]\}$.

Proof: We only prove for $[*] = [+]$, the proof of the rest are similar. By hypothesis, for any $\bar{\varepsilon} > \bar{0}$, there exists $\bar{\delta} > \bar{0}$, when $\tilde{\Gamma}([\tilde{Z}], [\tilde{W}]) \prec \bar{\delta}$, $\tilde{\Gamma}\left(\tilde{F}([\tilde{Z}]), \tilde{F}([\tilde{W}])\right) \prec \bar{\varepsilon}/2$ and $\tilde{\Gamma}\left(\tilde{G}([\tilde{Z}]), \tilde{G}([\tilde{W}])\right) \prec \bar{\varepsilon}/2$. Therefore, we have

$$\begin{aligned}
&\tilde{\Gamma}\left((\tilde{F}[+]\tilde{G})([\tilde{Z}]),\ \tilde{F}([\tilde{W}])[+]\tilde{G}([\tilde{W}])\right)\\
&\quad = \tilde{\Gamma}\left(\tilde{F}([\tilde{Z}]), \tilde{F}([\tilde{W}])[+]\tilde{G}([\tilde{W}])[-]\tilde{G}([\tilde{Z}])\right)\\
&\quad \preccurlyeq \tilde{\Gamma}\left(\tilde{F}([\tilde{Z}]), \tilde{F}([\tilde{W}])\right)[+]\tilde{\Gamma}\left(\tilde{F}([\tilde{W}]), \tilde{F}([\tilde{W}])[+]\tilde{G}([\tilde{W}])[-]\tilde{G}([\tilde{Z}])\right)\\
&\quad = \tilde{\Gamma}\left(\tilde{F}([\tilde{Z}]), \tilde{F}([\tilde{W}])\right)[+]\tilde{\Gamma}\left(\tilde{G}([\tilde{W}]), \tilde{G}([\tilde{Z}])\right) \prec \bar{\varepsilon}.
\end{aligned}$$

Theorem 3.4. Let $\tilde{F}$ be fuzzy continuous at $[\tilde{W}] \in [\tilde{\mathcal{F}}_{\neg N}^{**}]$. Then there exist GRVBCCCNFNs $[\tilde{Z}_0] = [\tilde{\mu}_0][+]i[\tilde{\mu}_1]$ and $[\tilde{Z}_1] = [\tilde{\lambda}_0][+]i[\tilde{\lambda}_1]$ satisfy ${}^{\gamma+}[\tilde{\mu}_0] < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right) < {}^{\gamma+}[\tilde{\lambda}_0]$ and ${}^{\gamma+}[\tilde{\mu}_1] < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right) < {}^{\gamma+}[\tilde{\lambda}_1]$ when $\tilde{\Gamma}([\tilde{Z}]), [\tilde{W}]) \prec \bar{\delta}$ for $\bar{\delta} > \bar{0}$.

Theorem 3.5. Let $\tilde{F}$ be fuzzy continuous function at $[\tilde{W}] \in [\tilde{\mathcal{F}}_{\neg N}^{**}]$, $[\tilde{Z}_0] \in [\tilde{\mathcal{F}}_{\neg N}^{**}]$, and there exists $\bar{\delta} > \bar{0}$ such that ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right) < {}^{\gamma+}(\mathrm{Re}[\tilde{Z}_0])$ (resp. ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right) > {}^{\gamma+}(\mathrm{Re}[\tilde{Z}_0])$) and ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right) < {}^{\gamma+}(\mathrm{Im}[\tilde{Z}_0])$ (resp. ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right) > {}^{\gamma+}(\mathrm{Im}[\tilde{Z}_0])$) when $\tilde{\Gamma}([\tilde{Z}]), [\tilde{W}]) \prec \bar{\delta}$ then ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right) < {}^{\gamma+}(\mathrm{Re}[\tilde{Z}_0])$ (resp. ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right) > {}^{\gamma+}(\mathrm{Re}[\tilde{Z}_0])$) and ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right) < {}^{\gamma+}(\mathrm{Im}[\tilde{Z}_0])$ (resp. ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right) > {}^{\gamma+}(\mathrm{Im}[\tilde{Z}_0])$).

Theorem 3.6. Let $\tilde{F}$ and $\tilde{G}$ are both fuzzy type I-continuous (resp. II-continuous) and $[*] \in \{[\vee], [\wedge]], [\cdot]\}$. Then

1. $\tilde{F}[*]\tilde{G}$ is also fuzzy type I-continuous (resp. II-continuous), such that ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l}$, ${}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u}$, ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u}$, ${}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l}$ are all greater than zero for all $\gamma \in I_{[0}^{1[}$.
2. $\tilde{F}[+]\tilde{G}$ is also fuzzy type I-continuous (resp. II-continuous).
3. $[-]\tilde{G}$ is fuzzy type II-continuous (resp. I-continuous).
4. $\tilde{1}[/]\tilde{G}$ is also fuzzy type I-continuous (resp. II-continuous) such that, ${}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-}$, ${}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l+}$, ${}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u+}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l-}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l+}$, ${}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u-}$, ${}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+}$ are all positive for any $\gamma \in I_{[0}^{1[}$.

Proof: By hypothesis, for any $\bar{\varepsilon} > \bar{0}$ there is a $\overline{\delta'} > \bar{0}$, $\overline{\delta''} > \bar{0}$ such that $\tilde{F}([\tilde{Z}]) \prec \tilde{F}([\tilde{W}])[+]\bar{\varepsilon}$ (resp. $\tilde{F}([\tilde{W}]) \prec \tilde{F}([\tilde{Z}])[+]\bar{\varepsilon}$) whenever $[\tilde{Z}][-][\tilde{W}] \prec \overline{\delta'}$ and $\tilde{G}([\tilde{Z}]) \prec \tilde{G}([\tilde{W}])[+]\bar{\varepsilon}$ (resp. $\tilde{G}([\tilde{W}]) \prec \tilde{G}([\tilde{Z}])[+]\bar{\varepsilon}$) whenever $[\tilde{Z}][-][\tilde{W}] \prec \overline{\delta''}$.

1. Let $\bar{\delta} = \overline{\delta'} \wedge \overline{\delta''}$ and $\gamma \in I_{[0}^{1[}$, we have

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} + \varepsilon\right).$$

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l+} + \varepsilon\right).$$

$^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{l-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{l+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l+} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{u-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{u+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u+} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u+} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l+} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l+} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u+} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u-} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-} + \varepsilon\right).$

$^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u+} + \varepsilon \left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+} + \varepsilon\right).$ Hence,

$$\begin{aligned}
&{}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} \\
&\quad < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \left({}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon\right) * \varepsilon \\
&\quad < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \left({}^{0+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l-} + {}^{0+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon\right) * \varepsilon \\
&\quad = {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon'
\end{aligned}$$

$$\begin{aligned}
&\Bigg(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} \\
&\quad < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \left({}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon\right) * \varepsilon \\
&\quad < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \left({}^{0+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} + {}^{0+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon\right) * \varepsilon \\
&\quad = {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon''\Bigg).
\end{aligned}$$

$^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l+} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l+} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l+} + \varepsilon' \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\widetilde{W}])\right)^{l+} *$
γ+ReGWl+ <γ+ReFZl+ ∗γ+ReGZl+ +ε''.

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l^-} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l^-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^-} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{l^-} + \varepsilon' \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^-} *$$
γ+ImGWl−<γ+ImFZl−∗γ+ImGZl−+ε''.

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l^+} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l^+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^+} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{l^+} + \varepsilon' \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^+} *$$
γ+ImGWl+<γ+ImFZl+∗γ+ImGZl++ε''

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u^-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u^-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^-} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{U^-} + \varepsilon' \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^-} *$$
γ+ReGWu−<γ+ReFZu−∗γ+ReGZu−+ε''.

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u^+} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u^+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^+} * {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{u^+} + \varepsilon' \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^+} *$$
γ+ReGWu+<γ+ReFZu+∗γ+ReGZu++ε''.

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u^-} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u^-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{u^-} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{u^-} + \varepsilon' \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{u^-} *$$
γ+ImGWu−<γ+ImFZu−∗γ+ImGZu−+ε''.

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u^+} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u^+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{u^+} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{u^+} + \varepsilon'$$
$$\left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{u^+} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{u^+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u^+} * {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u^+} + \varepsilon''\right).$$

2. For any $\gamma \in I_{[0}^{1[}$, we have

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l^-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l^-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l^-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{l^-} + 2\varepsilon \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l^-} +$$
γ+ReGWl−<γ+ReFZl−+γ+ReGZl−+2ε.

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{l^+} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l^+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l^+} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{l^+} + 2\varepsilon \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{l^+} +$$
γ+ReGWl+<γ+ReFZl++γ+ReGZl++2ε.

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l^-} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l^-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^-} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{l^-} + 2\varepsilon \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^-} +$$
γ+ImGWl−<γ+ImFZl−+γ+ImGZl−+2ε.

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{l^+} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l^+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^+} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{W}])\right)^{l^+} + 2\varepsilon \quad \Big(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{W}])\right)^{l^+} +$$
γ+ImGWl+<γ+ImFZl++γ+ImGZl++2ε

$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u^-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u^-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{U^-} + 2\varepsilon$$
$$\left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{u^-} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u^-} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u^-} + 2\varepsilon\right).$$
$${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u^+} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u^+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^+} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{u^+} + 2\varepsilon$$
$$\left(\text{resp. } {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{W}])\right)^{u^+} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{W}])\right)^{u^+} < {}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right)^{u^+} + {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u^+} + 2\varepsilon\right).$$

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u-} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u-} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u-} + 2\varepsilon$$

$$\left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u-} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u-} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u-} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-} + 2\varepsilon\right).$$

$${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u+} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u+} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u+} + 2\varepsilon$$

$$\left(\text{resp. } {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\widetilde{W}])\right)^{u+} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u+} < {}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right)^{u+} + {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+} + 2\varepsilon\right).$$

3. For any $\gamma \in I_{[0}^{1[}$, we have

$$-{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l-} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l+} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l+} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{l+} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{l+} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l-} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l-} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l-} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l-} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l+} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l+} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{l+} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{l+} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u-} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u-} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u-} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u-} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u+} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u+} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\widetilde{W}])\right)^{u+} > -{}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)^{u+} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u-} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u-} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u-} + \varepsilon\right).$$

$$-{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u+} + \varepsilon \left(\text{resp.} -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\widetilde{W}])\right)^{u+} > -{}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)^{u+} + \varepsilon\right).$$

4. Obvious.

Theorem 3.7. If $\tilde{F}$ and $\tilde{G}$ are a fuzzy type I-continuous and II-continuous such that ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right) \leq {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)$ and ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right) \leq {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)$ for every $[\tilde{Z}] \in [\mathcal{F}^{**}_{\neg N}]$ and $\gamma \in I_{[0}^{1[}$, then there is a fuzzy continuous function $\tilde{H}$ satisfy ${}^{\gamma+}\left(\mathrm{Re}\tilde{F}([\tilde{Z}])\right) \leq {}^{\gamma+}\left(\mathrm{Re}\tilde{H}([\tilde{Z}])\right) \leq {}^{\gamma+}\left(\mathrm{Re}\tilde{G}([\tilde{Z}])\right)$ and ${}^{\gamma+}\left(\mathrm{Im}\tilde{F}([\tilde{Z}])\right) \leq {}^{\gamma+}\left(\mathrm{Im}\tilde{H}([\tilde{Z}])\right) \leq {}^{\gamma+}\left(\mathrm{Im}\tilde{G}([\tilde{Z}])\right)$.

Here, an open problem is presented for further investivigations: One can study that $\tilde{F}$ is both fuzzy type I-continuous and II-continuous if and only if $\tilde{F}$ is fuzzy continuous.

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