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Investigations of Certain Estimators for Modeling Panel Data Under Violations of Some Basic Assumptions

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Abstract
This paper investigates the efficiency of four methods of estimating panel data models (Pooling (OLS), First-Differenced (FD), Between (BTW) and Feasible Generalized Least Squares (FGLS)) when the assumptions of homoscedasticity, no autocorrelation and no collinearity are jointly violated. Monte-Carlo studies were carried out at different sample sizes, at varying degrees of heteroscedasticity, different levels of collinearity and autocorrelation all at different time periods. The results from this work showed that in small sample situation, irrespective of number of time length, FGLS estimator is efficient when heteroscedasticity is severe regardless of levels of autocorrelation and multicollinearity. However, when heteroscedasticity is low or mild with moderate autocorrelation level, both FD and FGLS are efficient, while BTW performs better only when there is no autocorrelation and low degree of heteroscedasticity. However, in large sample with short time periods, both FD and BTW could be used when there is no autocorrelation and low degree of heteroscedasticity, while FGLS is preferred elsewise. Meanwhile, Pooling estimator performs better when the assumptions of homoscedasticity, independent of error terms and orthogonality among the explanatory variables are justifiably valid.
Key words: Panel data, heteroscedasticity, autocorrelation, Multicollinearity, CLRM

1. Introduction

Panel data is a kind of data in which observations are obtained on the same set of entities at several periods of time. A panel dataset is one where there are repeated observations on the same units. The units may be individuals, households, firms, regions or countries. It has the combination of the characteristics of both time-series and cross-sectional data. Hence, problems that generally afflict time-series data (i.e. autocorrelation) and cross-sectional data (i.e. heteroscedasticity) need to be addressed while analyzing panel data. Because of many distinctive features that usually characterise panel data as abound in many econometrics settings, the use of classical ordinary least squares (OLS) estimator for modelling such data becomes grossly inefficient.

One of the critical assumptions of the classical linear regression model (CLRM) is that the error terms in the model are independent. If this assumption is violated, then serial correlation (or autocorrelation) is suspected (i.e. $cov(u_{it}, u_{is}) \neq 0, for\ t \neq s$). Also, the error terms are expected to have the same variance. If this is not satisfied, there is heteroscedasticity (i.e. $var(u_{it}) = \sigma_i^2$). (See Schmidt, 2005; Greene, 2008; Maddala, 2008; Creel, 2011; Wooldridge, 2012). Multicollinearity ensues when the assumption of “no linear dependencies in the explanatory variables” is violated. (See Chatterjee, 2006; Maddala, 2008; Gujarati & Porter, 2009). When multicollinearity is present in a model, there will be deterministic relationship among the exogenous variables such that one of the variables can be expressed as a linear function of at least one of the other variables.

Not only this, perfect collinearity could render the estimates of the parameters indeterminate. For the cases where the estimates of the parameters are obtained, the associated confidence intervals tend to be too wide and the standard error becomes infinitely large, an indication of inherent inconsistence in the estimated parameters.

In the presence of both autocorrelation and heteroscedasticity, the usual OLS estimators, although linear, unbiased, and asymptotically normally distributed, are no longer having minimum variance among all linear unbiased estimators. See Greene (2008), Baltagi et al. (2008), Olofin et al. (2010). Thus, the OLS estimator is not efficient relative to other linear and unbiased estimators under such situations.

A number of works on the methodologies and applications of panel data modelling have appeared in the literature (Li and Stengos, 1994; Roy, 2002; Baltagi et al., 2005, 2008; Bresson et al., 2006; Olofin et al., 2010). Situations where all the necessary assumptions underlying the use of classical linear regression methods are satisfied are rarely found in real life situations. Most of the studies that discussed panel data modelling considered the violation of each of the classical assumptions separately. For instance, Lillard and Wallis (1978), Bhargava et al. (1983), Baltagi and Li (1995), Galbraith and Zinde-Walsh (1995) and Roy (1999; 2002) at different times did appreciable works on panel data with autocorrelated disturbances. Also, the studies of Mazodier and Trognon (1978), Rao et al. (1981), Magnus (1982), Baltagi and Griffin (1988) and Wansbeek (1989) focused on the existence of heteroscedasticity in panel data modelling. Notably among the works that considered the joint violation of the assumptions of homoscedasticity and no autocorrelation are those of Baltagi

et al. (2008) and Olofin et al. (2010). The major distinction in their studies is that Baltagi et al. (2008) considered one-way error component model while Olofin et al. (2010) considered two-way error component model.
In this study, the methodologies of panel data modelling when the assumptions of no serial correlation, no multicollinearity and homoscedasticity are jointly violated are investigated. The efficiency of four methods of estimating panel data models is studied. The best estimator that is robust to the violations of the basic assumptions highlighted above among those considered are determined using absolute biases, variances and root mean square errors (RMSE) of parameter estimates. Results from this work would serve as useful guides to econometricians and students while modelling panel data that are characterized by the structure conjectured here.

2. Materials and Methods

This work considers one-way error component model with two exogenous and one endogenous variables. Heteroscedasticity was implanted into the model via the individual-specific error component. This is in line with the works of Mazodier and Trognon (1978), Baltagi and Griffin (1988), Roy (2002) and many others. We considered first-order serial correlation as did Lillard and Wallis (1978) and Bhargava et al (1983) to mention but few. Most of the earlier works on panel data with autocorrelated disturbances and heteroscedasticity focused on single exogenous variable. We, however, considered two exogenous variables with the possibility of existence of collinearity between them and its effects with respect to stability and efficiency of the estimation methods for panel data models.

2.1 A Classical Panel Data Model

A general panel data model is given as

$$Y_{it} = \alpha_i + X'_{it}\,\beta + u_{it} \qquad (1)$$

where Y_{it} is the response for unit *i* at time *t*, α_i is the individual-specific intercept, vector X'_{it} contains *k* regressors for unit *i* at time *t*, vector β contains *k* regression coefficients to be estimated and u_{it} is the error component for unit *i* at time *t, i* = 1, 2, …., *n* and *t* = 1, 2, …, T.
Specifically, we considered the panel data model that has two exogenous and one endogenous variables as shown below;

$$Y_{it} = \alpha_i + \beta_1 X_{1it} + \beta_2 X_{2it} + u_{it} \qquad (2)$$

where $\alpha_i = \alpha + \varepsilon_i$. The individual-specific intercept (α_i) captures the effects of those variables that are peculiar to the i^{th} individual and that are time-invariant.
The model therefore becomes

$$Y_{it} = \alpha + \beta_1 X_{1it} + \beta_2 X_{2it} + \varepsilon_i + u_{it} \qquad (3)$$

where ε_i is the individual-specific error component and u_{it} is the combined time-series and cross-section error component with variances σ_ε^2 and σ_u^2 respectively.
Suppose we let $w_{it} = \varepsilon_i + u_{it}$, then, model (3) becomes

$$Y_{it} = \alpha + \beta_1 X_{1it} + \beta_2 X_{2it} + w_{it} \qquad (4)$$

2.2 Brief Overview of Some Estimators of Panel Data Models Considered

In this section, we provide brief theoretical formulations of the four estimators of panel data models as considered in this study.
i.) **Pooled Estimator (OLS):** This Estimator stacks the data over *i* and *t* into one long regression with *nT* observations, and estimates of the parameters are obtained by OLS using the model (Greene, 2008).

$$\mathbf{y} = \mathbf{X}'\boldsymbol{\beta} + \mathbf{w} \qquad (5)$$

where **y** is an $nT \times 1$ column vector of response variables, **X** is an $nT \times$ k matrix of regressors, **β** is a (k+1) × 1 column vector of regression coefficients, **w** is an nT × 1 column vector of the combined error terms (i.e $\varepsilon_i + u_{it}$)
The Pooled estimator is given as

$$\hat{\boldsymbol{\beta}}_{pooled} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \qquad (6)$$

ii.) **Between Estimator (BTW):** This regresses the group means of Y on the group means of X's in a regression of ***n*** observations. It uses cross-sectional variation by averaging the observations over period *t* (Creel, 2011; Wooldridge, 2012). Explicitly, it converts all the observations into individual-specific averages and performs OLS on the transformed data.
Averaging model (7) above over *t* gives

$$\bar{Y}_{i.} = \alpha + \beta_1 \bar{X}_{1i.} + \beta_2 \bar{X}_{2i.} + \bar{w}_{i.} \qquad (7)$$

where $\bar{Y}_{i.} = T^{-1}\sum_t Y_{it}$, $\bar{X}_{ji.} = T^{-1}\sum_t X_{jit}$ and $\bar{w}_{i.} = T^{-1}\sum_t w_{it}$ for *i* = 1, 2, 3, . . . *n* and *j* = 1, 2.
iii.) **First-Differenced Estimator (FD):** This is the ordinary least squares estimation of the difference between the original model and its one-period-lagged model (Arellano, 2003; Baltagi, 2005). The FD model is given as

$$\Delta Y_{it} = \beta_1\, \Delta X_{1it} + \beta_2\, \Delta X_{2it} + \Delta w_{it} \qquad (8)$$

where $\Delta Y_{it} = Y_{it} - Y_{i,t-1}$; $\Delta X_{1it} = X_{1it} - X_{1i,t-1}$; $\Delta X_{2it} = X_{2it} - X_{2i,t-1}$; and $\Delta w_{it} = w_{it} - w_{i,t-1}$, for *i* = 1, 2, . . ., *n* and *t* = 2, 3, . . ., T.

iv.) **Feasible Generalized Least Squares Estimator (FGLS):** The generalized least squares estimator for the model parameters is obtained from the OLS estimation of the transformed model as shown below:

$$\mathbf{y}_{it}^{*} = \boldsymbol{\alpha}_i^{*} + \mathbf{X'}_{it}^{*}\,\boldsymbol{\beta} + \mathbf{w}_{it}^{*}\,; \tag{9}$$

where $y_{it}^{*} = y_{it} - \lambda\bar{y}_{i.}$; $X'^{*}_{it} = X'_{it} - \lambda\bar{X}_{i.}$; $w_{it}^{*} = \lambda\bar{w}_i$ $\alpha_i^{*} = 1 - \lambda$ and $\lambda = 1 - \sqrt{\sigma_u^2/\{\sigma_u^2 + T\sigma_\Sigma^2\}}$ for $i = 1, 2, ..., n$ and $t = 1, 2, \dots, T$

The term λ gives a measure of the relative sizes of the within and between unit variances. The generalized least square estimator (GLSE) of the regression parameters using this method is given by

$$\hat{\boldsymbol{\beta}}_{GLS} = (\mathbf{X}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{X}^{*})^{-1}\,\mathbf{X}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{y}^{*} \tag{10}$$

However, since the elements of the error variance-covariance matrix Ω are often unknown, their values are estimated from the data to have $\hat{\Omega}$. Therefore, by replacing Ω in (10) with $\hat{\Omega}$ we have the feasible generalized least squares estimator (FGLSE) of model (9) given as

$$\hat{\boldsymbol{\beta}}_{FGLS} = (\mathbf{X}^{*\prime}\hat{\Omega}^{-1}\mathbf{X}^{*})^{-1}\,\mathbf{X}^{*\prime}\hat{\Omega}^{-1}\mathbf{y}^{*} \tag{11}$$

which is more frequently used rather than the GLSE.

One of the ways to estimate Ω is to first estimate λ using $\hat{\sigma}_u^2$ and $\hat{\sigma}_\Sigma^2$ such that $\lambda = \sqrt{\frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + T\hat{\sigma}_\Sigma^2}}$. Here, $\hat{\sigma}_u^2$ could be obtained from the error sum of squares (SSE) of the within effects or from the derivations of residuals from group means of the residuals (Hsiao, 2002; Arellano, 2003). Also, $\hat{\sigma}_\Sigma^2$ is derived from the between effects model (i.e group mean regression). See Baltagi (2005), Greene (2008), Creel (2011) and Wooldridge (2012) for more details.

2.3 The Simulation Scheme

The datasets used for this work were simulated using Monte Carlo experiments in the environment of R statistical package (www.cran.org). Two sizes of cross-sectional units (50 and 250), three time periods (10, 40, and 100), five levels of autocorrelation (ρ = ±0.9, ±0.5, 0), five levels of collinearity (r = ±0.9, ±0.5, 0) and three degrees of heteroscedasticity (low, mild & severe) were used for simulation. Each of the combinations was iterated 1000 times and the assessments of the various estimators considered in this work were based on the absolute bias, variance and RMSE of parameter estimates.

The absolute bias (AB) of parameter $\hat{\beta}_k$ estimated over r replicates is defined by

$$\text{AB}\,(\hat{\beta}_k) = \frac{1}{r}\sum_{j=1}^{r}\left|\hat{\beta}_{kj} - \beta_k\right| \tag{12}$$

The variance of the estimator $\hat{\beta}_k$ is given by

$$\text{Var}\,(\hat{\beta}_k) = \frac{1}{r}\sum_{j=1}^{r}\left(\hat{\beta}_{kj} - \bar{\hat{\beta}}_k\right)^2 \tag{13}$$

while its root mean square error is given by

$$\text{RMSE}\,(\hat{\beta}_k) = \sqrt{\frac{1}{r}\sum_{j=1}^{r}\left(\hat{\beta}_{kj} - \beta_k\right)^2} \tag{14}$$

where $\hat{\beta}_k$ indicates the k^{th} parameter being estimated for j = 1, 2, 3, . . ., r (number of iterations).

After the above criteria were evaluated for each estimator, the performances were ranked and the best method was identified. A test of significance was carried out to test if the sums of ranks of other estimators are actually significantly different from that of the estimator proclaimed best. Because the ranks are on ordinal scale of measurements, a non-parametric statistical test developed by Milton Friedman (1939) was employed to perform this task.

Thereafter, the pair-wise comparison test between a pair of estimators, say p and q, was performed using the least significant difference (LSD) with the test statistic given by:

$$LSD = t_{(n-1)(k-1)},\alpha\sqrt{2\left\{n\sum_i^k\sum_j^n r_{ij}^2 - \sum_i^k R_i^2\right\} \div (n-1)(k-1)} \tag{15}$$

where k is the number of estimators, n is the number of repetitions (levels of collinearity), R_i is the sum of ranks for each estimator and r_{ij}^2 is the square of the rank for each estimator at each level of multicollinearity.

Based on this formulations, any two estimators, p and q, are declared to be significantly different in terms of their performances at a chosen type I error rate α, if the absolute difference of their ranks is greater than the estimated LSD value.

3. Results and Discussion

The results of the performances of the estimators considered at various levels of autocorrelation, multicollinearity, and heteroscedasticity considered in this work are presented and discussed here. The four estimators were assessed using each of the criteria stated in Section 2.3. These estimators were ranked using the ranks 1, 2, 3 and 4 with rank 1 assigned to the best estimator that has the lowest value of the absolute bias, variance and root mean square error. A rank of 2 is assigned to the second best estimator and so on.

For instance, the performances of the estimators using variance criterion when we have 50 cross-sectional units and 10 time periods at all levels of autocorrelation and multicollinearity and degrees of heteroscedasticity are presented in Table 1.
The ranks for each of the estimators were summed for all levels of multicollinearity to determine an estimator with lowest sum of ranks at each of the autocorrelation levels and degrees of heteroscedasticity. Here, the effects of multicollinearity are repressed to assess the upshots of autocorrelation and heteroscedasticity which are respectively the problems that are peculiar to time-series and cross-sectional data that are brass tacks of panel data.
We present in Table 2, the sum of ranks that resulted from Table 1 for the four estimators considered in this study.
The preference of these estimators at different sample sizes, different levels of autocorrelation and varying degrees of heteroscedasticity for each of the sample sizes used are presented in Table 3.
It can be deduced from the above table that for N = 50; T = 10 at high level of autocorrelation and high level of multicollinearity, FD and FGLS estimators are not significantly different at low degree of heteroscedasticity, but FGLS outperforms others at mild and severe degrees of heteroscedasticity. Meanwhile, for low level of collinearity and high autocorrelation level, FD is preferred regardless of degree of heteroscedasticity. Any of FD and FGLS could be used for moderate level of collinearity at high autocorrelation level irrespective of the degree of heteroscedasticity.
At moderate autocorrelation level and high level of collinearity, FD/BTW, FD and FGLS are preferred respectively at low, mild and severe degrees of heteroscedasticity. For middling levels of autocorrelation and multicollinearity, either of BTW and FD would produce efficient estimates for low degree of heteroscedasticity, FD is preferable for mild while any of FD and FGLS could be employed for severe degree of heteroscedasticity. Moreover, when there is no autocorrelation, BTW estimator which regresses the group means of the dependent variable on the group means of independent variables using OLS would return efficient estimates in spite of the degree of heteroscedasticity and level of collinearity excepting few cases where FD competes with it.
Apparently, the precedence of the estimators considered in this study based on the Monte-Carlo experiments carried out at varying degree of heteroscedasticity and levels of autocorrelation and multicollinearity for various sample sizes are displayed in the Table 3. The behaviours of the estimators differ as we change the levels and degree of the assumptions being violated for different sample sizes.

4. Conclusion

Various results obtained in this work generally showed that the behaviours of the four estimators investigated for modeling various panel data vary as the violations are varied. Failure of the orthogonality assumption makes the OLS estimators to be biased and imprecise. For OLS to be accurately used in estimating the parameters of panel data models, errors have to be independent and homoscedastic. These conditions are so atypical and mostly unrealistic in many real life situations that would have warranted the use of OLS for modeling panel data efficiently.
The efficiency of four methods of estimating panel data models with violations of homoscedasticity, no autocorrelation and no collinearity assumptions is principally and thoroughly addressed in this work. Our findings from Monte Carlo experiments for several combinations of violations show that in small sample, irrespective of number of time periods, FGLS is preferable when heteroscedasticity is severe regardless of autocorrelation level. But when heteroscedasticity is low or mild with moderate autocorrelation level, both FD and FGLS are preferred, while BTW performs better only when there is no autocorrelation and low degree of heteroscedasticity. That is, in large sample with little time periods, both FD and BTW could be used when there is no autocorrelation and low degree of heteroscedasticity.
Also when the degree of heteroscedasticity is mild and there is no autocorrelation in large sample with little time periods, any of the FD and FGLS would produce efficient results. Finally, when severe degree of heteroscedasticity is present and autocorrelation is apparent in large sample regardless of time periods, FGLS is superior. Meanwhile, both FD and FGLS are suitable when there is low heteroscedasticity despite the existence of autocorrelation and multicollinearity.
As general remark given the various results obtained in this study, it is always necessary to assess the degree of heteroscedasticity and level of autocorrelation while developing panel data models in order to ensure efficient results. In case there is more than one predictor in the model, the strength of relationship between or among the predictors needs to be examined for possible presence of multicollinearity in the data as to avoid erroneous inferences that may arise from the use of wrong method for estimating the model.

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Appendix

Table 1: Ranks of the Estimators Using Variance Criterion when N = 50 and T = 10

Auto-correlation level	Estimator	Multi = -0.9			Multi = -0.5			Multi = 0			Multi = 0.5			Multi = 0.9		
		Heteroscedasticity			Heteroscedasticity			Heteroscedasticity			Heteroscedasticity			Heteroscedasticity		
		Low	Mild	Sev	Low	Mild	Sev	Low	Mild	Sev	Low	Mild	Sev	Low	Mild	Sev
-0.9	OLS	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	FD	2	2	2	1	1	2	1	1	1	1	1	2	2	2	2
	BTW	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	FGLS	1	1	1	2	2	1	2	2	2	2	2	1	1	1	1
-0.5	OLS	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	FD	1	1	2	2	1	1	2	2	1	2	1	1	1	1	2
	BTW	2	3	3	1	2	3	1	1	3	1	2	3	2	3	3
	FGLS	3	2	1	3	3	2	3	3	2	3	3	2	3	2	1
0	OLS	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	FD	2	3	1	2	2	3	2	2	3	2	2	3	2	3	1
	BTW	1	1	2	1	1	1	1	1	1	1	1	1	1	1	2
	FGLS	3	2	3	3	3	2	3	3	2	3	3	2	3	2	3
0.5	OLS	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	FD	1	1	2	2	1	1	2	2	1	2	1	1	1	1	2
	BTW	2	3	3	1	2	3	1	1	3	1	2	3	2	3	3
	FGLS	3	2	1	3	3	2	3	3	2	3	3	2	3	2	1
0.9	OLS	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	FD	2	2	2	1	1	2	1	1	1	1	1	2	2	2	2
	BTW	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	FGLS	1	1	1	2	2	1	2	2	2	2	2	1	1	1	1

Table 2: Sum of Ranks of the Estimators Using Variance Criterion when N = 50 and T = 10

Autocorrelation Level	Estimator	Degree of Heteroscedasticity		
		Low	Mild	Severe
-0.9	OLS	20	20	20
	FD	7	7	9
	BTW	15	15	15
	FGLS	8	8	6
-0.5	OLS	20	20	20
	FD	8	6	7
	BTW	7	11	15
	FGLS	15	13	8
0	OLS	20	20	20
	FD	10	12	11
	BTW	5	5	7
	FGLS	15	13	12
0.5	OLS	20	20	20
	FD	8	6	7
	BTW	7	11	15
	FGLS	15	13	8
0.9	OLS	20	20	20
	FD	7	7	9
	BTW	15	15	15
	FGLS	8	8	6

Table 3: Preference of the estimators at varying degrees of heteroscedasticity and autocorrelation and multicollinearity levels for the various sample sizes considered

Cross-sectional units	Time Periods	Multicollinearity Level	Degree of Heteroscedasticity	Autocorrelation Level		
				± 0.9	± 0.5	0
50	10	± 0.9	Low	**FD**, FGLS	**FD**, BTW	**BTW**, FD
			Mild	FGLS	FD	**BTW**, FD
			Severe	FGLS	FGLS	FD
		± 0.5	Low	**FD**, FGLS	**BTW**, FD	BTW
			Mild	**FGLS**, FD	FD	BTW
			Severe	**FD**, FGLS	**FD**, FGLS	**BTW**, FD
		0	Low	FD	BTW	BTW
			Mild	FD	**BTW**, FD	BTW
			Severe	FD	**FD**, FGLS	BTW
	40	± 0.9	Low	FGLS	FD	BTW
			Mild	FGLS	FD	FD
			Severe	FGLS	FGLS	**FD**, FGLS
		± 0.5	Low	**FGLS**, FD	**FD**, BTW	**BTW**, FD
			Mild	**FGLS**, FD	**FD**, BTW	**BTW**, FD
			Severe	FGLS	**FD**, FGLS	FD
		0	Low	**FGLS**, FD	**BTW**, FD	**BTW**, FD
			Mild	**FGLS**, FD	**BTW**, FD	BTW
			Severe	**FGLS**, FD	FD	BTW
	100	± 0.9	Low	FGLS	FGLS	**FGLS**, FD
			Mild	FGLS	FGLS	FD
			Severe	FGLS	FGLS	FGLS
		± 0.5	Low	FGLS	**FGLS**, FD	FD
			Mild	FGLS	**FGLS**, FD	**FGLS**, FD
			Severe	FGLS	**FGLS**, FD	FGLS, FD
		0	Low	**FGLS**, FD	FD	BTW
			Mild	**FGLS**, FD	**FGLS**, FD	**BTW**, FD, FGLS
			Severe	**FGLS**, FD	**FGLS**, FD	**FGLS**, FD
250	10	± 0.9	Low	FGLS	**FD**, FGLS	**FD**, FGLS
			Mild	FGLS	FGLS	**FD**, FGLS
			Severe	FGLS	FGLS	**FD**, FGLS
		± 0.5	Low	**FD**, FGLS	**FD**, FGLS	**BTW**, FD, FGLS
			Mild	FGLS	FGLS	**FD**, FGLS
			Severe	FGLS	FGLS	**FD**, FGLS
		0	Low	**FD**, FGLS	**FD**, FGLS	**BTW**, FD
			Mild	**FD**, FGLS	**FD**, FGLS	**BTW**, FD, FGLS
			Severe	**FD**, FGLS	**FD**, FGLS	**FD**, FGLS
	40	± 0.9	Low	FGLS	FD	**FD**, FGLS
			Mild	FGLS	FGLS	**FD**, FGLS
			Severe	FGLS	FGLS	FGLS
		± 0.5	Low	FGLS	**FD**, FGLS	**BTW**, FD, FGLS
			Mild	FGLS	FGLS	**FD**, FGLS
			Severe	FGLS	FGLS	**FD**, FGLS
		0	Low	**FD**, FGLS	**FD**, FGLS	**BTW**, FD, FGLS
			Mild	FGLS	**FD**, FGLS	**BTW**, FD, FGLS
			Severe	FGLS	FGLS	**FD**, FGLS
	100	± 0.9	Low	FGLS	FGLS	**FD**, FGLS
			Mild	FGLS	FGLS	FGLS
			Severe	FGLS	FGLS	FGLS
		± 0.5	Low	FGLS	FGLS	**FD**, FGLS
			Mild	FGLS	FGLS	**FD**, FGLS
			Severe	FGLS	FGLS	**FD**, FGLS
		0	Low	FGLS	**FD**, FGLS	**BTW**, FD, FGLS
			Mild	FGLS	**FD**, FGLS	**FD**, FGLS
			Severe	FGLS	FGLS	**FD**, FGLS

Note that the boldfaced estimators are the ones with lowest sums of ranks, while other estimator(s) in the same cell with them are not significantly different from them as established by their respective least significant difference (LSD) statistical tests.