A Note on Estimation of Population Mean in Sample Survey using Auxiliary Information

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Abstract
In this paper we have suggested a class of estimators for population mean using auxiliary information in two-phase sampling. When the population mean $\overline{X}$ is not known, a class of estimators for finite population mean $\overline{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$ of the study variable y has been suggested. Expressions of bias and mean squared error are obtained upto the first order of approximation. Asymptotically optimum estimators (AOE's) are also identified with its mean squared error formula. We found that proposed class of estimator are better than usual ratio and other estimators.

1.1 Introduction

Whenever there is auxiliary information available, the investigator wants to use it in the method of estimation which yields larger efficiency. Ratio, regression and product methods of estimation are good examples in this context. When the population mean $\bar{X}$ of the auxiliary variable x is known a large number of estimators for the population mean $\bar{Y}$ of the study variable y is available in the literature for instance see Singh, H. P. (1986) and Singh, S. (2003) and the references cited therein. It is observed in the literature that the efficiency of the ratio and product estimators may be increased to the efficiency of regression estimator by making use of prior knowledge of $k=\rho\left(\frac{C_y}{C_x}\right)$, where ρ is the correlation coefficient between the study variable y and auxiliary variable x, C_y and C_x are the coefficients of variation of y and x respectively. Sacrificing the consistency of estimators researchers including Upadhyaya and Singh (1985), Srivastava (1974), Prasad (1989) and Lui (1990) have found the way by which the efficiency an estimator can be increased beyond the regression estimator of the mean $\bar{Y}$ in case of known population mean $\bar{X}$.

When the population mean $\bar{X}$ of the auxiliary variable x is not known, it is often estimated from a preliminary large sample on which only the auxiliary variable x is observed. The value of the population mean $\bar{X}$ of the auxiliary variable x is then replaced by this estimate. This procedure is known as the double sampling or two-phase sampling. Throughout, samples have been drawn by the method of simple random sampling without replacement (SRSWOR). The sample survey statisticians have presented several modifications of the classical two-phase sampling ratio and product estimators and studied their properties. They have shown that the efficiency of their estimators can be increased maximum upto usual two-phase sampling regression estimator by using of the well known optimum choice $k=\rho\left(\frac{C_y}{C_x}\right)$, for instance see Srivastava (1970, 81), Gupta (1978) and Adhvaryu and Gupta (1983).

In this paper we have made an effort to suggest a class of estimators using auxiliary information in two-phase sampling and studied its properties. When the population mean $\bar{X}$ is not known, a class of estimators for finite population mean $\overline{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$ of the study variable y has been suggested. A large number of estimators are identified as the member of the proposed class of estimators. Expressions of bias and mean squared error are obtained upto the first order of approximation. Asymptotically optimum estimators (AOE's) are also identified with its mean squared error formula.

1.2 Two-Phase Sampling Procedure

Consider the finite population $U=(U_1,U_2,\dots,U_N)$ of N identifiable units. Let y and x be the variable under study and the auxiliary variable respectively. Further let y_i be the unknown real variable value of y and x_i be the known variable value of x associated with $U_i,(i=1,2,\dots,N)$. The problem of estimating the population mean $\overline{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$ of the study variable y when the population mean $\overline{X}=\frac{1}{N}\sum_{i=1}^{N}x_i$ of the auxiliary variable

x is known has been dealt at a grater length. However, in many practical situations when the population mean $\bar{X}$ is unknown a prior, the procedure of double sampling is used. Allowing, simple random sampling without replacement (SRSWOR) design in each phase, the two-phase (or double) sampling scheme will be as follows:

(i) The first-phase sample $s_1(s_1 \subset U)$ of fixed size n_1 is drawn to observe only x in order to furnish a good estimate of the population mean $\bar{X}$.

(ii) Given s_1, the second phase sample $s(s \subset s_1)$ of fixed size n is drawn to observe y only.

Let $\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

Further we write

$$\bar{y} = \bar{X}(1+e_0)$$
$$\bar{x} = \bar{X}(1+e_1)$$
$$\bar{x}_1 = \bar{X}(1+e_2) \qquad 1.2.1$$

Such that

$$E(e_0) = E(e_1) = E(e_2) = 0$$

and

$$\left.\begin{aligned} E(e_0^2) &= \left(\frac{1}{n}-\frac{1}{N}\right)C_y^2, & E(e_1^2) &= \left(\frac{1}{n}-\frac{1}{N}\right)C_x^2, \\ E(e_2^2) &= \left(\frac{1}{n_1}-\frac{1}{N}\right)C_x^2, & E(e_0\,e_1) &= \left(\frac{1}{n}-\frac{1}{N}\right)k\,C_x^2 \\ E(e_0\,e_2) &= \left(\frac{1}{n_1}-\frac{1}{N}\right)k\,C_x^2, & E(e_1\,e_2) &= \left(\frac{1}{n_1}-\frac{1}{N}\right)C_x^2 \end{aligned}\right\} \qquad 1.2.2$$

where

$$C_y = \frac{S_y}{\bar{Y}}, C_x = \frac{S_x}{\bar{X}} , S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(y_i - \bar{Y})^2$$

and

$$S_x^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(x_i - \bar{X})^2.$$

1.3 Proposed Class of Estimators

Motivated by Gangele (1995) and Gupta (1978), we define a class of estimators for population mean $\bar{Y}$ in two-phase sampling as

$$\hat{\bar{Y}}_{\alpha d} = \sum_{i=0}^{3} \alpha_i\, \bar{y}\left(\frac{\bar{x}}{\bar{x}_1}\right)^{i\delta}, \qquad 1.3.1$$

where $\alpha_i's(i = 0,1,2,3)$ are suitably chosen constants whose sum need not be unity, δ is a suitably chosen scalar takes value $+1$ for product-type estimator and -1 for ratio-type estimator; $\bar{x}_1$ is a first phase sample mean based on n_1 observations and $(\bar{y}, \bar{x})$ are second phase sample means of (y, x) respectively based on n observations. Some members of the proposed class of estimators are shown in Table-1.3.1.

Table-1.3.1

Some members of the class of estimators $\hat{\hat{Y}}_{\alpha d}$

S. No.	Estimator	Choice of constants				
		α_0	α_1	α_2	α_3	δ
1	The mean per unit $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$	1	0	0	0	-
2	The classical two-phase sampling ratio estimator $\bar{y}_{Rd} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}\right)$	0	1	0	0	-1
3	The classical two-phase sampling product estimator $\bar{y}_{Pd} = \bar{y}\left(\frac{\bar{x}}{\bar{x}_1}\right)$	0	1	0	0	1
4	Srivastava (1970) estimator $\bar{y}_{1d} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}\right)^{\delta}$	0	1	0	0	$-\delta$
5	Chakraberty (1968), Vos (1980), Adhvaryu and Gupta (1983) type-estimator $\bar{y}_{2d} = \alpha_0\bar{y} + (1-\alpha_0)\bar{y}\left(\frac{\bar{x}_1}{\bar{x}}\right)$	α_0	$(1-\alpha_0)$	0	0	-1
6	Vos (1980), Adhvaryu and Gupta (1983) type estimator $\bar{y}_{3d} = \alpha_0\bar{y} + (1-\alpha_0)\bar{y}\left(\frac{\bar{x}}{\bar{x}_1}\right)$	α_0	$(1-\alpha_0)$	0	0	1

(α, δ) being constants.

1.4 Bias and Mean Squared Error

To obtain the bias and mean squared error of the proposed class of estimators $\hat{\hat{Y}}_{\alpha d}$, we express it in terms of $e's$ we have

$\hat{\hat{Y}}_{\alpha d} = \sum_{i=0}^{3} \alpha_i \,\bar{Y}(1+e_0)(1+e_1)^{i\delta}(1+e_1')^{-i\delta}$.

or

$\hat{\hat{Y}}_{\alpha d} = \bar{Y}(1+e_0)\sum_{i=0}^{3} \alpha_i \,(1+e_1)^{i\delta}(1+e_1')^{-i\delta}$. 1.4.1

We assume that the sample size n and $n_1(n_1 > n)$ are large enough so that

$|e_1| < 1$ and $|e_1'| < 1$.

i.e. $\left|\frac{\bar{x}-\bar{X}}{\bar{X}}\right| < 1$ and $\left|\frac{\bar{x}_1-\bar{X}}{\bar{X}}\right| < 1$.

and $(1+e_1)$ and $(1+e_1')$ are expandable.

Expanding the right hand side of (1.4.1) we have

$$\hat{\hat{Y}}_{\alpha d} = \bar{Y}\sum_{i=0}^{3} \alpha_i \,(1+e_0)\left\{1 + i\delta e_1 + \frac{i\delta(i\delta-1)}{2}e_1^2 + \cdots\right\} \times \left\{1 - i\delta e_1' + \frac{i\delta(i\delta+1)}{2}e_1'^2 + \cdots\right\}$$

or

$$\hat{\hat{Y}}_{\alpha d} = \bar{Y}\sum_{i=0}^{3} \alpha_i \Big[1 + e_0 + i\delta e_1 + i\delta e_0 e_1 + \frac{i\delta(i\delta-1)}{2}e_1^2 - i\delta e_1' - i\delta e_0 e_1' - i^2\delta^2 e_1 e_1' + \frac{i\delta(i\delta+1)}{2}e_1'^2 + \frac{i\delta(i\delta-1)}{2}e_0 e_1^2 - i^2\delta^2 e_0 e_1 e_1' - \frac{i^2\delta^2(i\delta-1)}{2}e_1^2 e_1' + \frac{i\delta(i\delta+1)}{2}e_0 e_1'^2 + \frac{i^2\delta^2(i\delta+1)}{2}e_1 e_1'^2 + \cdots\Big].$$

Neglecting terms of $e's$ having power greater than two we have

$\hat{\hat{Y}}_{\alpha d} = \bar{Y}\sum_{i=0}^{3} \alpha_i \left[1 + e_0 + i\delta(e_1 - e_1') + i\delta(e_0 e_1 - e_0 e_1') - i^2\delta^2 e_1 e_1' + \frac{i\delta(i\delta-1)}{2}e_1^2 + \right.$
*iδiδ+12e1'2.*or

$\hat{\hat{Y}}_{\alpha d} - \bar{Y}) = \bar{Y}\left[\sum_{i=0}^{3} \alpha_i \left[1 + e_0 + i\delta(e_1 - e_1') + i\delta(e_0 e_1 - e_0 e_1') - i^2\delta^2 e_1 e_1' + \frac{i\delta(i\delta-1)}{2}e_1^2 + \right.\right.$
iδiδ+12e1'2−1.

1.4.2

Taking expectation of both sides of (1.4.2) and using the expected values given by (1.2.2) we get the bias of $\hat{\hat{Y}}_{\alpha d}$ to the first degree of approximation as

$$B\left(\hat{\hat{Y}}_{\alpha d}\right)=\bar{Y}\left[\sum_{i=0}^{3}\alpha_i\begin{bmatrix}1+\left(\frac{1}{n}-\frac{1}{n'}\right)i\delta kC_x^2+\frac{i\delta(i\delta-1)}{2}\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2\\-\frac{i\delta(i\delta-1)}{2}\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2\end{bmatrix}-1\right]$$

$$=\bar{Y}\left[\sum_{i=0}^{3}\alpha_i\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)i\delta kC_x^2+\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{i\delta(i\delta+1)}{2}C_x^2\right]-1\right]$$

i.e.

$$B\left(\hat{\hat{Y}}_{\alpha d}\right)=\bar{Y}\left[\sum_{i=0}^{3}\alpha_i\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)i\delta C_x^2\left(k+\frac{(i\delta-1)}{2}\right)\right]-1\right]. \qquad 1.4.3$$

Squaring both sides of (1.4.2) and neglecting term of $e's$ having power greater than two we have

$$\left(\hat{\hat{Y}}_{\alpha d}-\bar{Y}\right)^2=\bar{Y}^2\begin{bmatrix}1+\sum_{i=0}^{3}\alpha_i^2\begin{Bmatrix}1+2e_0+2i\delta(e_1-e_1')\\+4i\delta(e_0e_1-e_0e_1')+e_0^2\\+i^2\delta^2(e_1-e_1')^2-2i^2\delta^2e_1e_1'\\+i\delta(i\delta-1)e_1^2+i\delta(i\delta+1)e_1'^2\end{Bmatrix}\\+2\sum_{i(i<j)=0}^{3}\alpha_i\alpha_j\begin{Bmatrix}1+2e_0+(i+j)\delta(e_1-e_1')\\+e_0^2+2(i+j)(e_0e_1-e_0e_1')\\-(i-j)^2\delta^2e_1e_1'+\frac{\delta(i+j)}{2}(\delta(i+j)-1)e_1^2\\+\frac{\delta(i+j)}{2}(\delta(i+j)-1)e_1'^2\end{Bmatrix}\\-2\sum_{i=0}^{3}\begin{Bmatrix}1+e_0+i\delta(e_1-e_1')\\+i\delta(e_0e_1-e_0e_1')\\-i^2\delta^2e_1e_1'+\frac{i\delta(i\delta-1)}{2}e_1^2+\frac{i\delta(i\delta+1)}{2}e_1'^2\end{Bmatrix}\end{bmatrix}.$$

1.4.4

Taking expectation of both sides of (1.4.4) using the result in (1.2.2) we get the MSE of $\hat{\hat{Y}}_{\alpha d}$ to the first degree of approximation as

$$MSE\left(\hat{\hat{Y}}_{\alpha d}\right)=\bar{Y}^2\left[1+\sum_{i=0}^{3}\alpha_i^2A_i+2\sum_{i(<j)=0}^{3}\alpha_i\alpha_jA_{ij}-2\sum_{i=0}^{3}\alpha_iB_i\right], \quad \mathbf{1.4.5}$$

where

$$A_i=\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)i\delta(4k+2i\delta-1)C_x^2\right]\ (i=0,1,2,3),$$

$$A_{ij}=\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{\delta(i+j)}{2}(4k+\delta(i+j)-1)C_x^2\right] \quad (i(<j)=0,1,2,3),$$

$$B_i=\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)i\delta\left(k+\frac{(i\delta-1)}{2}\right)C_x^2\right](i=0,1,2,3).$$

Expression (1.4.5) can be rewritten as

$$MSE\left(\hat{\hat{Y}}_{\alpha d}\right)=\bar{Y}^2\begin{bmatrix}1+\alpha_0^2A_0+\alpha_1^2A_1+\alpha_2^2A_2+\alpha_3^2A_3\\+2(\alpha_0\alpha_1A_{01}+\alpha_0\alpha_2A_{02}+\alpha_0\alpha_3A_{03}+\alpha_1\alpha_2A_{12}+\alpha_1\alpha_3A_{13}+\alpha_2\alpha_3A_{23})\\-2(\alpha_0B_0+\alpha_1B_1+\alpha_2B_2+\alpha_3B_3)\end{bmatrix},$$

1.4.6

where

$$B_0=1,$$
$$B_1=\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)\alpha\left\{k+\frac{(\delta-1)}{2}\right\}C_x^2\right],$$
$$B_2=\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)2\alpha\left\{k+\frac{(2\delta-1)}{2}\right\}C_x^2\right],$$
$$B_3=\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)3\alpha\left\{k+\frac{(3\delta-1)}{2}\right\}C_x^2\right],$$
$$A_0=\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2\right],$$
$$A_1=\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)\delta(4k+2\delta-1)C_x^2\right],$$
$$A_2=\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)2\delta(4k+4\delta-1)C_x^2\right],$$

$$A_3 = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) 3\delta(4k + 6\delta - 1)C_x^2\right],$$
$$A_{01} = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) \frac{\delta}{2}(4k + \delta - 1)C_x^2\right],$$
$$A_{02} = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) \delta(4k + 2\delta - 1)C_x^2\right],$$
$$A_{03} = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) \frac{3\delta}{2}(4k + 3\delta - 1)C_x^2\right],$$
$$A_{12} = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) \frac{3\delta}{2}(4k + 3\delta - 1)C_x^2\right],$$
$$A_{13} = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) 2\delta(4k + 4\delta - 1)C_x^2\right],$$

and

$$A_{23} = \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) \frac{5\delta}{2}(4k + 5\delta - 1)C_x^2\right].$$

Differentiating the $MSE\left(\hat{\hat{Y}}_{\alpha d}\right)$ given in (1.4.6) with respect to $\alpha_i's, (i = 0,1,2,3)$ partially and equating them to zero we have

$$\begin{bmatrix} A_0 & A_{01} & A_{02} & A_{03} \\ A_{01} & A_1 & A_{12} & A_{13.} \\ A_{02} & A_{12} & A_2 & A_{23} \\ A_{03} & A_{13} & A_{23} & A_3 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 1 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} \qquad 1.4.7$$

Solving (1.4.6) we get the optimum values of constants $\alpha_i's, (i = 0,1,2,3)$ as

$$\left.\begin{aligned} \alpha_0^* = \frac{\Delta_0}{\Delta}, \quad \alpha_1^* = \frac{\Delta_1}{\Delta} \\ \alpha_2^* = \frac{\Delta_2}{\Delta} \quad \alpha_3^* = \frac{\Delta_3}{\Delta} \end{aligned}\right\} \qquad 1.4.8$$

where

$$\Delta = \begin{bmatrix} A_0 & A_{01} & A_{02} & A_{03} \\ A_{01} & A_1 & A_{12} & A_{13.} \\ A_{02} & A_{12} & A_2 & A_{23} \\ A_{03} & A_{13} & A_{23} & A_3 \end{bmatrix} \qquad \Delta_0 = \begin{bmatrix} 1 & A_{01} & A_{02} & A_{03} \\ B_1 & A_1 & A_{12} & A_{13.} \\ B_2 & A_{12} & A_2 & A_{23} \\ B_3 & A_{13} & A_{23} & A_3 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} A_0 & 1 & A_{02} & A_{03} \\ A_{01} & B_1 & A_{12} & A_{13.} \\ A_{02} & B_2 & A_2 & A_{23} \\ A_{03} & B_3 & A_{23} & A_3 \end{bmatrix} \qquad \Delta_2 = \begin{bmatrix} A_0 & A_{01} & 1 & A_{03} \\ A_{01} & A_1 & B_1 & A_{13.} \\ A_{02} & A_{12} & B_2 & A_{23} \\ A_{03} & A_{13} & B_3 & A_3 \end{bmatrix}$$

$$\Delta_3 = \begin{bmatrix} A_0 & A_{01} & A_{02} & 1 \\ A_{01} & A_1 & A_{12} & B_{1.} \\ A_{02} & A_{12} & A_2 & B_2 \\ A_{03} & A_{13} & A_{23} & B_3 \end{bmatrix}.$$

Putting the optimum values α_i^* of α_i $(i = 0,1,2,3)$ in (1.6) we get the minimum MSE of proposed estimator $\hat{\hat{Y}}_{\alpha d}$ as

$$MSE\left(\hat{\hat{Y}}_{\alpha d}\right) = \bar{Y}^2[1 - \alpha_0^* - \alpha_1^* B_1 - \alpha_2^* B_2 - \alpha_3^* B_3]. \qquad 1.4.9$$

or

$$MSE\left(\hat{\hat{Y}}_{\alpha d}\right) = \bar{Y}^2\left[1 - \frac{(\Delta_0 + \Delta_1 B_1 + \Delta_2 B_2 + \Delta_3 B_3)}{\Delta}\right]. \qquad 1.4.10$$

Thus we established the following theorem.

Theorem 1.4.1. To the first degree of approximation,

$$MSE\left(\hat{\hat{Y}}_{\alpha d}\right) \geq \bar{Y}^2\left[1 - \frac{(\Delta_0 + \Delta_1 B_1 + \Delta_2 B_2 + \Delta_3 B_3)}{\Delta}\right],$$

with equality holding if

$$\alpha_0^* = \frac{\Delta_0}{\Delta}, \alpha_1^* = \frac{\Delta_1}{\Delta}, \alpha_2^* = \frac{\Delta_2}{\Delta}, \alpha_3^* = \frac{\Delta_3}{\Delta}.$$

It is to be noted that the biases and mean squared errors of the estimators belonging to the class $\hat{\hat{Y}}_{\alpha d}$ can be obtained easily from (1.4.3) and (1.4.6) respectively just by putting the suitable values of the scalars $\alpha_i (i = 0,1,2,3)$ and δ. It is to be noted that any member of the class will not have MSE/minimum MSE smaller than that of the proposed class of estimators $\hat{\hat{Y}}_{\alpha d}$.

It is to be noted that some empirical studies need to be conducted in order to find out situations where some members of the proposed class $\hat{\hat{Y}}_{\alpha d}$ are found to be better than usual unbiased estimator $\bar{y}$, double sampling ratio $\bar{y}_{Rd}$, product $\bar{y}_{Pd}$, regression estimator ($\bar{y}_{lrd} = \bar{y} + \hat{\beta}(\bar{x}_1 - \bar{x}), \hat{\beta}$ being the sample estimate of the population regression coefficient β), Srivastava (1970) estimator ($\bar{y}_{ld}$) and other existing estimators. Probably a well designed Monte Carlo study will be quite useful and may throw some light on it.

Remark 1.4.1 In similar fashion the properties of the proposed class of estimator $\hat{\hat{Y}}_{\alpha d}$ can be discussed in the case "when the second phase sample of size n is drawn independently of the first phase sample of size n_1".

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