# Generarlized Operations on Fuzzy Graphs

D.Venugopalam, Nagamurthi Kumar, M.vijaya kumar

**Abstract**
To discuss the Cartesian Product Composition, union and join on Interval-valued fuzzy graphs. We also introduce the notion of Interval-valued fuzzy complete graphs. Some properties of self complementary graph.
**Key Words :** Interval-valued fuzzy graph self complementary Interval valued fuzzy complete graphs
**Mathematics Subject Classification 2000: 05C99**

## 1.Introduction

It is quite well known that graphs are simply model of relations. A graphs is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and realtions by edges. When there is vagueness in the descriptionof the objects or in its relationships or in both, it is natural that we need to design a Fuzy Graph Model. Application of fuzzy relations are widespread and important; especially in the field of clustering analysis, neural networks, computer networks, pattern recognition, decision making and expert systems. In each of these the basic mathematical structure is that of a fuzzy graph.

We know that a graphs is a symmetric binry relation on a nonempty set V. Similary, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann [18] in 1973, based on Zadeh's fuzzy relations [46]. But it was Azriel Rosenfeld [35] who considered fuzzy relations onf uzzy sets and developed the theory of fuzzy graphs in 1975. During the sam etime R.T.Yeh and S.Y.Bang [44] have also introduced various connectedness concepts in fuzzy graphs.

## 2 Preliminaries

**Definition 2.1** : Let V be a nonempty set. A fuzzy graphs is a pair of functions.
G : (σ, μ) where σ is a fuzzy subset of v and μ is a symmetric fuzzy relation on σ i.e. $\sigma: V\rightarrow[0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V.

We denot the underlying (crisp) graph of G: (σ, μ) by G*:(σ*, μ*) where σ* is referred to as the (nonempty) set V of nodes and $\mu^* = E \subseteq V \times V$. Note that the crisp graph (V, E) is a special case of a fuzzy graph with each vertex and edge of (V, E) having degree of membership 1. We need not consider loops and we assume that μ is reflexive. Also, the underlyign set V is assumed to be finite and σ can be chosen in any manner so as to satisfy the definition of a fuzzy graphs in all the examples.

**Definition 2.2** : The fuzzy graph H: (τ, v) is called a partial fuzzy subgraph of
G : (σ, μ) if $\tau \subseteq \sigma$ and $v \subseteq \mu$. In particular, we call H: (τ, v) a fuzzy subgraph of
G : (σ, μ) if $\tau(u) = \sigma(u)\ \forall u \in \tau^*$ and $v(u,v) = \mu(u, v)\ \forall (u, v) \in v^*$.For any threshold t, $0 \leq t \leq 1$, $\sigma' = \{u \in V : \sigma(u) \geq t\}$ and $\mu' = \{(u, v) \in V \times V : \mu(u, v) \geq t\}$. Since $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V we have $\mu' \subseteq \sigma'$, so that (σ′, μ′) is a graph with vertex set σ′ and edge set μ′ for $t \in [0, 1]$.

**Note**1.: Let G : (σ, μ) be a fuzzy graph. If $0 \leq t1 \leq t2 \leq 1$, then $(\sigma'^2, \mu'^2)$ is a subgraph of $(\sigma'^1, \mu'^1)$.

**Note** 2.: Let H : (τ, v) be a partial fuzzy subgraph of G: (σ, μ). For any threshold $0 \leq t \leq 1$, (τ′, v′) is a subgraph of (σ′, μ′).

**Definition 2.3** : For any fuzzy subset τ of V such that $\tau \subseteq \sigma$, the partial fuzzy subgraph of (σ, μ) induced by τ is the maximal partial fuzzy subgraph of (σ, μ) that has fuzzy node set τ. This is the partial fuzzy subgraph (τ, v) where
$T(u, v) = \tau(u) \wedge \mu(u, v)$ for all $u, v \in V$.

**Definition 2.4** : The fuzzy graph H: (τ, v) is called a fuzzy subgraph of G: (σ, μ) induced by P if $P \subseteq V$, $\tau(u) = \sigma(u)\ \forall\ u, v \in P$.

**Definition 2.5** : A partial fuzy subgraph (τ, v) spans the fuzzy graph (σ, μ) if σ = τ. In this case (τ, v) is called a aprtial fuzzy spanning subgraph of (σ, μ).

Next we introduce the concept of a fuzzy spanning subgrph as a special case of partial fuzzy spanning subgraph.

**Operations 2.6**: Graphs g = (D, E) are simple : no multiple edges and no loops.
An unordered pair {x, y} is deonte by xy or x – y

**Operations 2.7**: Graphs g = (D, E) are simple: no multiple edges and no loops.
An unordered pair {x, y} is denote by xy or x – y
An operation is a permutation on the set of graphs on D :

$$\alpha : g \rightarrow h$$

**Operations 2.8**: Graphs g = (D, E) are simple: no multiple edges and no loops.
An unordered pair {x, y} is denote by xy or x – y

An operation is a permutation on the set of graphs on D :

$$\alpha : g \to h$$

Let

$$\langle \Gamma \rangle_D = \langle \alpha_1, \alpha_2, ... \alpha_n \rangle_D$$

Be the subgroup of the symmetric group generated by

$$\Gamma = \{\alpha_1, \alpha_2, ..., \alpha_n\}$$

**Transitivity 2.8** : The problem setting : Given operations $\Gamma = \{\alpha_1, \alpha_2, ..., \alpha_n\}$

And any two graphs g, h on DDoes there exist a composition $\alpha \in \langle \Gamma \rangle_D$

$$\alpha = \alpha_{ik}, \alpha_{ik-1}, ..., \alpha_1 \text{ such that } \alpha(g) = h.$$

**Complement 2.9** : Let $\binom{D}{2}$ be the set of all 2-subsets {x, y}. $C(g) = \left( D, \binom{D}{2} \setminus E \right)$

Edges ↔ nonedges

**Neighbours 2.10** : Neighbours of x $N_g(x) = \{y \mid xy \in E\}$ $N'_g(x) = D \setminus (N_g(x) \cup \{x\})$

Nonneighbours of x

**Subgraphs 2.11** : The symmetric difference : A + B = (A \ B) U (B \ A)

The sub graph of g induced by $A \subseteq D : g[A] = \left( A, E \cap \binom{A}{2} \right)$

**Complementing Subgraphs 2.12** : Denote by g $g \oplus A = \left( D, E + \binom{A}{2} \right)$

**3.Main Results**

**Theorem 3.1**

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two Interval Valued Fuzzy Graphs. Then

(i) $\overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2}$

(ii) $\overline{G_1 \cup G_2} \cong \overline{G_1} + \overline{G_2}$

**Proof**

Consider the identity map I : $V_1 \cup V_2 \to V_1 \cup V_2$,

To prove (i) it is enough to prove

(a) (i) $\overline{\mu_1 \cup \mu'_1}(v_i) = \overline{\mu_1} \cup \overline{\mu'_1}(v_i)$

(ii) $\overline{\gamma_1 + \gamma'_1}(v_i) = \overline{\gamma_1} \cup \overline{\gamma'_1}(v_i)$

(b) (i) $\overline{\mu_2 \cup \mu'_2}(v_i, v_j) = \overline{\mu_2} \cup \overline{\mu'_2}(v_i, v_j)$

(ii) $\overline{\gamma_2 + \gamma'_2}(v_i, v_j) = \overline{\gamma_1} \cup \overline{\gamma'_1}(v_i, v_j)$

(a) (i) $\left(\overline{\mu_1 + \mu'_1}\right)(v_i) = (\mu_1 + \mu'_1)(v_i)$, by Definition 4.1

$$= \begin{cases} \mu_1(v_i) & if\ v_1 \in V_1 \\ \mu'_1(v_i) & if\ v_1 \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_1}(v_i) & if\ v_1 \in V_1 \\ \overline{\mu'_1}(v_i) & if\ v_1 \in V_2 \end{cases}$$

$$= \left(\overline{\mu_1} \cup \overline{\mu'_1}\right)(v_i)$$

(ii) $\left(\overline{\gamma_1 + \gamma'_1}\right)(v_i) = (\gamma_1 + \gamma'_1)(v_i)$,

$$= \begin{cases} \gamma_1(v_i) & if\ v_1 \in V_1 \\ \gamma_1^{'}(v_i) & if\ v_1 \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_1}(v_i) & if\ v_1 \in V_1 \\ \overline{\gamma_1^{'}}(v_i) & if\ v_1 \in V_2 \end{cases}$$

$$= \left(\overline{\gamma_1} \cup \overline{\gamma_1^{'}}\right)(v_i)$$

(b) (i) $\left(\overline{\mu_2 + \mu_2^{'}}\right)(v_i, v_j) = \left(\mu_1 + \mu_1^{'}\right)(v_i).\left(\mu_1 + \mu_1^{'}\right)(v_j) - \left(\mu_2 + \mu_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} \left(\mu_1 \cup \mu_1^{'}\right)(v_i).\left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \left(\mu_2 \cup \mu_2^{'}\right)(v_i, v_j) & if\ (v_i, v_j) \in E_1 \cup E_2 \\ \left(\mu_1 \cup \mu_1^{'}\right)(v_i).\left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \mu_1(v_i).\mu_1^{'}(v_j) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} (\mu_1)(v_i).\mu_1(v_j) - \mu_2(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \mu_1^{'}(v_i).\mu_1^{'}(v_j) - \mu_2^{'}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ (\mu_1)(v_i).\mu_1^{'}(v_j) - \mu_1(v_i).\mu_1^{'}(v_j) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \overline{\mu_2}(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \overline{\mu_2^{'}}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ 0 & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \left(\overline{\mu_2} \cup \overline{\mu_2^{'}}\right)(v_i, v_j)$$

(b) (ii) $\left(\overline{\gamma_2 + \gamma_2^{'}}\right)(v_i, v_j) = \left(\gamma_1 + \gamma_1^{'}\right)(v_i).\left(\gamma_1 + \gamma_1^{'}\right)(v_j) - \left(\gamma_2 + \gamma_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i).\left(\gamma_1 \cup \gamma_1^{'}\right)(v_j) - \left(\gamma_2 \cup \gamma_2^{'}\right)(v_i, v_j) & if\ (v_i, v_j) \in E_1 \cup E_2 \\ \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i).\left(\gamma_1 \cup \gamma_1^{'}\right)(v_j) - \gamma_1(v_i).\gamma_1^{'}(v_j) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} (\gamma_1)(v_i).\gamma_1(v_j) - \gamma_2(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \gamma_1^{'}(v_i).\gamma_1^{'}(v_j) - \gamma_2^{'}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ (\gamma_1)(v_i).\gamma_1^{'}(v_j) - \gamma_1(v_i).\gamma_1^{'}(v_j) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \overline{\gamma_2}(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \overline{\gamma_2^{'}}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ 0 & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \left(\overline{\gamma_2} \cup \overline{\gamma_2^{'}}\right)(v_i, v_j)$$

To prove (ii) it is enough to prove

(a) (i) $\left(\overline{\mu_1 \cup \mu_1^{'}}\right)(v_i) = \left(\overline{\mu_1} \cup \overline{\mu_1^{'}}\right)(v_i)$

(ii) $\left(\overline{\gamma_1 \cup \gamma_1^{'}}\right)(v_i) = \left(\overline{\gamma_1} + \overline{\gamma_1^{'}}\right)(v_i)$

(b) (i) $\left(\overline{\mu_2 \cup \mu_2^{'}}\right)(v_i, v_j) = \left(\overline{\mu_2} + \overline{\mu_2^{'}}\right)(v_i, v_j)$

(ii) $\left(\overline{\gamma_2 \cup \gamma_2^{'}}\right)(v_i, v_j) = \left(\overline{\gamma_2} \cup \overline{\gamma_2^{'}}\right)(v_i, v_j)$

Consider the identity map $I : V_1 \cup V_2 \to V_1 \cup V_2$

(a) (i) $\left(\overline{\mu_1 \cup \mu_1^{'}}\right)(v_i) = \left(\mu_1 \cup \mu_1^{'}\right)(v_i)$

$$= \begin{cases} \mu_1(v_i) & if\ v_i \in V_1 \\ \mu_1^{'}(v_i) & if\ v_i \in V_2 \end{cases} = \begin{cases} \overline{\mu_1}(v_i) & if\ v_i \in V_1 \\ \overline{\mu_1^{'}}(v_i) & if\ v_i \in V_2 \end{cases}$$

$$= \left(\overline{\mu_1} \cup \overline{\mu_1^{'}}\right)(v_i) = \left(\overline{\mu_1} \cup \overline{\mu_1^{'}}\right)(v_i)$$

(a) (ii) $\left(\overline{\gamma_1 \cup \gamma_1^{'}}\right)(v_i) = \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i)$

$$= \begin{cases} \gamma_1(v_i) & if\ v_i \in V_1 \\ \gamma_1^{'}(v_i) & if\ v_i \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_1}(v_i) & if\ v_i \in V_1 \\ \overline{\gamma_1^{'}}(v_i) & if\ v_i \in V_2 \end{cases}$$

$$= \left(\overline{\gamma_1} \cup \overline{\gamma_1^{'}}\right)(v_i) = \left(\overline{\gamma_1} \cup \overline{\gamma_1^{'}}\right)(v_i)$$

(b) (i) $\left(\overline{\mu_2 \cup \mu_2^{'}}\right)(v_i, v_j) = \left(\mu_1 \cup \mu_1^{'}\right)(v_i).\left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \left(\mu_2 \cup \mu_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} (\mu_1)(v_i).\mu_1(v_j) - \mu_2(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \mu_1^{'}(v_i).\mu_1^{'}(v_j) - \mu_2^{'}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1^{'}(v_j) - 0 & if v_i \in v_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_2}(v_i, v_j) & if (v_i, v_j) \in E_1 \\ \overline{\mu_2^{'}}(v_i, v_j) & if (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1^{'}(v_j) & if\ v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_2} \cup \overline{\mu_2^{'}}(v_i, v_j) & if (v_i, v_j) \in E_1\ or\ E_2 \\ \mu_1(v_1).\mu_1^{'}(v_1) & if (v_i, v_j) \in E' \end{cases}$$

$$= \overline{\mu_2} + \overline{\mu_2^{'}}(v_i, v_j)$$

(b) (ii) $\left(\overline{\gamma_2 \cup \gamma_2^{'}}\right)(v_i, v_j) = \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i).\left(\gamma_1 \cup \gamma_1^{'}\right)(v_j) - \left(\gamma_2 \cup \gamma_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} (\gamma_1)(v_i).\gamma_1(v_j) - \gamma_2(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \gamma_1^{'}(v_i).\gamma_1^{'}(v_j) - \gamma_2^{'}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \gamma_1(v_i).\gamma_1^{'}(v_j) - 0 & if v_i \in v_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_2}(v_i, v_j) & if (v_i, v_j) \in E_1 \\ \overline{\gamma_2^{'}}(v_i, v_j) & if (v_i, v_j) \in E_2 \\ \gamma_1(v_i).\gamma_1^{'}(v_j) & if\ v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_2} \cup \overline{\gamma_2^{'}}(v_i, v_j) & if (v_i, v_j) \in E_1\ or\ E_2 \\ \gamma_1(v_1).\gamma_1^{'}(v_1) & if (v_i, v_j) \in E' \end{cases}$$

$$= \overline{\gamma_2} + \overline{\gamma_2^{'}}(v_i, v_j)$$

**Theorem 3.2**

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two Interval Valued Fuzzy Graphs. Then $G_1 \circ G_2$ is a strong Interval Valued Fuzzy Graphs

Proof

Let $G_1 \circ G_2 = G = \langle V, E \rangle$ where $V_1$ x $V_2$ and

$$E = \{(u,u_2)(u,v_2) : u \in V_1, u_2v_2 \in E_2\} \cup \{(u_1,w):(v_1,w): w \in V_2, u_1v_1 \in E_1\}$$
$$\cup \{(u_1,u_2)(v_1,v_2) : u_1v_1 \in E_1, u_2 \neq v_2\}.$$

(i) $\mu_2(u,u_2)(u,v_2) = \mu_1(u).\mu_2(u_2v_2)$

$= \mu_1(u).\mu_1^{'}(u_2).\mu_1^{'}(v_2)$, since $G_2$ is strong

$= \mu_1(u).\mu_1^{'}(u_2).\mu_1(u).\mu_1^{'}(v_2)$

$= (\mu_1 \circ \mu_1^{'})(u,u_2).(\mu_1 \circ \mu_1^{'})(u,v_2)$

$\gamma_2(u,u_2)(u,v_2) = \gamma_1(u).\gamma_2(u_2v_2)$

$= \gamma_1(u).\gamma_1^{'}(u_2).\gamma_1^{'}(v_2)$, since $G_2$ is strong

$= \gamma_1(u).\gamma_1^{'}(u_2).\gamma_1(u).\gamma_1^{'}(v_2)$

$= (\gamma_1 \circ \gamma_1^{'})(u,u_2).(\gamma_1 \circ \gamma_1^{'})(u,v_2)$

(ii) $\mu_2((u_1,w)(v_1,w)) = \mu_1^{'}(w).\mu_2(u_1,v_1)$

$= \mu_1^{'}(w).\mu_1(u_1).\mu_1(v_1)$, since $G_1$ is strong

$= \mu_1^{'}(w).\mu_1(u_1).\mu_1^{'}(w).\mu_1(v_1)$

$= (\mu_1 \circ \mu_1^{'})(u_1,w).(\mu_1 \circ \mu_1^{'})(v_1,w)$

$\gamma_2((u_1,w)(v_1,w)) = \gamma_1^{'}(w).\gamma_2(u_1,v_1)$

$= \gamma_1^{'}(w).\gamma_1(u_1).\gamma_1(v_1)$, since $G_1$ is strong

$= \gamma_1^{'}(w).\gamma_1(v_1).\gamma_1^{'}(w).\gamma_1(v_1)$

$= (\gamma_1 \circ \gamma_1^{'})(u_1,w).(\gamma_1 \circ \gamma_1^{'})(v_1,w)$

(iii) $\mu_2(u,u_2)(v_1,v_2) = \mu_2(u_1,v_1).\mu_1^{'}(u_2).\mu_1^{'}(v_2)$

$= \mu_1(u_1).\mu_1(v_1).\mu_1^{'}(u_2).\mu_1^{'}(v_2)$, since $G_1$ is strong

$= \mu_1(u_1)|.\mu_1^{'}(u_2).\mu_1(v_1).\mu_1^{'}(v_2)$

$= (\mu_1 \circ \mu_1^{'})(u_1,u_2).(\mu_1 \circ \mu_1^{'})(v_1,v_2)$

$\gamma_2(u_1,u_2)(v_1,v_2) = \gamma_2(u_1,v_1).\gamma_2(u_1,v_1).\gamma_1^{'}(v_2)$

$= \gamma_1(u_1).\gamma_1(v_1).\gamma_1^{'}(u_2).\gamma_1^{'}(v_2)$, since $G_1$ is strong

$= \gamma_1(u_1).\gamma_1^{'}(u_2).\gamma_1(v_1).\gamma_1^{'}(v_2)$

$= (\gamma_1 \circ \gamma_1^{'})(u_1,u_2).(\gamma_1 \circ \gamma_1^{'})(v_1,v_2)$

From (i), (ii), (iii), G is a strong Interval valued Fuzzy Graphs.

**References**

[1] A. Nagoorgani, K. Radha, Isomorphism on fuzzy graphs, International J. Computational Math. Sci. 2 (2008) 190-196.

[2] A. Perchant, I. Bloch, Fuzzy morphisms between graphs, Fuzzy Sets Syst. 128 (2002) 149-168.

[3] A. Rosenfeld, Fuzzy graphs, Fuzzy Sets and their Applications( L.A.Zadeh, K.S.Fu, M.Shimura, Eds.), Academic Press, New York, (1975) 77-95.

[4] A.Alaoui, On fuzzification of some concepts of graphs, Fuzzy Sets Syst. 101 (1999) 363-389.

[5] F. Harary, Graph Theory, 3rd Edition, Addison-Wesley, Reading, MA, 1972.

[6] I.B. Turksen, Interval valued fuzzy sets based on normal forms, Fuzzy Sets Syst. 20 (1986) 191-210.
[7] J. Hongmei, W. Lianhua, Interval-valued fuzzy subsemigroups and subgroups associated by intervalvalued suzzy graphs, 2009 WRI Global Congress on Intelligent Systems, 2009, 484-487.
[8] J.M. Mendel, Uncertain rule-based fuzzy logic systems: Introduction and new directions, Prentice-Hall, Upper Saddle River, New Jersey, 2001.
[9] J.M. Mendel, X. Gang, Fast computation of centroids for constant-width interval-valued fuzzy sets, Fuzzy Information Processing Society, NAFIPS (2006)621-626.
[10] J.N. Mordeson, C.S. Peng, Operations on fuzzy graphs, Information Sci. 79 (1994) 159-170.
[11] J.N. Mordeson, Fuzzy line graphs, Pattern Recognition Letter 14 (1993) 381-384.
[12] J.N. Mordeson, P.S. Nair, Fuzzy graphs and fuzzy hypergraphs, Physica Verlag, Heidel¬berg 1998; Second Edition 2001.
[13] K.P. Huber, M.R. Berthold, Application of fuzzy graphs for metamodeling, Proceedings of the 2002 IEEE Conference, 640-644
[14] K.R. Bhutani, A. Battou, On M-strong fuzzy graphs, Information Sci. 155 (2003) 103-109.
[15] K.R. Bhutani, A. Rosenfeld, Strong arcs in fuzzy graphs, Information Sci. 152 (2003) 319-322.
[16] K.R. Bhutani, On automorphism of fuzzy graphs, Pattern Recognition Letter 9 (1989) 159-162.
[17] K.T. Atanassov, Intuitionistic fuzzy sets: Theory and applications, Studies in fuzziness and soft computing, Heidelberg, New York, Physica-Verl., 1999.
[18] L.A. Zadeh, Fuzzy sets, Information Control 8 (1965) 338-353.
[19] L.A. Zadeh, Similarity relations and fuzzy orderings, Information Sci. 3 (1971) 177-200.
[20] L.A. Zadeh, The concept of a linguistic and application to approximate reasoning I, Information Sci. 8 (1975) 199-249
[21] M. Akram, K.H. Dar, Generalized fuzzy K-algebras, VDM Verlag, 2010, pp.288, ISBN 978-3-639-27095-2.
[22] M.Akram, Fuzzy Lie ideals of Lie algebras with interval-valued membership functions, Quasigroups Related Systems 16 (2008) 1-12.
[23] M.B. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets Syst. 21 (1987) 1-17.
[24] M.B. Gorzalczany, An interval-valued fuzzy inference method some basic properties, Fuzzy Sets Syst. 31 (1989) 243-251.
[25] M.K. Roy, R. Biswas, I-V fuzzy relations and Sanchez's approach for medical diagnosis, Fuzzy Sets Syst. 47 (1992) 35-38.
[26] M.S. Sunitha, A. Vijayakumar, Complement of a fuzzy graph, Indian J. Pure Appl. Math. 33 (2002) 1451-1464.
[27] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letter 6 (1987) 297-302.
[28] S. Mathew, M.S. Sunitha, Node connectivity and arc connectivity of a fuzzy graph, Information Sciences, 180(4)(2010) 519-531.