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Contra QPI- Continuous Functions in Ideal Bitopological Spaces

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Abstract
In this paper, we apply the notion of qpI-open sets and qpI-continuous functions to present and study a new class of functions called contra qpI-continuous functions in ideal bitopological spaces.
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1. Preliminaries

In 1961 Kelly [6] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2 [6]. The study of quasi open sets in bitopological spaces was initiated by Dutta [1] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [1] if it is a union of a τ_1-open set and a τ_2-open set. Complement of a quasi open set is termed quasi closed. Every τ_1-open (resp. τ_2-open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A. It is denoted by qcl(A) [9]. The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A) [9].

Mashhour [10] introduced the concept of preopen sets in topology. A subset A of a topological space (X, τ) is called preopen if $A \subset Int(Cl(A))$. Every open set is preopen but the converse may not be true. In 1995, Tapi [12] introduced the concept of quasi preopen sets in bitopological spaces. A set A in a bitopological space (X, τ_1, τ_2) is called quasi preopen [12] if it is a union of a τ_1-preopen set and a τ_2-preopen set. Complement of a quasi preopen set is called quasi pre closed. Every τ_1-preopen (τ_2-preopen, quasi open) set is quasi preopen but the converse may not be true. Any union of quasi preopen sets of X is a quasi preopen set in X. The intersection of all quasi pre closed sets which contains A is called quasi pre closure of A. It is denoted by qpcl(A). The union of quasi preopen subsets of A is called quasi pre interior of A. It is denoted by qpInt(A).

The concept of ideal topological spaces was initiated Kuratowski [8] and Vaidyanathaswamy [13]. An Ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: i)A $\in$ I and B $\subset$ A $\Rightarrow$ B $\in$ I and ii) A $\in$ I and B $\in$ I $\Rightarrow$ A$\cup$B $\in$ I If $\mathcal{P}(X)$ is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*:\mathcal{P}(X) \rightarrow \mathcal{P}(X)$ called the local function [3] of A with respect to τ and I and is defined as follows:

$$A^*(\tau, I) = \{x\in X \mid U \cap A \notin I, \forall U\in \tau(x)\}, \text{ where } \tau(x) = U\in \tau \mid x\in U\}.$$

Given an ideal bitopological space (X,τ_1,τ_2I) the quasi local function [3] of A with respect to τ_1, τ_2 and I denoted by A^*_q (τ_1,τ_2,I) (in short A^*_q) is defined as follows:

$$A^*_q(\tau_1,\tau_2, I) = \{x\in X \mid U \cap A \notin I, \forall \text{ quasi open set U containing x}\}.$$

A subset A of an ideal bitopological space (X, τ_1, τ_2) is said to be qI- open [3] if $A \subset$ qInt A^*_q. A mapping f: (X,τ_1,τ_2I) $\rightarrow$ (Y,σ_1,σ_2) is called qI-continuous [3] if $f^{-1}(V)$ is qI-open in X for every quasi open set V of Y .

In 1996 Dontchev [2] introduced a new class of functions called contra-continuous functions. A function f: X $\rightarrow$ Y to be contra continuous if the pre image of every open set of Y is closed in X.

Recently the authors of this paper [4 & 5] defined quasi pre local functions, qpI- open sets and qpI-continuous mappings, qpI- irresolute mappings in ideal bitopological spaces.

Definition1.1. [4] Given an ideal bitopological space (X, τ_1, τ_2 I) the quasi pre local mapping of A with respect to τ_1, τ_2 and I denoted by A^*_{qp}(τ_1, τ_2, I) (more generally as A^*_{qp}) is defined as follows: $A^*_{qp}(\tau_1, \tau_2, I) = \{x\in X \mid U \cap A \notin I, \forall$ quasi pre-open set U containing x$\}$

Definition1.2. [4] A subset A of an ideal bitopological space (X, τ_1, τ_2, I) is qpI- open if $A \subset$ qpInt(A^*_{qp}). Complement of a qpI- open set is qpI- closed. If the set A is qpI-open and qpI-closed, then it is called qpI-clopen

Definition1.3. [4] A mapping f: (X, τ_1, τ_2, I) $\rightarrow$ (Y, σ_1,σ_2) is called a qpI- continuous if $f^{-1}(V)$ is a qpI- open set in X for every quasi open set V of Y

Definition1.4. [5] A mapping f: (X,τ_1,τ_2,I) $\rightarrow$ (Y,σ_1,σ_2) is called qI- irresolute if $f^{-1}(V)$ is a qI- open set in X for every quasi open set V of Y.

Definition1.5. [5] A mapping f: $(X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is called qpI- irresolute if $f^{-1}(V)$ is a qpI- open set in X for every quasi semi open set V of Y.

2. Contra qpI-continuous functions

Definition 2.1. A function f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$ is called contra qpI- continuous if $f^{-1}(V)$ is qpI-closed in X for each quasi open set V in Y.

Theorem 2.1. For a function f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:

a) f is contra qpI-continuous .
b) For every quasi closed subset F of Y, $f^{-1}(F)$ is qpI-open in X.
c) For each $x \in X$ and each quasi closed subset F of Y with $f(x) \in F$, there exists a qpI-open subset U of X with $x \in U$ such that $f(U) \subset F$.

Proof: (a) $\Rightarrow$ (b) and (b) $\Rightarrow$ (c) are obvious.

(c) $\Rightarrow$ (b) Let F be any quasi closed subset of Y. If $x \in f^{-1}(F)$ then $f(x) \in F$, and there exists a qpI- open subset U_x of X with $x \in U_x$ such that $f(U_x) \subset F$. Therefore, $f^{1}(F) = \cup \{U_{x:}\ x \in f^{1}(F)\}$. Hence we get $f^{-1}(F)$ is qpI-open. [4]

Remark 2.1. . Every contra qpI-continuous function is contra qI-continuous, but the converse need not be true

Example2.1. Let X ={ a, b, c } and I = {ϕ,{a}} be an ideal on X. Let τ_1 = {X, ϕ,{c}}, τ_2 = {X, ϕ, {a, b}}, σ_1 = {X, ϕ,{b}}and σ_2 ={ ϕ, X} be topologies on X. Then the identity mapping f: $(X,\tau_1, \tau_2 ,I) \to (X,\sigma_1,\sigma_2)$ is contra qI- continuous but not contra qpI- continuous as A= {b} is qI- open but not qpI-open in (X, τ_1, τ_2, I).

Theorem 2.2. If a function f: $(X, \tau1, \tau2, I) \to (Y, \sigma_1, \sigma_2)$ is contra qpI-continuous and Y is regular, then f is qpI-continuous

Proof: Let $x \in X$ and let V be a quasi open subset of Y with $f(x) \in V$ Since Y is regular, there exists an quasi open set W in Y such that $f(x) \in W \subset cl(W) \subset V$. Since f is contra qpI-continuous, by Theorem 2.1.there exists a qpI-open set U in X with $x \in U$ such that $f(U) \subseteq Cl(W)$. Then $f(U) \subseteq Cl(W) \subseteq V$. Hence f is qpI-continuous [4].

Definition 2.2. A topological space $(X, \tau1, \tau2, I)$ is said to be qpI -connected if X is not the union of two disjoint non-empty qp I-open subsets of X.

Theorem 2.3. If f: $(X, \tau1, \tau2, I) \to (Y, \sigma_1, \sigma_2)$ is a contra qpI-continuous function from a qpI-connected space X onto any space Y , then Y is not a discrete space.

Proof: Suppose that Y is discrete. Let A be a proper non-empty quasi clopen set in Y. Then $f^{-1}(A)$ is a proper non-empty qpI- clopen subset of X, which contradicts the fact that X is qpI-connected.

Theorem 2.4. A contra qpI-continuous image of a qpI-connected space is connected.

Proof: Let f: $(X, \tau1, \tau2, I) \to (Y, \sigma_1, \sigma_2)$ be a contra qpI- continuous function from a qpI-connected space X onto a space Y . Assume that Y is disconnected. Then $Y = A \cup B$, where A and B are non-empty quasi clopen sets in Y with $A \cap B = \emptyset$. Since f is contra qpI-continuous , we have that $f^{-1}(A)$ and $f^{-1}(B)$ are qpI-open non-empty sets in X with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = f^{-1}(Y) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. This means that X is not semi-I-connected, which is a contradiction. Then Y is connected.

Definition 2.6. A mapping f: $(X, \tau_1, \tau_2, I) \to (Y, \sigma_1, \sigma_2)$ is called contra qpI- irresolute if $f^{-1}(V)$ is a qpI- closed set in X for every quasi semi open set V of Y.

Theorem 2.8. Let f: $(X, \tau_1, \tau_2, I_1) \to (Y, \sigma_1, \sigma_2, I_2)$ and g: $(Y, \sigma_1, \sigma_2, I_2) \to (Z, \rho_1, \rho_2, I_3)$ Then,
gof is contra qpI- continuous if g is continuous and f is contra qpI- continuous.

Proof: Obvious.

3. Conclusion

Ideal Bitopological Spaces is an extension for both Ideal Topological Spaces and Bitopological Spaces. It has opened new areas of research in Topology and in the study of topological concepts via Fuzzy ideals in Ideal Bitoplogical spaces. The application of the results obtained would be remarkable in other branches of Science too.

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