Mathematical Theory and Modeling www.iiste.org
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.3, No.9, 2013-Special issue, International Conference on Recent Trends in Applied Sciences with Engineering Applications

Steady State Analysis of 5-Phase Symmetric Power System Networks Displaying Rotational Symmetries

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Introduction

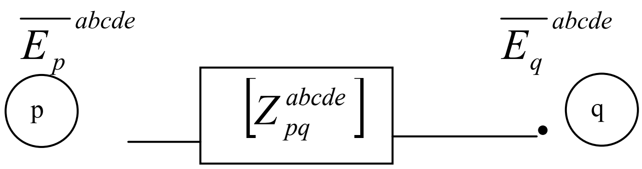
Power system network consists of generation, transmission and distribution sub networks and on the whole it is balanced i.e. symmetric. When faults occur, even though excitation may become unbalanced but the network remains normally balanced. These symmetries which are inherent in the power system satisfy mathematical group theoretical axioms. Feasibility multiphase power transmission was discussed by Stewart and Wilson (1978).

A few balanced faults occur on a real power transmission system and their fault analysis is easy. On the other hand line-to-ground faults, which are unsymmetrical in nature, are more likely to occur and their analysis requires an important tool, namely, the theory of symmetrical components. L.P.Singh and his coworkers(1979,81) employed the inherent symmetries of a power system network satisfying group theoretic axioms. This makes possible to derive the symmetric and Clarke's component transformation in a unified and systematic manner which can be applied for 4-phase and 6-phase networks.

Although five-phase system are not at present in practice, they serve to indicate the type of symmetrical sets which result from the resolution of a system having more than three-phases. When the number of vectors or phases of a system are prime, as in the three and five-phase cases, each component set of sequence higher than zero has n-individual vectors where as when the number of phases is not prime, as in the six-phase system, some of the component sets contain superimposed vectors and thus appear not to have an symmetrical component vectors.

Consider a 5-phase symmetric sub network between buses p and q as shown in fig. The network equation in the impedance form for this sub network is

$$\overline{E_p}^{abcde} - \overline{E_q}^{abcde} = \left[Z_{pq}^{abcde}\right]\overline{i_{pq}}^{abcde} \qquad (1)$$



$$\overline{V_{pq}}^{abcde} = \overline{E_p}^{abcde} - \overline{E_q}^{abcde}$$

Fig-1:Five phase power system network

Where $\overline{V_{pq}}^{abcde}$ and $\overline{i_{pq}}^{abcde}$ are the column vectors of voltage drops and current through the 5-phase element p-q for the phases a, b, c, d, and e and z_{pq}^{abcde} is the impedance matrix (5 X 5 coefficient matrix) of the element p-q and $\overline{E_p}^{abcde}$ and $\overline{E_q}^{abcde}$ are the column vectors of bus voltages at the buses p and q respectively.

Or
$$\begin{bmatrix} V_{pq}^{a} \\ V_{pq}^{b} \\ V_{pq}^{c} \\ V_{pq}^{d} \\ V_{pq}^{e} \end{bmatrix} = \begin{bmatrix} Z_{pq}^{aa} & Z_{pq}^{ab} & Z_{pq}^{ac} & Z_{pq}^{ad} & Z_{pq}^{ae} \\ Z_{pq}^{ba} & Z_{pq}^{bb} & Z_{pq}^{bc} & Z_{pq}^{bd} & Z_{pq}^{be} \\ Z_{pq}^{ca} & Z_{pq}^{cb} & Z_{pq}^{cc} & Z_{pq}^{cd} & Z_{pq}^{ce} \\ Z_{pq}^{da} & Z_{pq}^{db} & Z_{pq}^{dc} & Z_{pq}^{dd} & Z_{pq}^{de} \\ Z_{pq}^{ea} & Z_{pq}^{eb} & Z_{pq}^{ec} & Z_{pq}^{ed} & Z_{pq}^{ee} \end{bmatrix} \begin{bmatrix} i_{pq}^{a} \\ i_{pq}^{b} \\ i_{pq}^{c} \\ i_{pq}^{d} \\ i_{pq}^{e} \end{bmatrix}$$

The symmetry operations for the 5-phase element p-q Constitute a group (cyclic) of order 5.They are represented by the following matrices D(R) which are proper orthogonal matrices

$$D(E)=\begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix} \quad D(C_5^1)=\begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \end{bmatrix} \quad D(C_5^2)=\begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \end{bmatrix} \quad D(C_5^3)=\begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \end{bmatrix}$$

$$D(C_5^4)=\begin{bmatrix} 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \end{bmatrix} \qquad \text{------------ (2)}$$

The coefficient matrix Z_{pq}^{abcde} commutes with D(R) hence

Z_{pq}^{abcde} =D(c_5^1)z $_{pq}^{abcde}$ D(c_5^4)

$$Z_{pq}^{abcde} = D(c_5^1) \begin{bmatrix} Z_{pq}^{aa} & Z_{pq}^{ab} & Z_{pq}^{ac} & Z_{pq}^{ad} & Z_{pq}^{ae} \\ Z_{pq}^{ba} & Z_{pq}^{bb} & Z_{pq}^{bc} & Z_{pq}^{bd} & Z_{pq}^{be} \\ Z_{pq}^{ca} & Z_{pq}^{cb} & Z_{pq}^{cc} & Z_{pq}^{cd} & Z_{pq}^{ce} \\ Z_{pq}^{da} & Z_{pq}^{db} & Z_{pq}^{dc} & Z_{pq}^{dd} & Z_{pq}^{de} \\ Z_{pq}^{ea} & Z_{pq}^{eb} & Z_{pq}^{ec} & Z_{pq}^{ed} & Z_{pq}^{ee} \end{bmatrix} \quad D(c_5^4) = Z_{pq}^{abcde} = \begin{bmatrix} Z_{pq}^{bb} & Z_{pq}^{bc} & Z_{pq}^{bd} & Z_{pq}^{be} & Z_{pq}^{ba} \\ Z_{pq}^{cb} & Z_{pq}^{cc} & Z_{pq}^{cc} & Z_{pq}^{ce} & Z_{pq}^{ca} \\ Z_{pq}^{db} & Z_{pq}^{dc} & Z_{pq}^{dd} & Z_{pq}^{de} & Z_{pq}^{da} \\ Z_{pq}^{eb} & Z_{pq}^{ec} & Z_{pq}^{ed} & Z_{pq}^{ee} & Z_{pq}^{ea} \\ Z_{pq}^{ab} & Z_{pq}^{ae} & Z_{pq}^{ad} & Z_{pq}^{ae} & Z_{pq}^{aa} \end{bmatrix}$$

From the equality of matrices in equation (2) we get

$$Z_{pq}^{abcde} = \begin{bmatrix} Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} \\ Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} \\ Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} \end{bmatrix}$$

The unitary matrix which diagonalizes the matrices (2) is nothing but the matrix formed Eigen vectors of D(R) as its columns and is given by

$$\alpha = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix} \qquad \text{------------ (3)}$$

The same unitary matrix α will also diagonalize the coefficient matrix Z_{pq}^{abcde} and α is called the symmetrical component transformation matrix T_s.

Using representation theory, the 5-phase symmetric network displaying only rotational symmetries form a cyclic group denoted by C_5. Since this group is cyclic Abelian group, it has 5 conjugate classes hence 5 one dimensional Irreducible representations. The permutation matrices in (2) form a reducible representation of C_5 and can be reduced to irreducible components using the formula (Hamermesh 1962).

$$a_i = \sum_R \frac{1}{g} \chi^i(R) \ \ \chi(R) \qquad \text{------------ (4)}$$

We can see each of the irreducible representations of C_5 appears once. This

$$\bar{D}(R) = \alpha^{*T} \ D(R)\alpha = \begin{bmatrix} D^1(R) & 0 & 0 & 0 & 0 \\ 0 & D^2(R) & 0 & 0 & 0 \\ 0 & 0 & D^3(R) & 0 & 0 \\ 0 & 0 & 0 & D^4(R) & 0 \\ 0 & 0 & 0 & 0 & D^5(R) \end{bmatrix} \qquad \text{------------ (5)}$$

The Matrix α of the similarity transformation can be constructed using orthogonality theorem of representations it comes out to be

$[D^1(R) D^3(R) D^5(R) D^4(R) D^2(R)]^*$ i.e.

$$\alpha = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix}$$

This is the same symmetrical component transformation matrix T_s derived in (3) using Eigen values approach therefore

$$T_S^{-1} \ Z_{pq}^{abcde} \ T_S$$

$$= \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega^4 & \omega & \omega^3 & \omega^2 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega^3 & \omega^2 & \omega^4 & \omega \\ 1 & \omega & \omega^3 & \omega^2 & \omega^4 \end{bmatrix} \begin{bmatrix} Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} \\ Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} \\ Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^4 & \omega^2 & \omega^3 \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^2 & \omega^3 & \omega & \omega^4 \\ 1 & \omega^4 & \omega^2 & \omega^3 & \omega \end{bmatrix}$$

$$= \begin{bmatrix} Z_{pq}^0 & 0 & 0 & 0 & 0 \\ 0 & Z_{pq}^1 & 0 & 0 & 0 \\ 0 & 0 & Z_{pq}^2 & 0 & 0 \\ 0 & 0 & 0 & Z_{pq}^3 & 0 \\ 0 & 0 & 0 & 0 & Z_{pq}^4 \end{bmatrix} \qquad \text{------------ (6)}$$

$where\ Z_{pq}^0 = Z_{pq}^S + Z_{pq}^{M_1} + Z_{pq}^{M_2} + Z_{pq}^{M_3} + Z_{pq}^{M_2}$ is the zero sequence impedance and $Z_{pq}^1, Z_{pq}^2, Z_{pq}^3, Z_{pq}^4$ are the first, second, third and fourth sequence impedances

Network having both rotational and reflection symmetries

The symmetry operations for the 5-phase network possessing rotations as well as reflection symmetries form a group C_{5v} whose elements are E, c_5 , c_5^2, c_5^3, c_5^4 , σ_{v_1} , σ_{v_2} , σ_{v_3} , σ_{v_4} , σ_{v_5} .

The network equation remains invariant for permutations of port variables hence

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$$Z_{pq}^{abcde} = D(R)\,[Z_{pq}^{abcde}]\,D^{-1}(R)$$

$$for\ R = E, c_5, c_5^2, c_5^3, c_5^4, \sigma_{v_1}, \sigma_{v_2}, \sigma_{v_3}, \sigma_{v_4}, \sigma_{v_5}$$

The symmetry operations can be represented by the following permutation matrices.

$$D(\sigma_{v_1}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad D(\sigma_{v_2}) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad D(\sigma_{v_3}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D(\sigma_{v_4}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad D(\sigma_{v_5}) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{------------ (7)}$$

Substituting for D(R) and comparing both sides of the coefficient matrix z_{pq}^{abcde} of a 5-phase element p-q displaying both rotational as well as reflection symmetries will be of the form

$$Z_{pq}^{abcde} = \begin{bmatrix} Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} \\ Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} \\ Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} \end{bmatrix}$$

According to representation theory, the group C_{5v} is having four conjugate classes hence from irreducible representations the matrices in (7) form a reducible representation of C_{5v} and can be decomposed into irreducible representations D^1, D^3, and D^4.

That is

$$\overline{D}(R) = \alpha^{-1}\ D(R)\alpha = \begin{bmatrix} D^1(R) & 0 & 0 \\ 0 & D^3(R) & 0 \\ 0 & 0 & D^4(R) \end{bmatrix} \qquad \text{------------ (8)}$$

And the matrix α which diagonalizes the matrix D(R) is called the Clarke's components transformation matrix Tc.

CLARKE'S COMPONENT " T_C"

We have that a unitary matrix which turnsout to be a clarke's components by using real basis for representation and which turns out to be symmetrical components by using a complex basis of representation, diagonalizes the coefficient matrix z_{pq}^{abcde} of a 5- phase stationary elements. The clarke's components transformation T_c as derived in the following equation (9),

$$\alpha = T_C = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & 0 & 0 \\ 1 & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5+\sqrt{5}}}{2} & \frac{\sqrt{5-\sqrt{5}}}{2} \\ 1 & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & -\frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} \\ 1 & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & \frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} \\ 1 & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & -\frac{\sqrt{5+\sqrt{5}}}{2} & -\frac{\sqrt{5-\sqrt{5}}}{2} \end{bmatrix} \quad \text{------------ (9)}$$

This is on orthogonal matrix, i.e., $T_C^T.T_C = U = I = T_C^{-1}.T_C$ and hence

$$T_C^{-1} = T_C^T = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ \sqrt{2} & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} \\ \sqrt{2} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} \\ 0 & \frac{\sqrt{5+\sqrt{5}}}{2} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & -\frac{\sqrt{5+\sqrt{5}}}{2} \\ 0 & \frac{\sqrt{5-\sqrt{5}}}{2} & -\frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} & \frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} & -\frac{\sqrt{5-\sqrt{5}}}{2} \end{bmatrix}$$

The Clarke's components transformation matrix T_c diagonalizes the coefficient matrix z_{pq}^{abcde} of a 5-phase stationary elements such as that of transposed transmission line. We have

$$T_C^{-1}\left[z_{pq}^{abcde}\right]T_C = T_C^T\left[z_{pq}^{abcde}\right]T_C$$

$$= \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & 0 & 0 \\ 1 & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5+\sqrt{5}}}{2} & \frac{\sqrt{5-\sqrt{5}}}{2} \\ 1 & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & -\frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} \\ 1 & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & \frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} \\ 1 & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & -\frac{\sqrt{5+\sqrt{5}}}{2} & -\frac{\sqrt{5-\sqrt{5}}}{2} \end{bmatrix} \begin{bmatrix} Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} \\ Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} & Z_{pq}^{M_1} \\ Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^{S} \end{bmatrix}$$

$$\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ \sqrt{2} & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} \\ \sqrt{2} & -\frac{\sqrt{5}+1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & \frac{\sqrt{5}-1}{2\sqrt{2}} & -\frac{\sqrt{5}+1}{2\sqrt{2}} \\ 0 & \frac{\sqrt{5+\sqrt{5}}}{2} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & \frac{\sqrt{5}}{\sqrt{5+\sqrt{5}}} & -\frac{\sqrt{5+\sqrt{5}}}{2} \\ 0 & \frac{\sqrt{5-\sqrt{5}}}{2} & -\frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} & \frac{\sqrt{5}}{\sqrt{5-\sqrt{5}}} & -\frac{\sqrt{5-\sqrt{5}}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} Z_{pq}^S + 4Z_{pq}^M & 0 & 0 & 0 & 0 \\ 0 & Z_{pq}^S - Z_{pq}^M & 0 & 0 & 0 \\ 0 & 0 & Z_{pq}^S - Z_{pq}^M & 0 & 0 \\ 0 & 0 & 0 & Z_{pq}^S - Z_{pq}^M & 0 \\ 0 & 0 & 0 & 0 & Z_{pq}^S - Z_{pq}^M \end{bmatrix} \quad \text{------------ (10)}$$

where $z_{pq}^S + 4z_{pq}^m$ is known as zero sequence component and $z_{pq}^S - z_{pq}^m$ is known as α-component That is same as β, γ, δ component.

SYMMETRICAL COMPONENTS"T_S"

We have developed earlier the unitary matrix α for 5-phase rotating element solely based upon symmetry considerations The unitary matrix α as derived in equation (9) is as shown below,

$$\alpha = T_S = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix} \quad \text{------------ (11)}$$

The matrix T_s is unitary i.e.,*

$T_S^{*T}.T_S = T_S.T_S^{*T} = T_S^{-1}.T_S = I = U$ i.e.,

$$T_s^{-1} = T_s^{*T} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \end{bmatrix} \quad \text{----------- (12)}$$

As discussed the symmetrical components transformation matrix diagonalizes the coefficient matrix z_{pq}^{abcde} of a 5-phase rotating elements.

i.e., $T_S^{*T}\left[Z_{pq}^{abcde}\right]T_S$ ------------ (13)

$$= \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \end{bmatrix} \begin{bmatrix} Z_{pq}^S & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^S & Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} \\ Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^S & Z_{pq}^{M_1} & Z_{pq}^{M_2} \\ Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^S & Z_{pq}^{M_1} \\ Z_{pq}^{M_1} & Z_{pq}^{M_2} & Z_{pq}^{M_3} & Z_{pq}^{M_2} & Z_{pq}^S \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix}$$

In equation (13), T_s and T_S^{*T} are substituted from equation(11) and (12)and z_{pq}^{abcde} which is the coefficient matrix for 5-phase rotating elements is substituted from equation (7). Here in the equation (13) $Z_{pq}^S + Z_{pq}^{M_1} + Z_{pq}^{M_2} + Z_{pq}^{M_3} + Z_{pq}^{M_2}$ is known as a zero sequence impedence and is denoted by Z_{pq}^0 .Similarly $Z_{pq}^S + \omega Z_{pq}^{M_1} + \omega^2 Z_{pq}^{M_2} + \omega^3 Z_{pq}^{M_3} + \omega^4 Z_{pq}^{M_2}$ is known as a positive sequence impedence and is denoted by Z_{pq}^1 .and $Z_{pq}^S + \omega^2 Z_{pq}^{M_1} + \omega^4 Z_{pq}^{M_2} + \omega Z_{pq}^{M_3} + \omega^3 Z_{pq}^{M_2}$. is known as a second order sequence impedence and is denoted by Z_{pq}^2 . and $Z_{pq}^S + \omega^3 Z_{pq}^{M_1} + \omega Z_{pq}^{M_2} + \omega^4 Z_{pq}^{M_3} + \omega^2 Z_{pq}^{M_2}$. is known as a third order sequence impedence and is denoted by Z_{pq}^{3}. and $Z_{pq}^{S} + \omega^4 Z_{pq}^{M_1} + \omega^3 Z_{pq}^{M_2} + \omega^2 Z_{pq}^{M_3} + \omega Z_{pq}^{M_2}$. is known as a fourth order or negative sequence impedence and is denoted by Z_{pq}^{4}.

Thus we conclude from equations (13) that the symmetrical component transformation matrix diagonalizes the coefficient matrix z_{pq}^{abcde} of the 5-phase rotating elements however, in this case $Z_{pq}^{0} \neq Z_{pq}^{1} \neq Z_{pq}^{2} \neq Z_{pq}^{3} \neq Z_{pq}^{4}$ i.e., sequence impedances are not equal.

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