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Modelling the Rate of Treasury Bills in Ghana

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Abstract

Treasury bills rate is a preeminent default-risk free rate asset in Ghana's money market whose existence can affect the purchasing power of other assets in the security market. Bank of Ghana sells its Bills to mop up excess liquidity and buys Bank of Ghana Bills to inject liquidity into the system. This paper empirically models the monthly Treasury bill rate of two short term Treasury bills (91 day and 182 day) from the year 1998 to 2012 from the BoG using ARIMA models. From the results, it was realized that ARIMA $(3,1,1)$ model is appropriate for modelling the 91-day Treasury bill rate with a log likelihood value of -328.58, and least AIC value of 667.17, AICc value of 667.52 and BIC value of 683.05. Also, ARIMA $(1,1,0)$ best models the 182-day Treasury bill rates with a log likelihood value of -356.50, and AIC value of 717.00, AICc value of 717.06 and least BIC value of 723.35. An ARCH-LM test and Ljung-Box test on the residuals of the models revealed that the residuals are free from heteroscedasticity and serial correlation respectively.

Keywords: Treasury bills, Ghana, Asset, Empirical, Short term.

1. Introduction

The acceptance of financial risk is inherent to the business of banking and insurance roles as financial intermediaries. To meet the demands of customers and communities and to execute business strategies, financial institutions make loans, purchase securities and deposit with different maturities and interest rates. A treasury bill which is one of the common securities being purchased in many societies is a default-risk free short-term bonds that matures within one year or less from their time of issuance. Treasury bills are sold with maturities of four weeks (1month), 13 weeks (3 months-91-day), 26 weeks (6 months-182-day), and 52 weeks (12 months-360-days) weeks, which are more commonly referred to as the one-, three-, six-, and 12-month T-bills, respectively. Like a zero-coupon bond, Treasury bills are sold at a discount to par. "Par" is the value at which all T-bills mature. Treasury Bills are issued to finance government deficits and Bank of Ghana sells Bank of Ghana Bills to mop up excess liquidity and buys Bank of Ghana Bills to inject liquidity in the system (Brilliant, 2011). Treasury Bills have so gained a high appeal among the population as securities with high returns and virtually no default risk.

Jacoby, *et al.,* (2000) provided theoretical arguments to show how treasury bills impacts stock market prices. Jones (2001) showed that stock prices predicted expected returns in the time-series. As more data has become available, recent work has shifted focus on studying time-series properties of risk in equity markets as well as in Treasury bills. Huang, *et al.,* (2001) related risk to return volatility, while Brandt (2002) studied the relationship between liquidity, order flow and the yield curve.

Aboagye *et al.,* (2008) in studying the performance of stocks in Ghana, using an investment of the same amount in treasury bills and shares over a period of 1991 and 2001, found out that investors in stock exchange traded shares earned on average 54% per annum, whereas treasury bill investors Earned 36.3%. The changing rate relationship across the spectrum of maturities is analyzed by some researchers by a yield curve risk: A yield curve is the graph of required interest rates for various maturity times.

Ghanaians are so impressed with the observable high rate of returns on treasury bills that many believe treasury bills offer the chance to earn higher returns than can be earned on other financial securities. Due to the impressive nature of treasury bills in the financial market, many researchers on the Ghanaian economy focus much on comparative study on the performance of treasury bills and other stock investments but none in this country has concentrated on modeling Treasury bills. This study therefore is focused on modeling the 91-day and 182-day Treasury bills, to determine an appropriate times series model for predicting these Treasury bills and forecast future outcomes. The Modelling of Treasury bills in useful to investors in their choice of Treasury bill to invest in. It will also be useful in the financial markets and the security agencies in the control of the different degrees of liquidity in their securities.

2. Materials and Methods of Analysis

We obtained monthly data on the 91-day and 182-day Treasury bills rate from the Bank of Ghana (BoG) from December, 1998 to October, 2012. The rates were modeled using Autoregressive Integrated Moving Average (ARIMA) models. The ARIMA (p,d,q) is a modified form of the Autoregressive Moving Average Model (ARMA (p,q)) model where the times series variable is non-stationary. An ARMA (p,q) model is the combination of an Autoregressive process and a Moving average process into a compact form in order to reduce the number of parameters. For an ARMA(p, q) model, p is the order of the Autoregressive process and q is the order of the Moving average process. An ARMA is used only if a time series variable is weakly stationary. If the times variable is non-stationary (that is has a unit root), the ARMA (p, q) model is extended to an ARIMA (p, d, q) model where d is the order of integration of the series (number of times the series is differenced to make it stationary).

The general form of the ARMA (p, q) model is:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + a_t - \sum_{j=1}^{q} \Theta_j a_{t-j}$$

whiles an ARIMA (p, d, q) model is represented by the backward shift operator as:

$$\left(1 - \phi_1 B - \phi_2 B^2 - \ldots\ldots - \phi_p B^p\right)(1 - B)^d y_t = \left[1 - \Theta_1 B - \Theta_2 B^2 - \ldots . - \Theta_q B^q\right) a_t$$

where $(1 - B)^d$ is the non-seasonal difference filters. $(1 - B)^d y_t = \Delta^d y_t$
a_t is a white noise series and p and q are positive integers. p is determined from the PACF plot whiles q is determined from the ACF plot.

The modelling of an ARIMA (p, d, q) model as outlined by Box–Jenkins consist of Model identification, Parameter Estimation and Diagnostic of selected model.

Model Identification: The order of the Autoregressive component (p), the Moving Average component (q) and the order of integration(d) were obtained by a model identification process. Generally, for an ARIMA (p, d, q) model, q is identified from the Autocorrelation function (ACF) plot and p is taken from the Partial Autocorrelation function (PACF) plot. For any ARIMA (p, d, q) process, the theoretical PACF has non-zeros partial autocorrelations at lag $1, 2, \ldots\ldots.., p$, but zero partial autocorrelation at all lags, whiles the theoretical ACF has non-zeros partial autocorrelations at lag$1, 2, \ldots\ldots.., q$, but zero partial autocorrelation at all lags. In this study, the non-zero lags were taken as the p and q for the model estimation as expected. Before the orders were identified the following test were carried out on the data.

Ljung Box test: We performed the Ljung Box test to test jointly whether or not several autocorrelations $(r_l\ )$ of the time series variable measured were zero. The Ljung Box statistic is given by:

$$Q(m) = T(T + 2) \sum_{l=1}^{m} \frac{r_l^2}{T - l}$$

$Q(m)$ is approximately a chi-square distribution with m degrees of freedom. We reject H_o and conclude there is serial correlation when the $\text{p} - \text{value} < 0.05$.

Augmented Dickey Fuller (ADF) Unit Root Test: We used the Augmented Dickey Fuller (ADF) Test to determine whether the times series has a unit root (non-stationary) or is weakly stationary. This test is based on the assumption that the series follows a random walk with model;

$$y_t = \phi_1 y_{t-1} + e_t$$

and hypothesis:

$$H_0 : \phi_1 = 1 \text{ (Non-stationary)}$$
$$H_1 : \phi_1 < 1 \text{(Stationary)}$$

Where $\phi_1$is the characteristic root of an AR polynomial and e_t is a white noise series.

An Autocorrelation plot of the series shows no serial correlation and randomness if all sample autocorrelations fall within the two standard error limits.

Estimation of Parameters: We selected the best model among all candidate models by the Akaike Information criterion (AIC), Akaike Information Corrected criterion (AICc), Normalized Bayesian Information Criterion (BIC) and the Log-likelihood values. The best model is the model with the maximum Log-likelihood value and least AIC, AICc and BIC value.

Model Diagnostics: The selected model was checked to determine whether or not it appropriately represented the data set. The diagnostic check on the residuals of the fitted model to check whether they are white noise series was done: These include an ACF plot of the residuals, a Ljung Box test and an ARCH-LM test on the residuals of the best model to determine whether they are random and their variance, homoscedastic (constant) or Heteroscedastic respectively.

3. Results and Discussion

Modelling of 91-Day Treasury Bill

The time series plot of the 91-day Treasury bill rate in figure 1 shows that the series does not fluctuate about a fixed point and thus gives an indication of non-stationary in the series. This is also seen from the ACF plot of the series which shows a slow decay and also from the PACF plot which has a very significant spike at lag 1. The Augmented Dickey-fuller test further confirms this assertion: The test is insignificant at the 0.05 significance level showing that the series has a unit root and hence not stationary. The series was therefore first differenced and tested for stationary with the Augmented Dickey-fuller test: The first difference was enough to make the series stationary as shown by the test.

Table 2 shows the different models fitted to the series, $\mathrm{ARIMA}(3,1,1)$ appears to be the best model as it has the least AIC, AICc, BIC values and the maximum Log-likelihood. The estimates of the parameters of the model, shown in table 3, indicates that $\mathrm{AR}(1)$, $\mathrm{AR}(2)$ and $\mathrm{MA}(1)$ models are significant at the 0.05 significance level whiles $\mathrm{AR}(3)$ is significant at 0.10 level of significance. Our diagnostic checking of the ARIMA $(3,1,1)$ model revealed that the model was adequate for the series. The ARCH-LM test showed that there was no ARCH effect; hence the residuals have a constant variance. The Ljung-Box p–values (> 0.05) showed that there is no serial correlation in the residuals of the model. The ACF plot of the residuals also shows that the residuals are white noise series.

Modelling of 182-Day Treasury Bill

A plot of the 182-day Treasury bill rate gave an indication that the series was not weakly stationary as shown in figure 6. This was also realized from the ACF and PACF plots in figure 7; the ACF plot of the series had a slow decay and the PACF plot showed a very significant spike at lag 1. The Augmented Dickey-fuller test which was also insignificant at the 0.05 level of significance further confirms the non-stationarity of the series. The series was therefore stationarized after the first differencing; the Augmented Dickey-fuller test of the differenced series was significant indicating that the series was stationary.

From table 7, $\mathrm{ARIMA}(1,1,0)$ model was selected as the best model among the different ARIMA $(p,1,q)$ models fitted for the 182-day series since it has the smallest BIC value. Even though the AIC and AICc value of the $\mathrm{ARIMA}(1,1,0)$ model was larger than other models fitted, the BIC criterion was used for selecting the best model because the BIC criterion is a consistent estimator and tends to select models with less parameters as compared to AIC criterion. The parameter estimate of ARIMA $(1,1,0)$ shown in table 8 indicates that $\mathrm{AR}(1)$ is significant at the 0.05 significance level. Our diagnostics of the ARIMA $(1,1,0)$ model showed that the model best fit the series. It was realized that there was no ARCH effect on the residuals of the selected model due to an insignificant ARCH-LM test statistic hence the residuals are homoscedastic. Also the Ljung-Box statistic was not significant thus gives an indication of no serial correlation among the residuals of the selected model at 0.05 level of significance. The ACF plot of the residuals further showed that the residuals were white noise series. These tests revealed that ARIMA $(1,1,0)$ model was adequate in representing the 182-day Treasury bill rate.

4. Conclusion

This study used time series to model the treasury bills in Ghana using data from the Bank of Ghana (BoG) from the year 1988 to 2012. The modeling of the treasury bills was done mainly by ARIMA model. The Study revealed that, the 91-day Treasury bill rate is best modeled with ARIMA $(3,1,1)$ whiles the 182-day Treasury bill rates is best modeled by ARIMA $(1,1,0)$. The diagnostics of these two models showed that these models adequately fits the two series hence are adequate for the forecasting of Treasury bill rate in Ghana.

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APPENDIX

Tables and Figures of 91-day Treasury bill

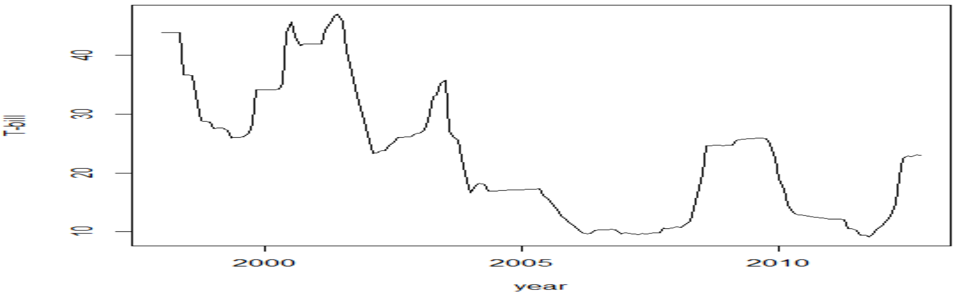

Figure1: Time series plot of 91-day Treasury bill rate

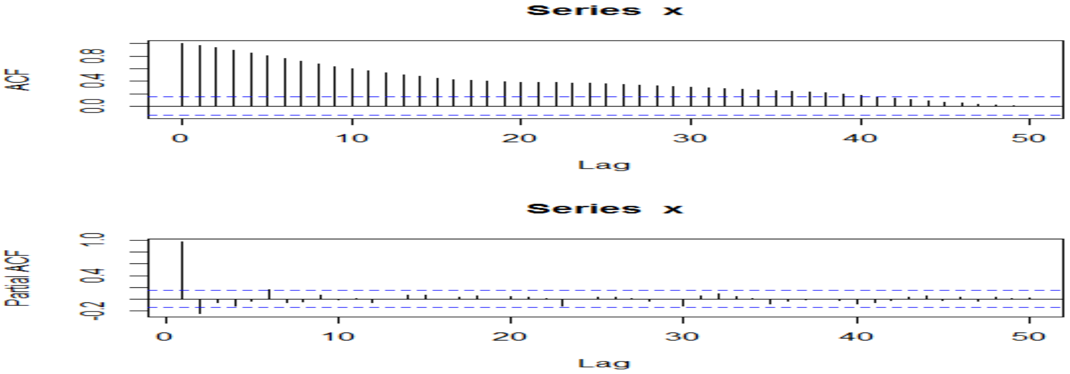

Figure 2: ACF AND PACF of undifferenced monthly 91-day T-bill rate

Table 1: Augmented Dickey-Fuller and Ljung-Box test Statistic

Differencing Order	ADF statistic	p-value	Ljung-Box statistic	p-value
0	-1.5037	0.1355	1288.621	0.000
1	-3.6914	0.01	51.8194	0.000

Figure 3: ACF and PACF of first difference series

Table 2: Different ARIMA $(p, 1, q)$ models fitted

model type	AIC	AICc	BIC	Log likelihood
ARIMA(1,1,1)	674.81	674.95	684.34	-334.41
ARIMA(1,1,2)	673.60	673.83	686.30	-332.8
ARIMA(1,1,3)	668.30	668.65	684.18	-329.15
ARIMA(1,1,4)	670.01	670.50	689.06	-329.00
ARIMA(3,1,1)*	667.17*	667.52*	683.05*	-328.58*
ARIMA(3,1,2)	668.44	668.94	687.50	-328.22
ARIMA(3,1,3)	669.78	670.44	692.01	-327.89
ARIMA(3,1,4)	671.22	672.08	696.63	-327.61

*: Model selected

Table 3: Estimates of ARIMA (3, 1, 1) model

Type	coefficient	standard error	t-statistic	p-value
AR(1)	-0.3817	0.0905	-4.06	0.000
AR(2)	0.2794	0.0831	3.280	0.001
AR(3)	0.1278	0.0784	1.640	0.103
MA(1)	0.8828	0.0555	4.730	0.000

Model Diagnostics

Table 4: Diagnostic test statistic

Test	statistic	p-value
ARCH LM	8.3945	0.7536
Ljung Box	5.696	0.9306

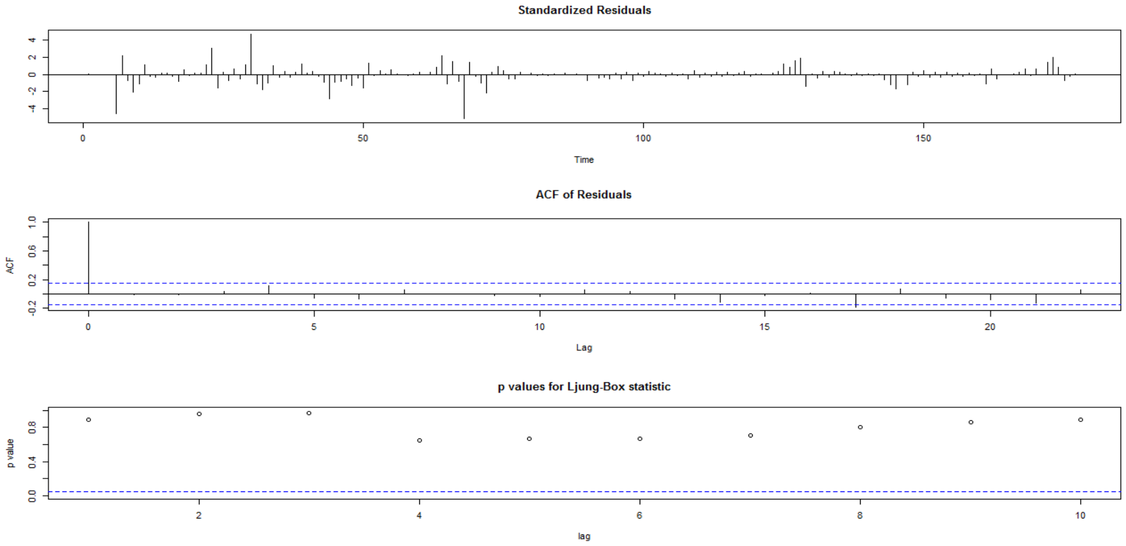

Figure 4: Diagnostic plot of residuals of ARIMA (3,1,1) model

Tables and Figures of 182-day Treasury bill

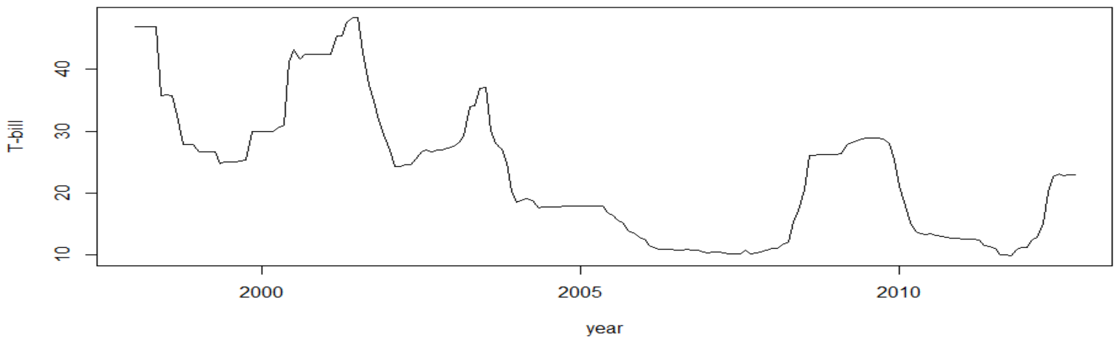

Figure6: Time series plot of 182-day Treasury bill rate

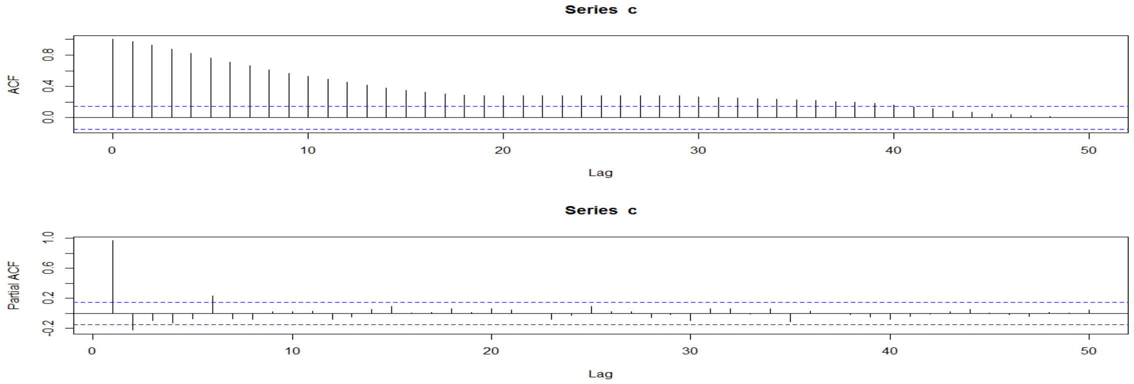

Figure 7: ACF AND PACF of undifferenced monthly 182-day T-bill rate

Table 6: Augmented Dickey-Fuller and Ljung-Box test Statistic

Differencing Order	ADF statistic	p-value	Ljung-Box statistic	p-value
0	-1.6229	0.09883	170.2105	0.000
1	-6.8901	0.01	25.3658	0.000

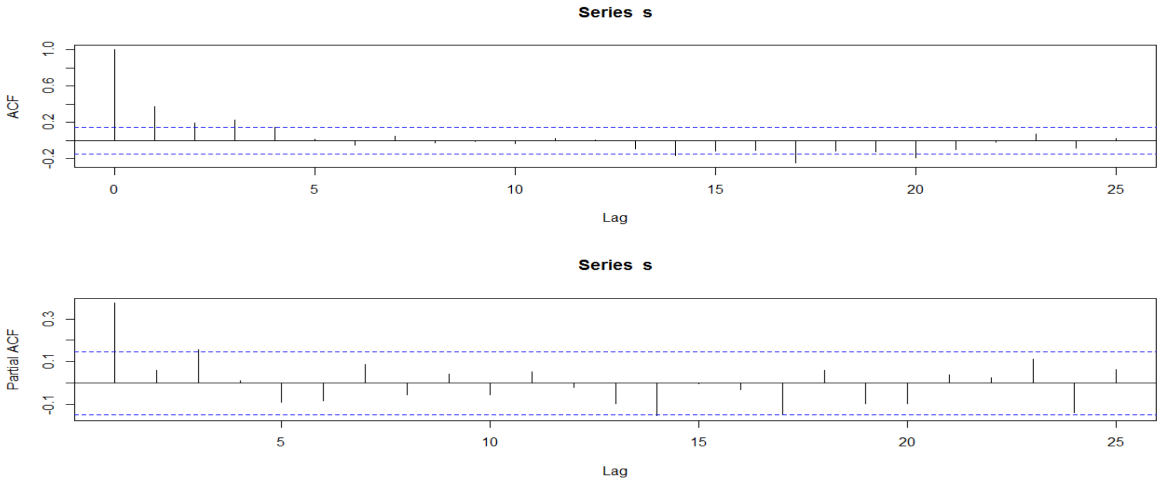

Figure 8: ACF AND PACF of differenced monthly 182-day T-bill rate

Table 7: Different ARIMA $(p, 1, q)$ models fitted for 182-day T-bill

model type	AIC	AICc	BIC	Log likelihood
ARIMA(1,1,0)*	717.00*	717.06*	723.35*	-356.50*
ARIMA(1,1,1)	717.15	717.29	726.68	-355.58
ARIMA(1,1,2)	718.36	718.59	731.06	-355.18
ARIMA(1,1,3)	717.14	717.49	733.02	-353.57
ARIMA(1,1,4)	718.35	718.84	737.4	-353.17
ARIMA(3,1,0)	715	716.17	728.65	-353.97
ARIMA(3,1,1)	717.93	718.28	733.81	-353.96
ARIMA(3,1,2)	715.64	716.14	734.7	-351.82

*: Model selected

Table 8: Estimates of ARIMA (1, 1, 0) model

type	coefficient	standard error	t-statistic	p-value
AR(1)	0.3763	0.0693	5.42	0.000

Model Diagnostics

Table 9: Diagnostic test statistic

Test	statistic	p-value
ARCH LM	2.2595	0.536
Ljung Box	10.2595	0.9989

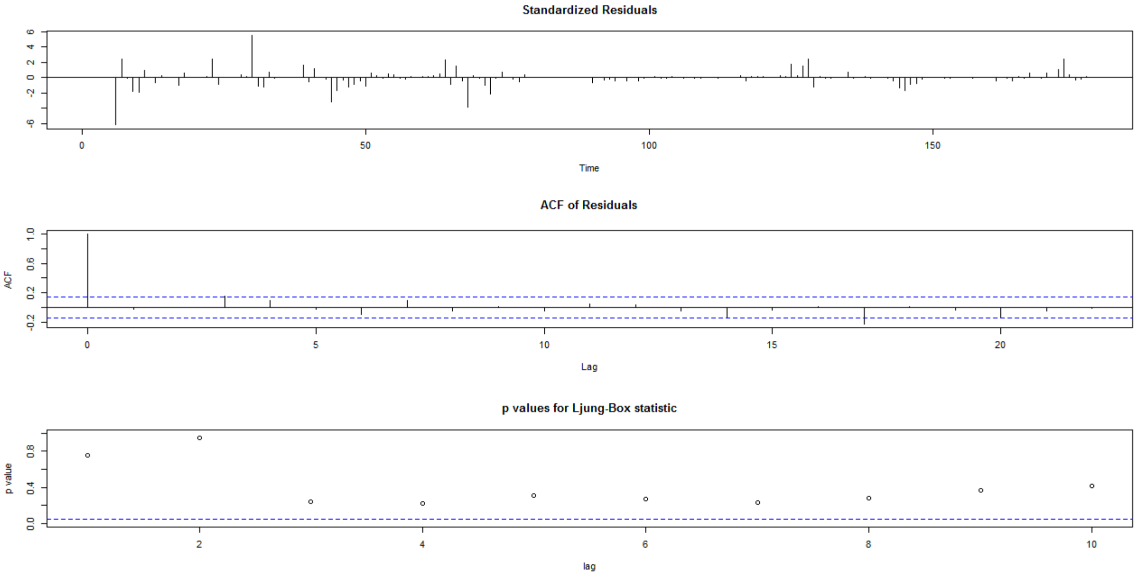

Figure 9: Diagnostic plot of residuals of ARIMA (1, 1, 0) model

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