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Fuzzy Inventory Model with Shortages in Man Power Planning

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Abstract

In this paper, an EOQ (Economical Order Quitting) model with shortages (of employees) can be studied. The cost due to decrease in real wage and the cost involved in moving to a new job are considered, with a constraint that, the decrease in the real income over a period of time is limited. In real life, these costs are uncertain to a certain extent. This uncertainty has been discussed by utilizing the concept of fuzzy set theory. Fuzzy non-linear programming technique using Lagrange multipliers is used to solve the problems in this model. The application of this model in man power planning is illustrated by means of a numerical example. The variations of the results with those of the crisp model have been compared. Further the sensitivity analysis is also presented.

Keywords: Inventory, Economical Order Quitting, Real Wage, Fuzzy Sets, Man Power Planning, Membership Function, Sensitivity Analysis.

1. Introduction

In the world, many researchers have worked on the EOQ model after the publication of classical lot-size formula by Haris in 1915. At present, one of the most promising reliable fields of research is recognized as fuzzy mathematical programming. Glenn.T.Wilson's[4] square Root Rule for employment change has been applied to develop an EOQ model in Man Power Planning to obtain the optimal time of quitting the present job for an employee of an organization based on the condition that the salary increase in the new job is equal to the decrease in the real income at the time of quitting[9].

In Inventory models we deal with costs that are crisp, that is, fixed and exact. But in realistic situations these costs are varying over a certain extent of predetermined level. However these uncertainties are due to fuzziness and in these cases the fuzzy set theory introduced by Zadeh[1] is applicable. In this paper an attempt is made to obtain a fuzzy model for Wilson's Paper[4] by adding an appropriate constraint. In 1995, T.K.Roy and M.Maiti[10] presented an EOQ model with constraint in a fuzzy environment. The model is solved by fuzzy non-linear programming (FNLP) method using Lagrange multipliers and illustrated with numerical examples. The solution is compared with solution of the corresponding crisp problem. Also sensitivity analysis is made on optimum increase in salary in the new job and on optimum quitting time of the present job for variations in the rate of decrease in real wages following Dutta.D, J.R.Rao and R.N.Tiwari[3].

As we know that the constant increase in the cost of living is always more than the increase in the salary of an employee, which in turn causes a decrease in his real income. In this situation, it is quite common that an employee thinks of quitting the present job and switching over to a new one. An EOQ model is analyzed using fuzzy set theory, which gives the optimal time for an employee to quit, at a minimum cost.

2. Notations And Terminology

I – Initial real income of an employee in the first year of our discussion (Holding cost)

S – The cost of resetting in the new place (Setup cost)

D – The cost of deficiting at the time of quitting (Shortage cost)

R – Rate of decline of real income per year, defined as a proportion of I

RI – Amount of decrease in real income per year, expressed as a proportion of I

Q_1 – Quantity of salary rise in the new job in a year

$Q_1 I$ – Quantity of salary rise, as a proportion of I, necessary to make it worthwhile to change jobs

Q_2 – Re-order quantity of employees (Re-order quantity of salary)

$Q = Q_1 + Q_2$

T – Time in years between changing jobs

B - The upper limit for decrease in salary (real income) at the time of quitting

Terminology:

$$\text{Cost of Living Index} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \text{ x } 100$$

where P_0 , P_1 are the prices of goods in base (year of reference) and current years and Q_0 being the quantities of goods bought in the base year.

$$\text{Real Wage} = \frac{\text{Income Drawn}}{\text{Cost of Living Index}} \text{ x } 100$$

We assume that the decrease in real income every year is uniform, so that after 'T' years it is reduced to RIT. If a person changes his job after T years, his salary increases in the new job, $Q_1 I$, must be atleast RIT.

Therefore $Q_1 I$= RIT implies that $T=Q_1/R$ (See figure - I)

3. Mathematical Analysis

A crisp non-linear programming problem may be defined as follows:

Min $g_0(X, C_0)$

Subject to:

$$g_i(X, C_i) \le b_i \qquad (3.1)$$

$X \ge 0,$

$$i = 1,2, \dots \dots \dots . m$$

where $X=(X_1, X_2, \dots \dots . . X_n)^T$ is a variable vector. g_0, g_i's are algebraic expressions in X with coefficients $C_0 \equiv (C_{01}, C_{02}, \dots \dots . , C_{0n})$ and $C_i = (C_{i1}, C_{i2}, \dots \dots . , C_{in})$ respectively.

Introducing fuzziness in the crisp parameters, the system (3.1) in a fuzzy environment is:

Min $g_0(X, \widetilde{C_0})$

Subject to:

$$g_i(X, \widetilde{C_i}) \le \widetilde{b_i} \qquad (3.2)$$

$X \ge 0,$

$$i = 1,2, \dots \dots \dots . m$$

where the wave bar (~) represents the fuzzification of the parameters.

In fuzzy set theory, the fuzzy objective, coefficients and constraints are defined by their membership functions which may be linear or non-linear. According to Bellman and Zadeh[1], Carlson and Korhonen[2] and Trappey et.al[11] the above problem can be written as,

$$\text{Max } \alpha$$

Subject to:

$$g_i(X, \mu_{c_i}{}^{-1}(\alpha)) \leq \mu_{b_i}{}^{-1}(\alpha) \tag{3.3}$$

$$X \geq 0,$$

$$i = 0,1,2, \ldots \ldots \ldots m$$

where $\mu_{c_i}(X)$ and $\mu_{b_i}(X)$ are the membership functions of fuzzy objective and fuzzy constraints and α is an additional variable wshich is known as aspiration level.

Therefore, the Lagrangian function is given by

$$L(\alpha, X, \lambda) = \alpha - \sum_{i=0}^{m} \lambda_i \{[g_i(X, \mu_{c_i}{}^{-1}(\alpha))] - [\mu_{b_i}{}^{-1}(\alpha)]\} \tag{3.4}$$

where λ_i , (i=0,1,2,……..,m) are Lagrange multipliers.

According to the Kuhn-Tucker[8] necessary conditions, the optimal values

$$(X_1{}^*, X_2{}^*, \ldots\ldots X_n{}^*, \lambda_0{}^*, \lambda_1{}^*, \ldots\ldots\ldots \lambda_m{}^*, \alpha^*)$$

should satisfy

$$\left.\begin{array}{l} \dfrac{\partial L}{\partial X_j} = 0, \\ \dfrac{\partial L}{\partial \alpha} = 0, \\ \lambda_i[g_i(X, \mu_{c_i}{}^{-1}(\alpha)) - \mu_{b_i}{}^{-1}(\alpha)] = 0 \\ g_i(X, \mu_{c_i}{}^{-1}(\alpha)) - \mu_{b_i}{}^{-1}(\alpha) \leq 0 \quad \text{and} \\ \lambda_i \leq 0, \; i = 0,1,2, \ldots \ldots m, \; j=1,2,3\ldots\ldots n \end{array}\right\} \tag{3.5}$$

The Kuhn – Tucker sufficient conditions demand that the objective function (for maximization) and constraints should be respectively concave and convex.

Solving equation (3.5), the optimal solution for the fuzzy non-linear programming problem is obtained.

4.EOQ Model where Fuzzy Goal, Costs are Represented by Linear Membership Functions

In a crisp EOQ Model, the problem is to choose the order level Q(>0) and $Q=Q_1+Q_2$ which minimizes the average total cost C(Q) per unit time. That is

$$\text{Min} C(Q) = \frac{1}{2} I \left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2} D \left(\frac{Q_2{}^2}{Q}\right) + S \left(\frac{R}{Q}\right) \tag{4.1}$$

Subject to:

$$Q_1 I \leq B$$
$$Q_1 > 0$$

The fuzzy version of the square root rule for employment change model with one constraint is written as follows:

$$\text{Maximize } \alpha$$

Subject to:

$$\frac{1}{2}\mu_I{}^{-1}(\alpha)\left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2}\mu_D{}^{-1}(\alpha)\left(\frac{Q_2{}^2}{Q}\right) + \mu_S{}^{-1}(\alpha)\left(\frac{R}{Q}\right) \leq \mu_C{}^{-1}(\alpha) \tag{4.2}$$

$$Q_1 I \leq \mu_B{}^{-1}(\alpha)$$
$$Q_1 > 0, \quad \alpha \in [0,1]$$

where $\mu_I(x), \mu_D(x), \mu_S(x), \mu_C(x)$ and $\mu_B(X)$ are linear membership functions for real income, deficit in cost, setup cost, objective function and upper bound for decrease in real wage respectively.

Assume that the linear membership functions are defined as follows:

$$\mu_I(x) = \begin{cases} 0 & for \quad x < I - I_0 \\ 1 - \left(\frac{I - x}{I_0}\right) & for \quad I - I_0 \le x \le I \\ 1 & for \quad x > I \end{cases}$$

$$\mu_D(x) = \begin{cases} 0 & for \quad x < D - D_0 \\ 1 - \left(\frac{D - x}{D_0}\right) & for \quad D - D_0 \le x \le D \\ 1 & for \quad x > D \end{cases}$$

$$\mu_S(x) = \begin{cases} 0 & for \quad x < S - S_0 \\ 1 - \left(\frac{S - x}{S_0}\right) & for \quad S - S_0 \le x \le S \\ 1 & for \quad x > S \end{cases}$$

$$\mu_C(x) = \begin{cases} 0 & for \quad x > C + C_0 \\ 1 - \left(\frac{x - C}{C_0}\right) & for \quad C \le x \le C + C_0 \\ 1 & for \quad x < C \end{cases}$$

$$\mu_B(x) = \begin{cases} 0 & for \quad x > B + B_0 \\ 1 - \left(\frac{x - B}{B_0}\right) & for \quad B \le x \le B + B_0 \\ 1 & for \quad x < B \end{cases} \qquad \text{------- (4.3)}$$

Here I_0, D_0, S_0, C_0 and B_0, are the maximal violations of the aspiration levels of I, D, S, C and B (or the permissible ranges). From the nature of parameters defined, it is observed that the setup cost, deficit cost and the real income are non-decreasing and the objective function and upper limit for decrease in real income are non-increasing. Hence we have

$$\mu_I^{-1}(\alpha) = I - (1 - \alpha)I_0 \; ; \qquad \mu_D^{-1}(\alpha) = D - (1 - \alpha)D_0 \; ;$$

$$\mu_S^{-1}(\alpha) = S - (1 - \alpha)S_0 \; ; \qquad \mu_C^{-1}(\alpha) = C + (1 - \alpha)C_0 \qquad \text{and}$$

$$\mu_B^{-1}(\alpha) = B + (1 - \alpha)B_0 \qquad \text{------(4.4)}$$

The Lagrangian equation takes the form as

$$L(\alpha, Q_1, Q_2, \lambda_1, \lambda_2) = \alpha - \lambda_1 \left[\frac{1}{2}(I - (1 - \alpha)I_0)\left(\frac{{Q_1}^2}{Q_1 + Q_2}\right) + \frac{1}{2}(D - (1 - \alpha)D_0)\left(\frac{{Q_2}^2}{Q_1 + Q_2}\right) + (S - (1 - \alpha)S_0)\left(\frac{R}{Q_1 + Q_2}\right) - (C + (1 - \alpha)C_0)\right] - \lambda_2[Q_1 I - (B + (1 - \alpha)B_0)] \qquad \text{--------- (4.5)}$$

By Kuhn-Tucker's necessary conditions, we have

$$\frac{\partial L}{\partial \alpha} = 0 \Rightarrow 1 - \lambda_1 I_0 \frac{{Q_1}^2}{2(Q_1 + Q_2)} - \lambda_1 D_0 \frac{{Q_2}^2}{2(Q_1 + Q_2)} - \lambda_1 S_0 \left(\frac{R}{Q_1 + Q_2}\right) - \lambda_1 C_0 - \lambda_2 B_0 = 0 \qquad \text{--------(4.6)}$$

$$\frac{\partial L}{\partial Q_1} = 0 \Rightarrow \lambda_1 \left[\frac{1}{2}(I - (1 - \alpha)I_0)\frac{Q_1(Q_1 + 2Q_2)}{(Q_1 + Q_2)^2} - \frac{1}{2}(D - (1 - \alpha)D_0)\frac{{Q_2}^2}{(Q_1 + Q_2)^2} - (S - (1 - \alpha)S_0)\left(\frac{R}{(Q_1 + Q_2)^2}\right)\right] + \lambda_2 I = 0 \qquad \text{--------(4.7)}$$

$$\frac{\partial L}{\partial Q_1} = 0 \Rightarrow \lambda_1 \left[\frac{1}{2}(I - (1 - \alpha)I_0)\frac{Q_1(Q_1 + 2Q_2)}{(Q_1 + Q_2)^2} - \frac{1}{2}(D - (1 - \alpha)D_0)\frac{{Q_2}^2}{(Q_1 + Q_2)^2} - (S - (1 - \alpha)S_0)\left(\frac{R}{(Q_1 + Q_2)^2}\right)\right] + \lambda_2 I = 0 \qquad \text{-------- (4.8)}$$

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow \frac{1}{2}(I-(1-\alpha)I_0)\left(\frac{{Q_1}^2}{Q_1+Q_2}\right) + \frac{1}{2}(D-(1-\alpha)D_0)\left(\frac{{Q_2}^2}{Q_1+Q_2}\right) + (S-(1-\alpha)S_0)\left(\frac{R}{Q_1+Q_2}\right) - (C+(1-\alpha)C_0) = 0$$

------------- (4.9)

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow IQ_1 - (B+(1-\alpha)B_0) = 0$$

$$\Rightarrow IQ_1 = (B+(1-\alpha)B_0)$$

------------- (4.10)

Solving the equations, the expression for optimum quantity of salary rise is

$${Q_1}^* = \frac{(B+(1-\alpha^*)B_0)}{I}$$

------------ (4.11)

and the optimum Re-order level is

$${Q_2}^* = \left[\frac{I-(1-\alpha)I_0}{D-(1-\alpha)D_0}\right] {Q_1}^*$$

------------ (4.12)

where α^* is a root of

$$(I-(1-\alpha)I_0)(B+(1-\alpha)B_0)^2 + (D-(1-\alpha)D_0){Q_2}^2I^2 + 2(S-(1-\alpha)S_0)RI^2 - 2(C+(1-\alpha)C_0)Q_1I^2 = 0 \; [I-(1-\alpha)I_0][B+(1-\alpha)B_0]^2 + [D-(1-\alpha)D_0]\,{Q_2}^2I^2 + 2[S-(1-\alpha)S_0]RI^2 - 2[C+(1-\alpha)C_0][B+(1-\alpha)B_0]I = 0$$

-------------(4.13)

which is a cubic equation in $(1-\alpha)$.

Solving this equation we obtain α and hence ${Q_1}^*, {Q_2}^*, I^*, D^*, S^*, B^*, T^*$, and C^*, the optimal values which lie within the tolerance limit of fuzzy range.

Remark: 4.1.1

When $Q_2 = 0$, Equation (4.13) reduces to

$$[I-(1-\alpha)I_0][B+(1-\alpha)B_0]^2 + 2[S-(1-\alpha)S_0]\,RI^2 - 2[C+(1-\alpha)C_0][B+(1-\alpha)B_0]I = 0$$

(4.14)

which is the equation derived in [7]

Equation (4.14) is cubic equation in $(1-\alpha)$ and the equation can be written as

$$I{B_0}^2(1-\alpha)^2 - \left({B_0}^2I - 2BB_0I - 2IC_0B\right)(1-\alpha)^2 - (2BB_0I - I_0B^2 - 2RI^2S_0 - 2IB_0C - 2II_0B)(1-\alpha) - (IB^2 + 2RI^2S - 2ICB) = 0 \qquad (4.15)$$

which is the equation derived in [6]

Example:

Let us assume that a person is possessed with the following fuzzy costs and goals

$I = 50000, I_0 = 13000$; $D = 11500, D_0 = 5000$;

$S = 20000, S_0 = 5000$; $B = 10000, B_0 = 1400$;

C=6000, $C_0 = 500$; R=0.05

Substituting these values in equation (4.13), we obtain the following optimal values $\alpha = 1$ (the maximum value)

${Q_1}^* = 0.2, T^* = 4, {Q_2}^* = 0.87$ and hence $I^* = 50000, S^* = 20000, D^* = 11500, B^* = 10000$,

$C^* = 6000$ which are original values of I, S, D, B and C. These are the best optimal values. We can ever get for this problem as the aspiration level α takes the maximum value ($\alpha = 1$). From this result we conclude that the person can leave this present job after 4 years provided that the increase in salary in the new job should be atleast 20% and the Re-order level of salary (Re-order level of employees) for a certain company should be atleast 87% of I, the initial income. With variations in R, the same problem can also be solved.

Comparison of the crisp model with the fuzzy model for various combinations of extreme limits of I, S, D, C and B for R=0.05 in the above example is given in the following table.

Analysis on Comparison Table – 1

With R=0.055, it is found by the fuzzy model that the aspiration level α to be 0.8419 and the optimal cost being Rs. 6016 with the optimal time of quitting as 3.9 years with new salary rise being 21% and the Re-order level in salary (Re-order level of employees)being 95%. But with different crisp parameter combinations, only three values (1,2 and 6) fall within the permissible expenditure range (6000-6500). These three expenditure values are greater than the fuzzy C^* value. Hence to get optimal solution for this problem by crisp model, the above eight different combinations are not sufficient, some more combinations of parameters had to be worked out to get this optimal value $C^* = 6016$, which is a cumbersome work, where as the fuzzy model simplifies the work of getting the optimal value. This is one of the advantages of fuzzy applications in day today life problems.

5. Sensitivity Analysis

Consider the sensitivity analysis [6, 7] on EOQ's and other costs with the variations in the tolerance limit of total cost C which is shown in the following table.

Analysis on Comparison Table – 2

In table – 2, we study the effect of variations on R. As the cost of living index changes from place to place, the rate of decrease in real income also varies from place to place. For the people dwelling in different places with same I, D, S, C and B along with the same permissible variations in I_0, D_0, S_0, C_0 and B_0, we observe from the above table that as R increases, α decreases to zero. Further $Q_1{}^*$ and $Q_2{}^*$ increases slightly. Also we observe that I^*, D^*, S^* and T^* decreases and C^* and B^* increases within the permissible range as R increases.

Tables similar to Table – 2 can also be constructed for variations in I_0, D_0, S_0, C_0 and B_0 individually and the corresponding changes in α, $Q_1{}^*$, $Q_2{}^*$, T^* and other values can be studied.

6. Conclusion

In this paper, we have seen a real life inventory problem faced by an employee in connection with his change of jobs, which could be solved easily by applying fuzzy inventory model. Sensitivity analysis is presented. This model can be extended to inventory problems under several constraints also.

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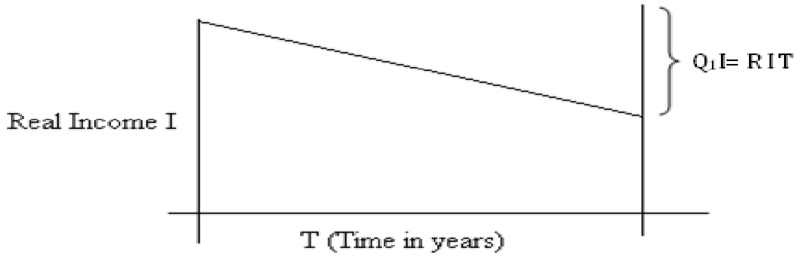

Figure I

Figure – I shows the decrease in real income till the time of quitting, that is, T years

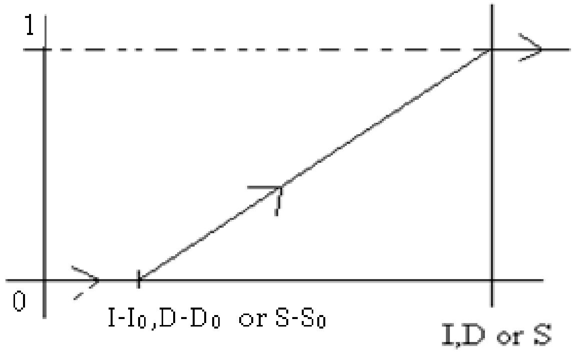

Figure - II

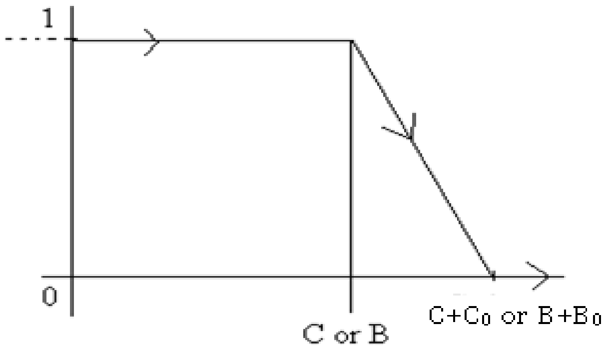

Figure - III

Figure – II represents the membership function for real wage, deficit in cost or setup cost.

Figure – III represents the membership function for objective function or upper limit for decrease in real income.

Table – 1

Comparision Table (R = 0.055)

S. No	Model	Real Income	Shortage (Deficit) Cost	Setup cost	Upper Limit for decrease in Salary	Optimal increase in new Salary	Optimal Re-order Level	Optimal Time of Quitting	Average Total Cost	Aspiration Level
		I	D	S	B	Q_1*	Q_2*	T*	C*	α
1	Crisp Model	50000	11500	20000	10000	0.20	0.87	3.6	6028	-
2	Crisp Model	50000	11500	20000	10500	0.21	0.91	3.8	6229	-
3	Crisp Model	50000	10000	15000	10000	0.20	1.00	3.6	5688	-
4	Crisp Model	50000	10000	15000	10500	0.21	1.05	3.8	5905	-
5	Crisp Model	40000	11500	20000	10000	0.25	0.87	4.5	5983	-
6	Crisp Model	40000	11500	20000	10500	0.26	0.91	4.8	6186	-
7	Crisp Model	40000	10000	15000	10000	0.25	1.00	4.5	5660	-
8	Crisp Model	40000	10000	15000	10500	0.26	1.05	4.8	5879	-
9	Fuzzy Model	47945	10710	19210	10221	0.21	0.95	3.9	6016	0.8419

Table – 2

Effect of Variations in R

S. No	R	α	Q_1*	Q_2*	T*	I*	D*	S*	C*	B*
1	0.050	1.0000	0.20	0.87	4.00	50000	11500	20000	6000	10000
2	0.055	0.8418	0.21	0.95	3.88	47943	10709	19209	6079	10221
3	0.056	0.8120	0.22	0.97	3.85	47556	10560	19060	6094	10263
4	0.057	0.7830	0.22	0.99	3.83	47179	10415	18915	6109	10304
5	0.058	0.7546	0.22	1.01	3.81	46810	10273	18773	6123	10344
6	0.059	0.7266	0.22	1.02	3.79	46446	10133	18633	6137	10383
7	0.060	0.6990	0.23	1.04	3.77	46087	9995	18495	6151	10421
8	0.065	0.5689	0.24	1.13	3.67	44396	9345	17845	6216	10604
9	0.070	0.4490	0.25	1.23	3.59	42837	8745	17245	6276	10771
10	0.080	0.2400	0.28	1.44	3.45	40120	7700	16200	6380	11064
11	0.090	0.1800	0.28	1.51	3.15	39340	7400	15900	6410	11148
12	0.092	0.0800	0.30	1.64	3.23	38040	6900	15400	6460	11288