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Heuristic Approach for n-Jobs, 3-Machines Flow Shop Scheduling Problem, Processing Time Associated With Probabilities Involving Transportation Time, Break-Down Interval, Weightage of Jobs and Job Block Criteria

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Abstract

This paper deals with a new simple heuristic algorithm for n jobs, 3 machines flow shop scheduling problem in which processing times are associated with their corresponding probabilities involving transportation time, break down interval and job block criteria. Further jobs are attached with weights to indicate their relative importance. A heuristic approach method to find optimal or near optimal sequence minimizing the total elapsed time whenever mean weighted production flow time is taken into consideration. The proposed method is very easy to understand and also provide an important tool for decision makers. A numerical illustration is also given to clarify the algorithm.

Keywords: Flow shop scheduling, Processing time, Transportation time, Breakdown interval, Weights of job, Optimal sequence

1. Introduction

Flow shop scheduling is an important process widely used in manufacturing, production, management, computer science, and so on. Appropriate scheduling not only reduces manufacturing costs but also reduces possibilities for violating the due dates. Finding good schedules for given sets of jobs can thus help factory supervisors effectively control job flow and provide solutions for job sequencing. In flow shop scheduling problems, the objective is to obtain a sequence of jobs which when processed on the machine will optimize some well defined criteria, The number of possible schedules of the flow shop scheduling problem involving n-jobs and m-machines is $(n!)^m$. Every job will go on these machines in a fixed order of machines. Early research on flow shop problems is based mainly on Johnson's theorem, which gives a procedure for finding an optimal solution with 2 machines, or 3 machines with certain characteristics. The research in to flow shop scheduling has drawn a great attention in the last decade with the aim to increase the effectiveness of industrial production. Now-a-days, the decision makers for the manufacturing plant must find a way to successfully manage resources in order to produce products in the most efficient way with minimum total flow time. The scheduling problem practically depends upon the important factors namely, Job Transportation which includes loading time, moving time and unloading time, Weightage of a job, Job block criteria which is due to priority of one job over the another and machine break down due to failure of a component of machine for a certain interval of time or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as non supply of electric current to the machines may be a government policy due to shortage of electricity production. These concepts were separately studied by various researchers Johnson (1954), Jakson (1956), Belman (1956), Baker (1974), Bansal (1986), Maggu and Das (1981), Miyazaki & Nishiyama (1980), Parker (1995), Singh, T.P. (1985), Chandramouli (2005), Belwal & Mittal (2008), Pandian & Rajendran (2010), khodadadi (2011), Gupta & Sharma (2011) . Maggu & Das (1977) introduce the concept of equivalent job blocking in the theory of scheduling. The concept is useful and significant in the sense to create a balance between the cost of providing priority in service to the customer and cost of giving services with non-priority. The decision maker may decide how much to charge extra from the priority customer.

Pandian & Rajendran (2010) proposed a heuristic algorithm for solving constrained flow shop scheduling problems with three machines. In practical situations, the processing time are always not be exact as has been taken by most of researchers, hence, we made an attempt to associate probabilities with processing time. In this paper, we propose a new simple heuristic approach to obtain an optimal sequence with three machines in which probabilities are associated with processing time involving transportation time, breakdown interval, job block criteria and weights of jobs. We have obtained an algorithm which minimizing the total elapsed time whenever means weighted production flow time is taken into consideration. Thus the problem discussed here is wider and practically more applicable and will have significant results in the process industry.

2. Practical Situation

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various qualities of paper, sugar and oil are produced with relative importance i.e. weight in jobs. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern. The concept of job block has many applications in the production situation where the priority of one job over the other is taken in to account as it may arise an additional cost for providing this facility in a given block. The break down of the machines (due to delay in material, changes in release and tails date, tool unavailability, failure of electric current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) has significant role in the production concern.

3. Notations

S : Sequence of jobs 1, 2, 3… n

S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, -------

M_j : Machine j, j= 1, 2, 3

M : Minimum makespan

a_{ij} : Processing time of i^{th} job on machine M_j

p_{ij} : Probability associated to the processing time a_{ij}

A_{ij} : Expected processing time of i^{th} job on machine M_j

A'_{ij} : Expected processing time of i^{th} job after break-down effect on j^{th} machine

$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k

$T_{i,j\rightarrow k}$: Transportation time of i^{th} job from j^{th} machine to k^{th} machine

w_i : Weight assigned to i^{th} job

L : Length of break down interval

β : Equivalent jobs for job-block.

4. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on three machines M_j (j = 1,2,3). let a_{ij} be the processing time of i^{th} job on j^{th} machine and p_{ij} be the probabilities associated with a_{ij}. Let $T_{i,j\to k}$ be the transportation time of i^{th} job from j^{th} machine to k^{th} machine. Let w_i be the weights assigned to the jobs. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the total elapsed time, and weighted mean-flow time whenever mean weighted production flow time is taken into consideration.. The mathematical model of the given problem P in matrix form can be stated as:

Jobs	Machine M_1		$T_{i,1\to 2}$	Machine M_2		$T_{i,2\to 3}$	Machine M_3		Weights of Jobs
i	a_{i1}	p_{i1}		a_{i2}	p_{i2}		a_{i3}	p_{i3}	
1	a_{11}	p_{11}	$T_{1,1\to 2}$	a_{12}	p_{12}	$T_{1,2\to 3}$	a_{13}	p_{13}	w_1
2	a_{21}	p_{21}	$T_{2,1\to 2}$	a_{22}	p_{22}	$T_{2,2\to 3}$	a_{23}	p_{23}	w_2
3	a_{31}	p_{31}	$T_{3,1\to 2}$	a_{32}	p_{32}	$T_{3,2\to 3}$	a_{33}	p_{33}	w_3
4	a_{41}	p_{41}	$T_{4,1\to 2}$	a_{42}	p_{42}	$T_{4,2\to 3}$	a_{43}	p_{43}	w_4
-	-	-	-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	$T_{n,1\to 2}$	a_{n2}	p_{n2}	$T_{n,2\to 3}$	a_{n3}	p_{n3}	w_n

(Table 1)

5. Algorithm

The following algorithm provides the procedure to determine an optimal sequence to the problem P.

Step 1 : Calculate the expected processing time $A_{ij} = a_{ij} \times p_{ij}; \forall i, j = 1,2,3.$

Step 2 : Check the structural condition

$$\text{Max } \{A_{i1} + T_{i.1\to 2}\} \geq \text{Min } \{A_{i2} + T_{i,1\to 2}\}$$

or $$\text{Max } \{A_{i3} + T_{i,2\to 3}\} \geq \text{Min } \{A_{i2} + T_{i,2\to 3}\}$$, or both.

If these structural conditions satisfied then go to step 3 else the data is not in standard form.

Step 3 : Introduce the two fictitious machines G and H with processing times G_i and H_i as give below:

$$G_i = |A_{i1} - A_{i2} - T_{i,1\to 2} - T_{i,2\to 3}| \text{ and } H_i = |A_{i3} - A_{i2} - T_{i,1\to 2} - T_{i,2\to 3}|.$$

Step 4 : Compute *Minimum* (G_i ,H_i)

- If *Min* (G_i , H_i)=G_i then define $G_i^{'}$ =G_i + w_i and $H_i^{'}$ =H_i
- If *Min* (G_i , H_i)=H_i then define $G_i^{'}$ =G_i and $H_i^{'}$ =H_i+ w_i

Step 5 : Define a new reduced problem with $G_i^{'}$ and $H_i^{'}$ where

$$G_i^{"} = G_i^{'}/w_i, H_i^{"} = H_i^{'}/w_i \quad \forall I = 1,2,3.....,n$$

Step 6 : Find the expected processing time of job block β= *(k, m)* on fictious machines G and H using equivalent job block criterion given by Maggu & Dass (1977). Find $G_\beta^{"}$ and $H_\beta^{"}$ using

$$G_\beta^{"} = G_k^{"} + G_m^{"} - \min(G_m^{"}, H_k^{"})$$

$$H_\beta^{"} = H_k^{"} + H_m^{"} - \min(G_m^{"}, H_k^{"})$$

Step 7 : Define a new reduced problem with the processing time $G_i^{"}$ and $H_i^{"}$ as defined in step 5 and replacing job block β= (k, m) by a single equivalent job β with processing time G''_β and H''_β as defined in step 6.

Step 8 **:** Using Johnson's procedure, obtain all the sequences S_k having minimum elapsed time. Let these be $S_1, S_2, ..., S_r$.

Step 9 : Prepare In-out tables for the sequences $S_1, S_2, ..., S_r$ obtained in step 8. Let the mean flow time is minimum for the sequence S_k. Now, read the effect of break down interval (a, b) on different jobs on the lines of Singh T.P. (1985) for the sequence S_k.

Step 10: Form a modified problem with processing time A'_{ij}; $i = 1, 2, 3, \ldots, n; j = 1, 2, 3$.

If the break down interval (a, b) has effect on job i then

$$A'_{ij} = A_{ij} + L \text{ ; Where L = b – a, the length of break-down interval}$$

If the break-down interval (a, b) has no effect on i^{th} job then

$$A'_{ij} = A_{ij} .$$

Step 11: Repeat the procedure to get the optimal sequence for the modified scheduling problem using. Determine the total elapsed time.

Step 12: Find the performance measure studied in weighted mean flow time defined as

$$F = \sum_{i=1}^{n} w_i f_i \Big/ \sum_{i=1}^{n} f_i \text{ , where } f_i \text{ is flow time of } i^{th} \text{ job.}$$

6. Numerical Illustration

Consider the following flow shop scheduling problem of 5 jobs and 3 machines problem in which the processing time with their corresponding probabilities, transportation time and weight of jobs is given as below:

Jobs	Machine M_1		$T_{i,1\to2}$	Machine M_2		$T_{i,2\to3}$	Machine M_3		Weights of jobs
i	a_{i1}	p_{i1}		a_{i2}	p_{i2}		a_{i3}	p_{i3}	w_i
1	50	0.1	5	20	0.4	3	40	0.2	4
2	40	0.2	3	45	0.2	2	60	0.1	3
3	50	0.2	1	40	0.1	4	35	0.2	2
4	30	0.3	4	35	0.2	5	25	0.2	1
5	35	0.2	5	60	0.1	1	30	0.3	5

(Table 2)

Find optimal or near optimal sequence when the break down interval is (a, b) = (30, 35) and jobs 2 & 4 are to be processed as an equivalent group job. Also calculate the total elapsed time and mean weighted flow time.

Solution: As per Step 1; The expected processing times for the machines M_1, M_2 and M_3 are as shown in **table 3.**

As per Step 2; Here Max $\{A_{i1} + T_{i.1\to2}\}$ =13, Min $\{A_{i2} + T_{i,1\to2}\}$ =5,

Max $\{T_{i,2\to3} + A_{i3}\}$ =11, Min $\{A_{i2} + T_{i,2\to3}\}$ =7.

Therefore, we have

Max $\{A_{i1} + T_{i.1\to2}\} \geq$ Min $\{A_{i2} + T_{i,1\to2}\}$ and Max $\{T_{i,2\to3} + A_{i3}\} \geq$ Min $\{A_{i2} + T_{i,2\to3}\}$.

As per Step. 3; The two fictitious machines G and H with processing times G_i and H_i are as shown in **table 4.**

As per Step 4 &5; The new reduced problem with processing time $G_i^{''}$ and $H_i^{''}$ are as shown in **table 5.**

As per Step 6; The expected processing time of job block β(2 ,4) on fictious machines G & H using equivalent job block criterion given by Maggu & Dass (1977) are

$$G_\beta^{''} = G_k^{''} + G_m^{''} - \min(G_m^{''}, H_k^{''}) = 3 + 8 - 2.66 = 8.34$$

$$H_\beta^{''} = H_k^{''} + H_m^{''} - \min(G_m^{''}, H_k^{''}) = 2.66 + 11 - 2.66 = 11$$

As per Step 7; The reduced problem with processing time $G_i^{''}$ and $H_i^{''}$ are as shown in **table 6.**

As per Step 8; The optimal sequence with minimum elapsed time using Johnson's technique is

S = 5 – 1 - β – 3 = 5 – 1 – 2 – 4 – 3.

As per Step 9 & 10; The In-Out flow table and checking the effect of break down interval (30, 35) on sequence S is as shown in **table 7.**

As per Step 11; On considering the effect of the break down interval the original problem reduces to as shown in **table 8**

Now, On repeating the procedure to get the optimal sequence for the modified scheduling problem, we get the sequence 5 - 2 – 4 – 3 – 1 which is optimal or near optimal. The In-Out flow table for the modified scheduling problem is as shown in **table 9.**

The mean weighted flow time = $\dfrac{28\times5+(40-7)\times3+(49-15)\times1+(56-24)\times2+(73-39)\times4}{5+3+2+4+1} = 31.53$

Hence the total elapsed time is 73 hrs and the mean weighted flow time is 31.53 hrs.

Conclusion

The new method provides an optimal scheduling sequence with minimum total elapsed time whenever mean weighted production flow time is taken into consideration for 3-machines, n-jobs flow shop scheduling problems. This method is very easy to understand and will help the decision makers in determining a best schedule for a given sets of jobs effectively to control job flow and provide a solution for job sequencing. The study may further be extended by introducing the concept of setup time and rental policy.

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Tables

Table 3: The expected processing times for the machines M_1, M_2 and M_3 are

Jobs	A_{i1}	$T_{i,1\to 2}$	A_{i2}	$T_{i,2\to 3}$	A_{i3}	w_i
1	5	5	8	3	8	4
2	8	3	9	2	6	3
3	10	1	4	4	7	2
4	9	4	7	5	5	1
5	7	5	6	1	9	5

Table 4: The two fictitious machines G and H with processing times G_i and H_i are

Jobs	G_i	H_i	w_i
1	11	8	4
2	6	8	3
3	1	2	2
4	7	11	1
5	5	3	5

Table 5: The new reduced problem with processing time G_i'' and H_i'' are

Jobs	G_i''	H_i''
1	2.75	3
2	3	2.66
3	1.5	1

4	8	11
5	1	1.66

Table 6: The reduced problem with processing time G_i'' and H_i'' are

Jobs	G_i''	H_i''
1	2.75	3
3	1.5	1
5	1	1.66
β	8.34	11

Table 7: The In-Out flow table for sequence S is

Jobs	Machine M_1	$T_{i,1\to 2}$	Machine M_2	$T_{i,2\to 3}$	Machine M_3	w_i
i	In – Out		In – Out		In - Out	
5	0 – 7	5	12 – 18	1	19 - 28	**5**
1	7 – 12	5	18 – 26	3	**29 – 37**	4
2	12 – 20	3	**26 – 35**	2	37 – 43	3
4	20 – 29	4	35 – 42	5	47 – 52	1
3	**29 – 39**	1	42 – 46	4	52 – 59	2

Table 8: On considering the effect of the break down interval the original problem reduces to

Jobs	A_{i1}	$T_{i,1\to 2}$	A_{i2}	$T_{i,2\to 3}$	A_{i3}	w_i
1	**5**	5	8	3	13	4
2	8	3	14	2	6	3
3	15	1	4	4	7	2
4	9	4	7	5	**5**	1
5	7	5	6	1	9	5

Table 9: The In-Out flow table for the modified scheduling problem is

Jobs	Machine M_1	$T_{i,1\to 2}$	Machine M_2	$T_{i,2\to 3}$	Machine M_3	w_i
i	In – Out		In – Out		In - Out	
5	0 – 7	5	12 – 18	1	19 – 28	**5**
2	7 – 15	3	18 -32	2	34 – 40	3
4	15 – 24	4	32 – 39	5	44 – 49	1
3	24 – 39	1	40 – 44	4	49 – 56	2
1	39 – 44	5	49 – 57	3	60 – 73	4