A Mathematical Model on the Dynamics of Poverty and Prostitution in Nigeria

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Abstract
In this paper, we propose a compartmental mathematical model that tracks the dynamic of poverty and prostitution in Nigeria. Our model incorporates an infected compartment that allows for a non-violent approach of government interventions. The stability of the system is analyzed for the existence of the prostitution free equilibrium. We established that there exist a prostitution free equilibrium point that is locally asymptotically stable when the reproduction number $\bar{R}_0 < 1$ and unstable when $\bar{R}_0 > 1$. Result in this paper shows that high rate of government interventions will reduce to the barest minimum the number of members in both poverty and prostitution class.
Keywords: Mathematical model, Poverty, Prostitution, Stability Analysis, Reproduction number

1. Introduction

There is a direct relationship between poverty and prostitution. Females generally resort to prostitution out of economic need and sometimes out of familial obligation (Davidoff et. al 2006). Those living in poverty, therefore, have a much greater chance of engaging in prostitution than the general population as observed in developing countries (Keller, 2006). The sex industry can be viewed as a market of supply and demand, where females are the primary supply and the males the main demand. There are studies in the literature that identifies different pathways into prostitution. These include child abuse (BBC, 1999), sexual abuse, running away from home (homelessness), family breakdown, poverty, educational underachievement, unemployment and peer pressure (McClanahan et. al. 1999). These pathways have been known to significantly increase a females odds of entering the sex industry (Keller, 2006). Our model focus primarily on the primary effects of prostitution due to poverty.

Davidoff et al, (2006) developed two mathematical models to explore the dynamics of the sex industry in terms of supply and demand. In their work, they perform qualitative analysis on these models separately, and explore the coupled system numerically. From their analysis, they provide explanations as to why the current system of arrest and detainment does little to control sex workers population. In addition they showed that if increase efforts of legal enforcement focus on making male arrests are made, it is possible to significantly reduce the number of women in prostitution. In this paper, we use a system of ODEs to try and obtain a more robust and realistic, dynamical solution to that same problem. Our model incorporates an infected compartment that allows for a non-violent approach of government interventions.

What this model aims to do is find an effective and non-violent approach at reducing the rate of prostitution and consequently the rate of poverty. This will eventually lower the cost of prostitution to society to the barest minimum. Previous works have addressed the problem of reducing the cost of crime in the Society (Becker, 1968, Ehrlich, 1973) using statistical and economic model approaches. Viewing prostitution as a crime, we shall use a mathematical model to understand the dynamic between poverty and prostitution and how both can be reduce to the barest minimum.

2. Model Formulation

The model seeks to enhance government interventions in such a way that prostitution is reasonably controlled and the cost of intervention is minimal. The population is divided into five compartment; the non-impoverished class (N), the poverty class (P), the prostitution class (S), the disease infected class (I) and the rehabilitated class (R) (from the poverty class or from the disease infected class). We denote β to be the rate of the flow from the non-impoverished class to poverty class. We make the assumption that β is dependent upon unemployment and underemployment rate, which normally is directly related to poverty. We denote ϖ as the conversion rate from the poverty class (P) to the rehabilitation class (R) due to government interventions and η the rate at which a prostitute get infected with any form of sexually transmitted diseases (STDs). Another major assumption is that a person in the P class will resort to prostitution after coming in contact with a prostitute over a given period of time. The term $\frac{\varphi PS}{T}$ represents the conversion from the poverty class (P) to the prostitution class (S), where φ is the transmission rate. A rehabilitated individual may also resort to prostitution again but at a reduced rate $\frac{\delta PS}{T}$, where $0 \leq \delta \leq 1$, is the reduction fraction that account for recidivism. According to Bureau of Justice Statistics, (2005), this rate is around 50%.

Those prostitutes, who have already been infected with STD, immediately move into the rehabilitation class (R) at a rate ϕ due to government intervention (non-violent) and then due to contact with other prostitute may revert back to prostitution at some reduced rate $\delta\varphi$ (Beckers, 1968). We assume a constant population, hence the per capital death rate is equal in magnitude to the per capital birth rate ($\mu = \lambda$). All parameters are assumed to be non-negative.

Given the above assumption, the relationship between poverty and prostitution is governed by the following system of ordinary differential equations (ODEs).

$$\frac{dN}{dt} = \mu T - (\mu + \beta)N \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\frac{dP}{dt} = \beta N - (\mu + \varpi)P - \frac{\varphi PS}{T} \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\frac{dS}{dt} = \frac{\varphi PS}{T} + \frac{\delta\varphi RS}{T} - (\mu + \eta)S \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$\frac{dI}{dt} = \eta s - (\mu + \alpha_1 + \phi)I \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$\frac{dR}{dt} = \varpi P + \phi I - (\mu + \alpha_2)R - \frac{\delta\varphi RS}{T} \qquad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$N + P + S + I + R = T \qquad \ldots \quad \ldots \quad \ldots \quad (6)$$

In the table below, variables and parameter used in the model are defined.

Table 1 Model variables and parameters

Variables	**Description**
$N(t)$	The size of the non-impoverished class
$P(t)$	The size of the poverty class
$S(t)$	The size of the prostitution class
$I(t)$	The size of the diseased infected class
$R(t)$	The size of the rehabilitation class
Parameters	**Description**
β	The rate of flow from the non-impoverish class (**N**) to the poverty class (**P**).
ϖ	The conversion rate from poverty class (**P**) to rehabilitation class (**R**) due to government intervention
μ	Per capita birth rate
μ	Per capita death rate
φ	The rate at which individuals in the poverty class (**P**) get into the prostitution class (**S**)
η	Infectious rate at which prostitute get infected with any form of STDs
ϕ	The rate at which infected prostitute are recruited into the rehabilitation class (**R**).
α_1	Disease induced death rate due to infection in the diseased infected class (**I**)
α_2	Disease induced death rate due to infection in the Rehabilitation class (**R**).

The flow diagram for the prostitution dynamic is given below

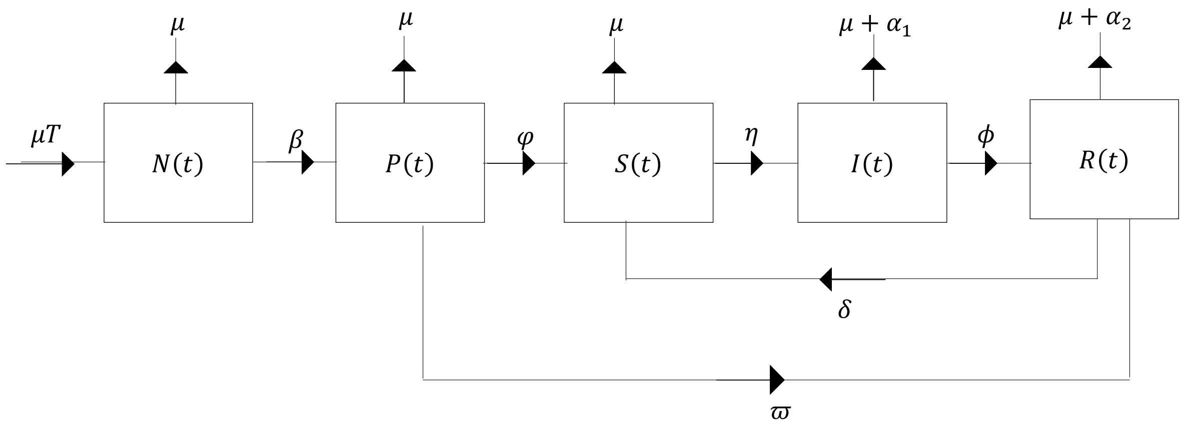

Figure 1: Flow diagram of the model

3. Mathematical Analysis

We discuss the existence and uniqueness of the disease free equilibrium (DFE) states of the model and their

stability analysis in this section. The prostitution free equilibrium (PFE) states of the model can be obtained if we consider the different compartment as proportions and setting the left hand side of equation (1) to (5) zero and then solving the resulting system of equations simultaneously, setting $S = 0, I = 0$.
The prostitution free equilibrium (PFE_S) is given as

$$PFE_S = \left(\frac{N_0}{T}, \frac{P_0}{T}, \frac{S_0}{T}, \frac{I_0}{T}. \frac{R_0}{T}\right)$$

$$PFE_S = \left(\frac{\mu}{\mu+\beta}, \frac{\mu\beta}{(\mu+\beta)(\mu+\varpi)}, 0, 0. \frac{\mu\beta\varpi}{(\mu+\beta)(\mu+\varpi)(\mu+\alpha_2)}\right) \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Using the approach presented by Castillo-Ch´avez et. al (2002), we derived the reproductive number $(\bar{R})$ to be given by

$$\bar{R} = \bar{R}_P + \bar{R}_R = \frac{\varphi\mu\beta(\mu+\alpha_2)+\delta\varphi\varpi\mu\beta}{(\mu+\beta)(\mu+\varpi)(\mu+\alpha_2)(\mu+\eta)} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where

$$\bar{R}_P = \frac{\varphi\mu\beta(\mu+\alpha_2)}{(\mu+\beta)(\mu+\varpi)(\mu+\alpha_2)(\mu+\eta)} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

$$\bar{R}_R = \frac{\delta\varphi\varpi\mu\beta}{(\mu+\beta)(\mu+\varpi)(\mu+\alpha_2)(\mu+\eta)} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

$\bar{R}_P$ and $\bar{R}_R$ represents the input from the poverty class (P) and the rehabilitation class (R) respectively. The factor $\frac{\varpi}{\mu+\eta}$ and $\frac{\delta\varphi}{\mu+\eta}$ represents the number of new prostitute (incidence rate) from the poverty class (P) and the rehabilitation class (R) by one individual prostitute during the entire prostitution period before being infected with an STD. The factor $\frac{\beta}{\mu+\beta}$ represents the probability that an individual in the non-impoverish class (N) survived and entered into the poverty class, $1-\frac{\varpi}{\mu+\varpi}$ represents the probability that an individual in the poverty class (P) remains in the poverty class and $\frac{\varpi}{\mu+\varpi}$ represents the probability that an individual in the poverty class move into the rehabilitation class (R).

We proceed to show the stability of the equilibria. To study the behaviour of the system of equations (1) to (5) around the prostitution free equilibrium states (PFE_S), we resort to the linearized stability principles. The question of interest is whether the equilibrium state is stable or unstable under small perturbation.

Let

$$f_1 = \mu T - (\mu+\beta)N \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$f_2 = \beta N - (\mu+\varpi)P - \frac{\varphi PS}{T} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$f_3 = \frac{\varphi PS}{T} + \frac{\delta\varphi RS}{T} - (\mu+\eta)S \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

$$f_4 = \eta s - (\mu+\alpha_1+\phi)I \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

$$f_5 = \varpi P + \phi I - (\mu+\alpha_2)R - \frac{\delta\varphi RS}{T} \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

To get the Jacobian matrix, we evaluate the partial derivative of the system (11) to (15) at the prostitution free equilibrium (PFE_S) states
The Jacobian $J_{[PFE_S]}$ is given by

$$J_{E_0} = \begin{bmatrix} -(\mu+\beta) & 0 & 0 & 0 & 0 \\ \beta & -(\mu+\varpi) & -\frac{\varphi P_0}{T} & 0 & 0 \\ 0 & 0 & \frac{\varphi P_0}{T} + \frac{\delta\varphi R_0}{T} - (\mu+\eta) & 0 & 0 \\ 0 & 0 & \mu & -(\mu+\alpha_1+\phi) & 0 \\ 0 & \varpi & -\frac{\delta\varphi R_0}{T} & \phi & -(\mu+\alpha_2) \end{bmatrix}$$

where $\frac{P_0}{T} = \frac{\mu\beta}{(\mu+\beta)(\mu+\varpi)}, \frac{R_0}{T} = \frac{\mu\beta\varpi}{(\mu+\beta)(\mu+\varpi)(\mu+\alpha_2)}$

Theorem 1
The system of equation (1) – (5) has a unique prostitution equilibrium (PE_S) if $\bar{R} > 1$ and has only the prostitution free equilibrium (PFE_S) if $\bar{R} < 1$
Theorem 2
The prostitution free equilibrium (PFE_S) is locally asymptotically stable if $\bar{R} < 1$ and unstable if $\bar{R} > 1$. Hence the prostitution equilibrium exist when $\bar{R} > 1$
Proof:
Using the method of linearized stability, we can show that the trace (J_{E_0}) is negative and the determinant (J_{E_0}) is positive under specified conditions.

$$\text{Trace}\,(J_{E_0}) = -(5\mu + \beta + \varpi + \eta + \phi + \alpha_1 + \alpha_2) + \left(\frac{\varphi P_0}{T} + \frac{\delta\varphi R_0}{T}\right) < 0$$

provided $\bar{R} < 1$, where $\bar{R} = \frac{\varphi P_0}{T} + \frac{\delta\varphi R_0}{T}$.
Also

$$\text{Determinant}\,(J_{E_0}) = (\mu + \beta)(\mu + \varpi)(\mu + \phi + \alpha_1)(\mu + \alpha_2)\left[\frac{\varphi P_0}{T} + \frac{\delta\varphi R_0}{T} - (\mu + \eta)\right] > 0$$

provided $\bar{R} > (\mu + \eta)$ and $\bar{R} < 1$, where $\bar{R} = \frac{\varphi P_0}{T} + \frac{\delta\varphi R_0}{T}$.

3.1 Numerical Experiments
The model (1)–(5) was solved numerically by using Maple 15 (Maplesoft, Waterloo Maple Inc, 2012). The baseline parameters for Nigeria were used as shown in Table 2. Other parameters are chosen in consonance with the threshold values obtained in the stability analysis of the prostitution free equilibrium state of the model as shown in Table 3

Table 2 Estimated values of the baseline parameters used in the experiments

Parameters	T	$N(0)$	$P(0)$	$S(0)$	$I(0)$	$R(0)$
Values	88875303	66656477	22218826	2221883	199970	39994

Source: NBS, (2011) USAID, 2011 , NACA (2011)

Table 3 Other parameters used in the experiments

Parameters	η	β	ϕ	α_1	α_2	ϖ	φ	δ	$\lambda = \mu$
Values	0.25	0.40	0.35	0.10	0.05	0.35	0.5	0.5	0.03923

Source: Field survey for research purpose.
The following experiment were carried out:
Experiment 1: Measuring the effect of Government Intervention (Low and High) on the Poverty class
Experiment 2: Measuring the effect of Government Intervention (Low and High) on the Infected class
Experiment 3: Measuring the effect of Government Intervention (Low and High) on the Rehabilitation class

3.2 Results
We present the following result in graphical representations.

Experiment 1: Measuring the effect of Government Intervention (Low and High) on the Poverty class ($\varpi = 0.20$, $\phi = 0.20$, $\varpi = 0.75$, $\phi = 0.75$)

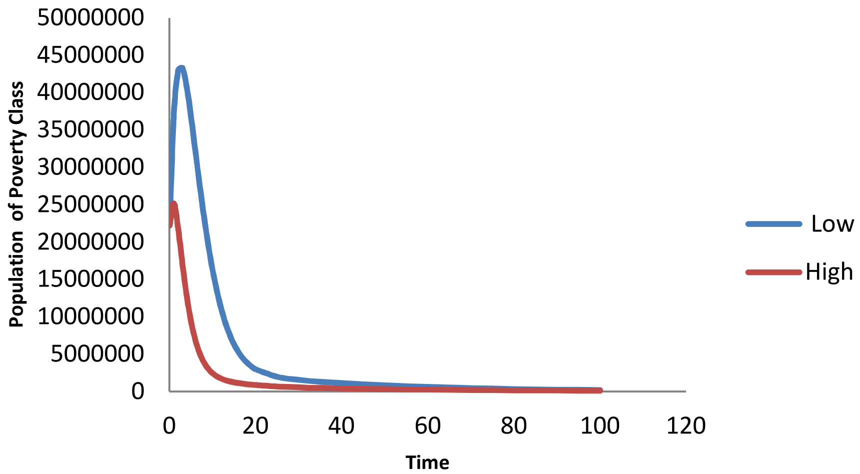

Figure 2 Effect of high and low Government intervention on the poverty class. ($\varpi = 0.20$, $\phi = 0.20$, $\varpi = 0.75, \phi = 0.75$ **Parameter values:** $\lambda = 0.03923, \beta = 0.25$, $T = 88875303$, $P(0) = 22218826$, $\alpha_1 = 0.10$, $\alpha_2 = 0.05$, $\delta = 0.5, \eta = 0.25, \beta = 0.40$

Experiment 2: Measuring the effect of Government Intervention (Low and High) on the Infected class ($\varpi = 0.20$, $\phi = 0.20, \varpi = 0.75$, $\phi = 0.75$)

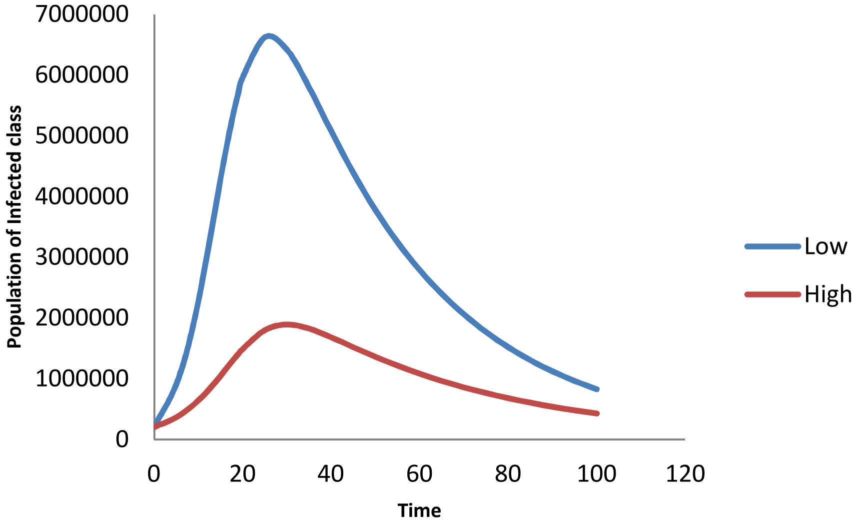

Figure 3 Effect of high and low Government intervention on the infected class. ($\varpi = 0.20$, $\phi = 0.20, \varpi = 0.75$, $\phi = 0.75$) .**Parameter values:** $\lambda = 0.03923$, $\eta = 0.25, \beta = 0.40$, $I(0) = 199970$, $T = 88875303$, $\alpha_1 = 0.10$, $\alpha_2 = 0.05$, $\delta = 0.5$

Experiment 3: Measuring the effect of Government Intervention (Low and High) on the Rehabilitation class ($\varpi = 0.20$, $\phi = 0.20, \varpi = 0.75$, $\phi = 0.75$)

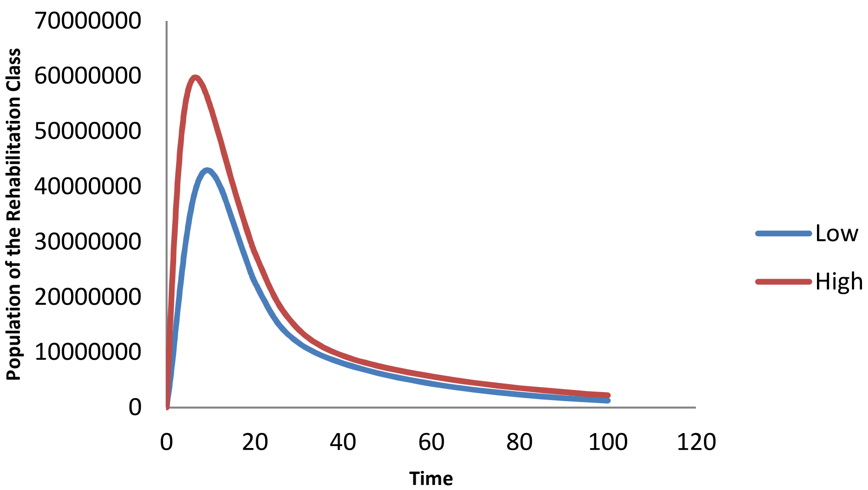

Figure 4 Effect of high and low Government intervention on the infected class. ($\varpi = 0.20$, $\phi = 0.20, \varpi = 0.75$, $\phi = 0.75$) . **Parameter values:** $\lambda = 0.03923$, $\eta = 0.25, \beta = 0.40, \delta = 0.5$, $I(0) = 199970$, $T = 88875303$, $\alpha_1 = 0.10$, $\alpha_2 = 0.05$

4. Discussion of Result

The conversion rate from poverty class (P) to the rehabilitation class (R) due to government intervention (ϖ) is of paramount importance in decreasing the rate of prostitution. In Figure 2, we observed that when the rate of government is high, the rate of poverty decreases faster. Consequently this will reduce the number of individual getting involved in prostitution. By government intervention we mean policies and programmes that are geared toward empowering the poverty class. These programmes include skill acquisition, provision of jobs, soft loan, educational opportunities and a host of others.

Prostitution as a career is a way of life that can adversely affects the health of those involve in it, irrespective of sex, hence the need to assist members of the prostitution class (S) with information on the dangers inherent in prostitution. This can be achieved with government non-violent intervention programmes. The rate at which infected prostitute (with STDs) are recruited into the rehabilitation class (R) due to government intervention (ϕ) will go a long way in reducing the number of prostitute. Figure 3 shows that when government assist infected prostitute health-wise and rehabilitate them, thus helping them to become self-employed, the number of prostitute in the long run will reduce drastically. This can be achieved through non-violent approach such as free counselling, financial assistance and religious preaching.

In this paper, the rehabilitation class provides important insight on the effectiveness of government intervention (ϖ and ϕ). Figure 4 shows that at the initial stage of government intervention the numbers of those rehabilitated grow higher and starts decreasing subsequently as a result of reduction in the number of individual in the poverty class and the prostitution class respectively. Government rehabilitation policies and programmes should be closely monitored for effectiveness if prostitution is to be reduce to its barest minimum

5. Conclusion

We have provided a realistic but simple compartmental mathematical model that help to understand the dynamics of prostitution in Nigeria. We introduce a non-violent compartment that focuses on rehabilitating both male and female prostitutes. The model presented in this paper is obviously unique out of many plausible models about the dynamics of prostitution as it's relates to many forms of poverty and crime. The effect of poverty in almost every other crime in the society cannot be underestimated. Poverty eradication and control should be the focus of good governance in developing countries as can be seen in the result of the model. In further research, it is necessary to involve other important stakeholders (the police, organizations representing or helping prostitutes, et cetera) or representing stakeholders that are hard to involve (e.g. organized crime). These future models can then be used together with the model described in this paper to provide the best option in monitoring prostitution prevention policies. The boundaries of the research of modeling prostitution should also include related illicit markets and related forms of organized crime that is related to prostitution.

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