

Assessment of Reliability and Availability of Series-Parallel Sub-Systems

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Abstract
The paper presents the explicit expressions for measuring reliability parameter of systems, Graphs were plotted to highlight important results. Results have shown that measures of system effectiveness such MTSF, system availability and profit increases with repair rates and decreases with failure rates.The developed model helps in determining the optimal maintenance strategies which ensure the maximum overall availability of the system. The optimum values of failure and repair rates for each subsystem were given. It is observed that the first subsystem is having the maximum availability with (97%). The optimum values of failure and repair rates for maximum availability level for each subsystem is also shown.
Keywords: Mean time to system failure, Availability, Reliability, Maintenance and Subsystem.

1. INTRODUCTION

Stochastic models of redundant systems as well as methods of evaluating system reliability indices such as mean time to system failure (MTSF), system availability, busy period of repairman, profit analysis, etc have been investigated by many researchers in order to improve the system effectiveness. Some systems are series-parallel design. These systems have wide application in the real world especially in industries. Furthermore, electrical substation, aircraft engine can be cited as a good example of series-parallel systems. Due to the importance of series-parallel nature of some complex systems in various industries, determination of their availability has become an increasingly important issue. System availability represents the percentage of time the system is available to users. Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order.

A large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar *et al* (1988) discussed the reliability analysis of the Feeding system in the paper industry, Kumar *el al*.(1989) discussed the availability analysis of the washing system in the paper industry, Kurien (1988), developed a simulation model for analyzing the reliability and availability of an aircraft training facility Kumar *el al.* (1993) deal with reliability, availability and operational behavior analysis for different systems in paper plant. Haggag (2009), discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Gupta[a] *et al* (2005), studied the design and cost analysis of a refining system in a Sugar industry. Srinath (1994), has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta[b] *el al*. (2005) has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta[b] *et al.* (2005) studied the behavior of Cement manufacturing plant. Arora and Kumar (2000), studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg (2005), perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates. Gupta and Tiwari (2009), study simulation modeling of complex system using thermal power plant as

2. SYSTEM DESCRIPTION/ NOTATIONS/ASSUMPTIONS

- The system considered in this study consist of four subsystems arranged in series connection, i.e. A, B, C and D are connected in series to each other with subsystem D having two units in parallel.

- The life time of the units $(A, B, C, Di; i=1,2)$ is exponentially distributed random variables with parameter $\lambda_{i,} i=1,2,3,4.$
- The repair time of the units is exponentially distributed random variables with parameter $\mu_j; j=1,2,3,4.$
- λ_i, μ_i Denote failure rate of $i^{th} and j^{th}$ units respectively.
- All failures are repairable ones.
- In state 8 D_2 has maintenance priority over D_1
- The system is attended by one repairman.
- Each unit is as good as new after repair.
- The repair is done at down time or at the time of failure
- There is no simultaneous failure among the four subsystems
- State 0 and 4 indicate the system is in fully operational states.
- Switching device is perfect for the units in subsystem D.
- Subsystems D_1 and D_2 are in cold standby.

3. SYSTEM STRUCTURE

A typical system consists of a number of subsystems connected to each other in series-parallel. The performance of the system depends on the configuration and performance of its subsystems. Before analyzing the failure data, it is better to describe the configuration of the system and classify it into various subsystems so that the failure can be categorized. The system under study consists of the following four subsystems:

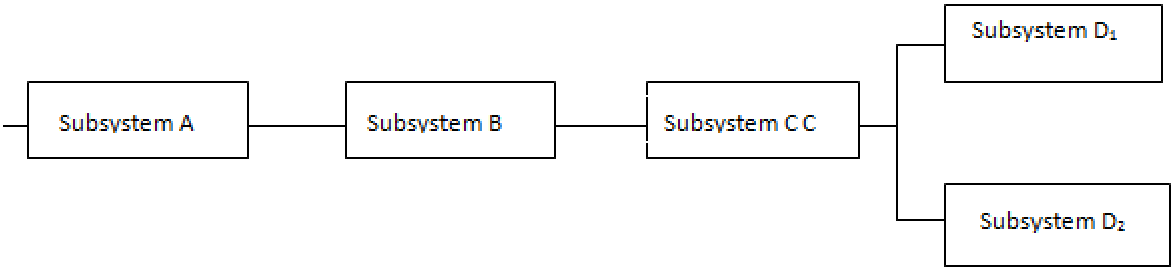


Fig.1 Reliability block diagram and transition diagram of the system

4. METHODOLOGY

4.1 MEAN TIME TO SYSTEM FAILURE FOR THE SYSTEM

Let $P(t)$ be the probability row vector at time t and $P_i(t)$ is the probability that the system is in state S_i at time $t \geq 0$,then the initial conditions for this problem are as follows:

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0)\right] = \left[1,0,0,0,0,0,0,0,0\right]$$

$$P_1'(t) = -\mu_1 P_1(t) + \lambda_1 P_0(t)$$

$$P_2'(t) = -\mu_2 P_2(t) + \lambda_2 P_0(t)$$

$$P_3'(t) = -\mu_3 P_3(t) + \lambda_3 P_0(t)$$

$$P_4'(t) = -(\mu_4 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) P_4(t) + \lambda_4 P_0(t) + \mu_1 P_5(t) + \mu_2 P_6(t) + \mu_3 P_7(t) + \mu_4 P_8(t)$$

$$P_5'(t) = -\mu_1 P_5(t) + \lambda_1 P_4(t)$$

$$P_6'(t) = -\mu_2 P_6(t) + \lambda_2 P_4(t)$$

$$P_7'(t) = -\mu_3 P_3(t) + \lambda_3 P_4(t) \qquad (1)$$

$$P_8'(t) = -\mu_4 P_8(t) + \lambda_4 P_4(t)$$

4.2 MODEL VALIDATION

In this section the range of values of the pair (λ, μ) were use for analysis, the range values used by Gupta and Tiwari (2009) in the modeling of air system of a thermal power plant to validate the model developed in fig.1.

The following transition probabilities were obtained by setting the left hand sides of equations (1) equal to zero and solve them recursively.

Therefore, following Kumar *et al* (1989), $P_i'(t)=0,\ i=1,2,3,...,8$ as $t\to\infty$. Equations (1) are solved recursively to obtain the following:

$$p_1=\frac{\lambda_1}{\mu_1}p_0 \quad (2)$$

$$P_2=\frac{\lambda_2}{\mu_2}p_0 \quad (3)$$

$$p_3=\frac{\lambda_3}{\mu_3}p_0 \quad (4)$$

$$p_4=\frac{\lambda_4}{\mu_4}p_0 \quad (5)$$

$$p_5=\frac{\lambda_1}{\mu_1}p_4=\frac{\lambda_1}{\mu_1}.\frac{\lambda_4}{\mu_4}p_0 \quad (6)$$

$$p_6=\frac{\lambda_2}{\mu_2}p_4=\frac{\lambda_2}{\mu_2}.\frac{\lambda_4}{\mu_4}p_0 \quad (7)$$

$$p_7=\frac{\lambda_3}{\mu_3}p_4=\frac{\lambda_3}{\mu_3}.\frac{\lambda_4}{\mu_4}p_0 \quad (8)$$

$$p_8=\frac{\lambda_4}{\mu_4}p_4=\frac{\lambda_4}{\mu_4}.\frac{\lambda_4}{\mu_4}p_0=(\frac{\lambda_4}{\mu_4})^2p_0 \quad (9)$$

Now by solving p_1 to p_8 recursively, following Gupta et al 2005, The sum of states probabilities equal to zero.

4.3 NORMALISING CONDITION

The probability of full working capacity, namely, p_0 and p_4 determined by using normalizing condition: (sum of the probabilities of all working states is equal to zero)

$\sum_{i=0}^{8}p_i=1$, therefore putting the values of p_0-p_8 and solving, one gets

$$p_0+\frac{\lambda_1}{\mu_1}p_0+\frac{\lambda_2}{\mu_2}p_0+\frac{\lambda_3}{\mu_3}p_0+\frac{\lambda_4}{\mu_4}p_0+\frac{\lambda_1\lambda_4}{\mu_1\mu_4}p_0+\frac{\lambda_2\lambda_4}{\mu_2\mu_4}p_0+\frac{\lambda_3\lambda_4}{\mu_3\mu_4}p_0+(\frac{\lambda_4}{\mu_4})^2p_0=1$$

$$p_0[1+\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_3}{\mu_3}+\frac{\lambda_4}{\mu_4}+\frac{\lambda_1\lambda_4}{\mu_1\mu_4}+\frac{\lambda_2\lambda_4}{\mu_2\mu_4}+\frac{\lambda_3\lambda_4}{\mu_3\mu_4}+(\frac{\lambda_4}{\mu_4})^2]=1$$

Therefore $p_0=\frac{1}{D}$ (10)

Where $D=1+\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_3}{\mu_3}+\frac{\lambda_4}{\mu_4}+\frac{\lambda_1\lambda_4}{\mu_1\mu_4}+\frac{\lambda_2\lambda_4}{\mu_2\mu_4}+\frac{\lambda_3\lambda_4}{\mu_3\mu_4}+(\frac{\lambda_4}{\mu_4})^2$

Therefore since normalizing condition is use to obtain p_0, then steady state availability would be computed in terms of p_0

4.4 **STEADY STATE AVAILABILITY**

Now, the steady state availability of the system may be obtained as summation of all working states probabilities as:

Availability = the sum of probabilities of operational states.

Thus, $AV = P_0 + P_4 = P_0 + \frac{\lambda_4}{\mu_4} P_0$ (11)

$= P_0(1 + \frac{\lambda_4}{\mu_4})$

And $P_0 = \frac{1}{1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_4}{\mu_4} + \frac{\lambda_1\lambda_4}{\mu_1\mu_4} + \frac{\lambda_2\lambda_4}{\mu_2\mu_4} + \frac{\lambda_3\lambda_4}{\mu_3\mu_4} + (\frac{\lambda_4}{\mu_4})^2} = \frac{1}{D}$

Where $D = 1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_4}{\mu_4} + \frac{\lambda_1\lambda_4}{\mu_1\mu_4} + \frac{\lambda_2\lambda_4}{\mu_2\mu_4} + \frac{\lambda_3\lambda_4}{\mu_3\mu_4} + (\frac{\lambda_4}{\mu_4})^2$

Therefore $AV = \frac{1 + \frac{\lambda_4}{\mu_4}}{D}$ (12)

Now, following Gupta and Tiwari (2009) to compute availability table 1 through 4. Taking the range of values of both failure and repair rates of sub-system A,B,C and D.

4.5 ANALYSIS OF SYSTEM MODEL

The simulation model is used to predict the availability/performance of the system for known input values of failure and repair rates of its subsystems. The performance of the system is mainly affected by the failure and repair rates of each subsystem. Appropriate failure and repair rates of all subsystems are taken and decision matrices (availability values) are prepared accordingly by putting these failure and repair rates values in availability expression, the availability simulation model (Av.). This model forms the foundation for all other performance improvement activities (e.g. solution design and development, implementation and analysis). These unit parameters ensure the high availability/performance of the system. This model includes all possible states of nature, that is, failure events (λ_i) and the identification of all the courses of action, i.e, repair priorities (μ_i). Tables 1-4 represent the availability matrices for various subsystems of the system. These matrices simply reveal the various availability levels for different combinations of failure and repair rates/priorities. On the basis of analysis made, the best possible combinations (λ, μ) may be selected. These availability values in availability matrices further help in identifying the subsystem which ensures the maximum availability, as shown in Table 5. The optimum vales of failure/repair rates of each subsystem of concerned system can easily be taken from Table 5.

PARAMETER	DESCRIPTION	SOURCE
λ_1	Failure rate of subsystem A	Gupta and Tewari (2009)
λ_2	Failure rate of subsystem B	Gupta and Tewari (2009)
λ_3	Failure rate of subsystem C	Gupta and Tewari (2009)
λ_4	Failure rate of subsystem D	Gupta and Tewari (2009)
μ_1	Repair rate of subsystem A	Gupta and Tewari (2009)
μ_2	Repair rate of subsystem B	Gupta and Tewari (2009)
μ_3	Repair rate of subsystem C	Gupta and Tewari (2009)
μ_4	Repair rate of subsystem D	Gupta and Tewari (2009)

6. RESULTS AND DISCUSSION

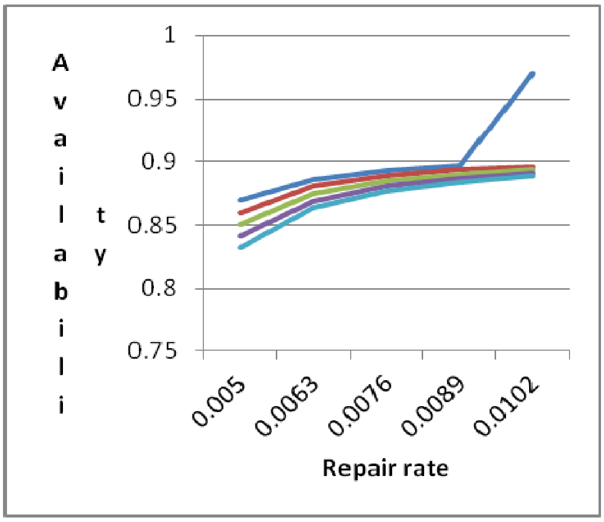
The result of the findings could be discussed in two ways as can be observed. Case 1 the system is considered as a whole and some arbitrary values were use to see the effect of failure/repair on the system effectiveness, meantime to system failure, and the effect of failure/repair on the system availability and lastly the optimum profit incurred when the system is receiving proper maintenance and vise-versa. While in the second case the four subsystems were analyzed, computing the availability level of each subsystem using range of values of failure and repair rates [see Gupta and Tiwari (2009)]. Lastly the maximum availability of each subsystem is computed to identify the subsystem to be given maintenance priority.

SECTION 2

The performance of each subsystem is analyzed using the developed model. On the basis of availability values, as given in Table 1-5 and plotted in Figure 2-9, the following observations are made using the pair values (λ, μ) as in Gupta and Tiwari (2009), which reveals the effect of failure and repair rates of various subsystems on the availability of the system.

Table 1 Availability matrix of the subsystem A of series-parallel system

λ_1 \ μ_1	0.1	0.175	0.250	0.325	0.4	
0.005	0.8694	0.8856	0.8927	0.8964	0.9704	$\lambda_2 = 0.0015$
0.0063	0.8597	0.8802	0.8886	0.8932	0.8961	$\mu_2 = 0.3$
0.0076	0.8502	0.8745	0.8845	0.8900	0.8935	$\lambda_3 = 0.0283$
0.0089	0.8409	0.8687	0.8805	0.8868	0.8909	$\mu_3 = 0.31$
0.0102	0.8318	0.8632	0.8764	0.8837	0.8883	$\lambda_4 = 0.0225$
						$\mu_4 = 0.35$



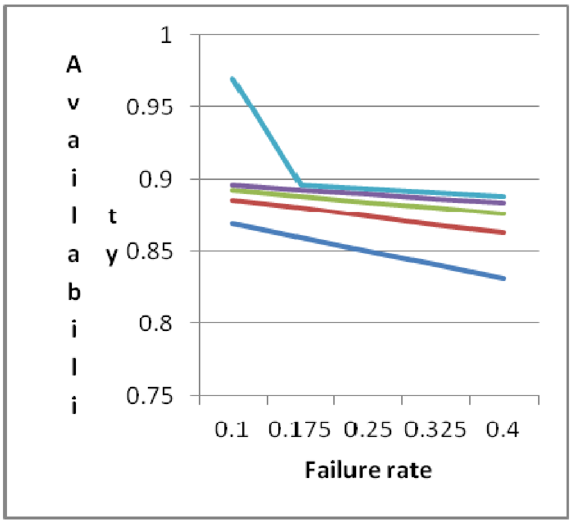


Table 1 and Figures 2 and 3 reveal the effect of failure and repair rates of subsystem A on the availability of the system. It is observed that for some known values of failure / repair rates of other three subsystems, as failure rate of first subsystem increases from 0.005 to 0.0102, the subsystem availability decreases .Similarly as repair rate of first subsystem increases from 0.1 to 0.4, the subsystem availability increases.

Table 2 Availability matrix of the subsystem A of series-parallel system

λ_2 \ μ_2	0.08	0.16	0.24	0.32	0.40	
						$\mu_1 = 0.25$
						$\lambda_1 = 0.0076$
0.001	0.8786	0.8835	0.8851	0.8860	0.8865	$\mu_3 = 0.31$
0.00125	0.8762	0.8823	0.8843	0.8854	0.8860	$\lambda_3 = 0.0283$
0.00150	0.8739	0.8810	0.8835	0.8847	0.8854	$\mu_4 = 0.35$
0.00175	0.8714	0.8799	0.8827	0.8841	0.8850	$\lambda_4 = 0.0225$
0.0020	0.8691	0.8786	0.8819	0.8835	0.8845	

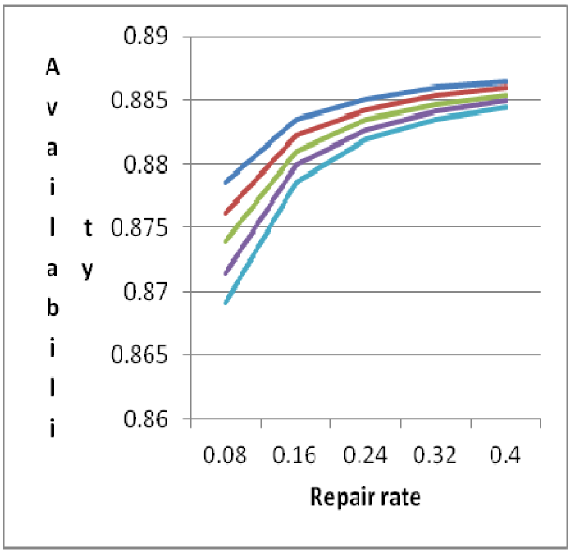


Fig. 4 Plot of availability vs μ_2

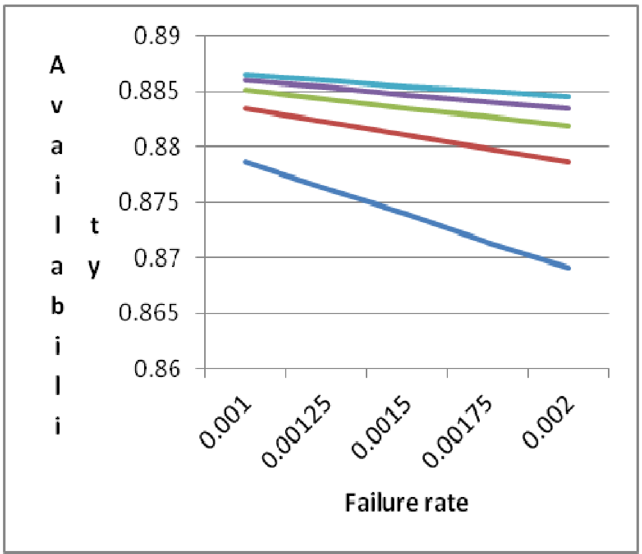


Fig. 5 Plot availability vs λ_2

Table 2 and Figure 4 and 5 reveal the effect of failure and repair rates of subsystem B on the availability of the system. It is observed that for some known values of failure / repair rates of other three subsystems, as failure rate of Subsystem B increases from 0.001 to 0.0020 the subsystem availability decreases. Similarly as repair rate of subsystem B increases from 0.08 to 0.40, the subsystem availability increases.

Table 3 Availability matrix of the subsystem C of series-parallel system

λ_3 \ μ_3	0.125	0.219	0.312	0.406	0.500	
0.0067	0.9150	0.9441	0.9523	0.9568	0.9598	$\mu_1 = 0.25$
0.0175	0.8558	0.9021	0.9220	0.9331	0.9402	$\lambda_1 = 0.0076$
0.0283	0.7968	0.8637	0.8934	0.9105	0.9215	$\mu_2 = 0.3$
0.0391	0.7455	0.8284	0.8667	0.8890	0.9035	$\lambda_2 = 0.0015$
0.0500	0.7000	0.7956	0.8411	0.8682	0.8860	$\mu_4 = 0.35$
						$\lambda_4 = 0.0225$

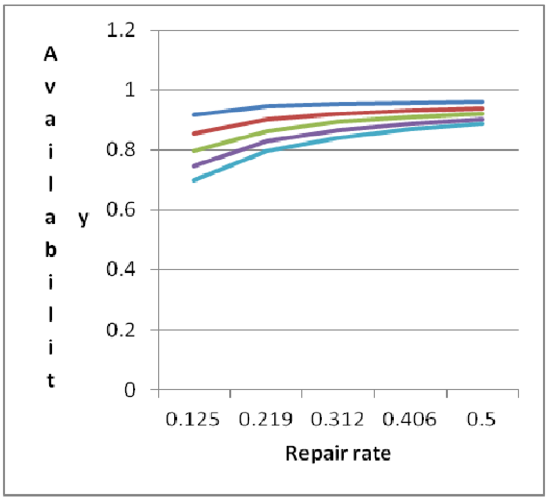


Fig. 6 Plot of availability vs μ_3

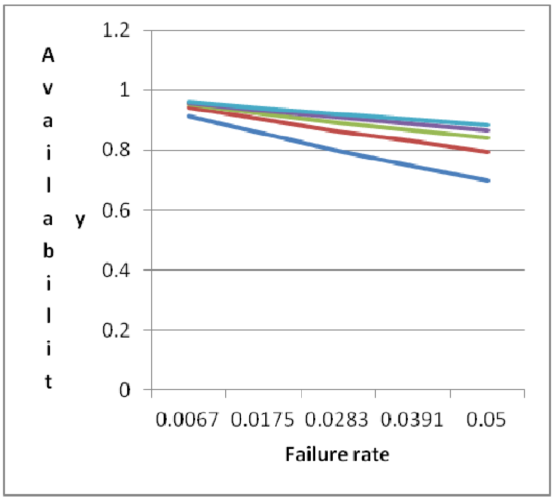


Fig. 7 Plot of availability Vs λ_3

Table 3 and Figures 6 and 7 reveal the effect of failure and repair rates of subsystem C on the availability of the system. It is observed that for some known values of failure/repair rates of other three subsystems, as failure rate of subsystem C increases from 0.0067 to 0.0500, the subsystem availability decreases. Similarly, as repair rate of subsystem C increases from 0.125 to 0.500, the subsystem availability increases.

Table 4 Availability matrix of the subsystem D of series-parallel system

λ_4 \ μ_4	0.2	0.275	0.350	0.425	0.5	
0.0050	0.8871	0.8873	0.8874	0.8875	0.8874	$\mu_1 = 0.25$, $\lambda_1 = 0.0076$, $\mu_2 = 0.3$, $\lambda_2 = 0.0015$, $\mu_3 = 0.31$, $\lambda_3 = 0.0283$
0.0137	0.8841	0.8857	0.8864	0.8868	0.8870	
0.0225	0.8788	0.8827	0.8845	0.8855	0.8860	
0.0313	0.8712	0.8785	0.8818	0.8836	0.8846	
0.04	0.8621	0.8732	0.8784	0.8812	0.8829	

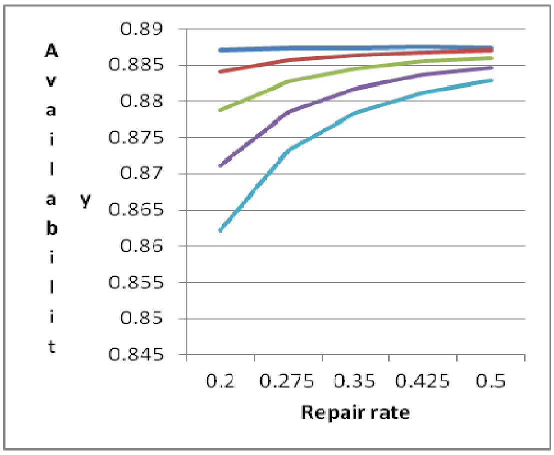


Fig. 8 Plot of availability vs μ_4

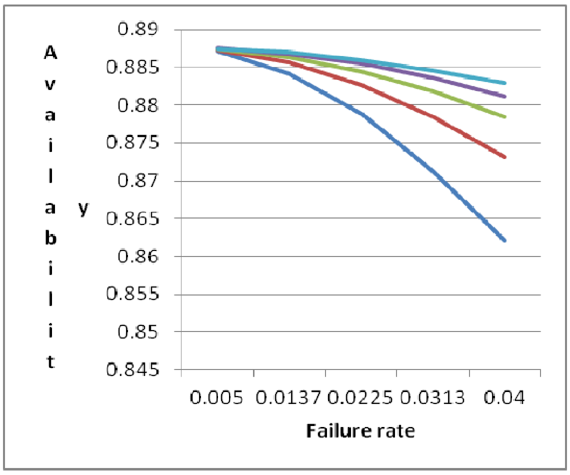


Fig. 9 Plot of availability vs λ_4

Table 4 and Figures 8 and 9 reveal the effect of failure and repair rates of subsystem D on the availability of the system. It is observed that for some known values of failure/repair rates of other three subsystems, as failure rate of subsystem 0.005 increases from 0.04 to 0.0500, the subsystem availability decreases. Similarly, as repair rate of subsystem C increases from 0.2 to 0.5, the subsystem availability increases.

Table 5 Optimum values of Failure/Repair rates of Subsystems of Series-Parallel system

S/N	Subsystem	Failure rate λ_i	Repair rate μ_i	Maximun Availability Level
1	A	$\lambda_1 = 0.005$	$\mu_1 = 0.40$	97%
2	B	$\lambda_2 = 0.001$	$\mu_2 = 0.40$	89%
3	C	$\lambda_3 = 0.0067$	$\mu_3 = 0.50$	96%
4	D	$\lambda_4 = 0.0102$	$\mu_4 = 0.5$	89%

Table 5 helps in identifying the subsystem with maximum availability. It is observed that the first subsystem is having the maximum availability with (97%). The optimum values of failure and repair rates for maximum availability level for each subsystem is also shown in Table 5.

7. CONCLUSIONS

In this presentation,the explicit expressions for measuring the effectiveness of system such as steady state availability, MTSF, and profit function. Graphs were plotted to highlight important results. Results have shown that measures of system effectiveness such MTSF, system availability and profit increases with repair rates and decreases with failure rates.

Secondly, it can be concluded from tables 1 through 4, that as failure rate increases, the availability goes on decreasing and as repair rate increases, the availability goes on increasing. The developed model helps in determining the optimal maintenance strategies, which will ensure the maximum overall availability of the system. The optimum values of failure and repair rates for each subsystem are given in table 5.It is also

concluded that first subsystem (A) is having maximum availability. Such results are found highly beneficial to the plant management for the availability analysis of their system.
Lastly, we have considered the problem so as to find out the optimum combination of failure/repair rates, which maximizes the system availability subject to the cost and system weight. In many situations, problem parameters are more competent to take, for real life examples to validate the model developed. Hence this work gives more significant contribution for reliability engineers for decision making. For practical situation, based on decision maker's choice, several combinations of different failure and repair rate values may be considered in the reliability model.

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