Non-Linear Static and Modal Analysis of Three Types of Cable-Stayed Bridges

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Abstract
This paper is concerned about the study of cable-stayed bridges having three spans with double plane of cables. Three common types of bridge arrangement are considered - harp, fan and radiating shapes. The influence of the arrangements of cables on the bridge deformation is examined. The relationship between the sag and the other parameters of cables for each type is presented. In the static analysis, the energy method, based on the minimization of the total potential energy of structural elements, via conjugate gradient technique is used. A computer program in FORTRAN language is used in the analysis of bridge model. Natural frequencies for the considered types of cable-stayed bridge are calculated, using the SAP2000 program.
Keywords: Non-linear static analysis, cable structures, cable stayed bridges, minimization of potential energy, conjugate gradient method.

1. Introduction.

Cable stayed bridges have acquired popularity around the world as a viable bridge design for medium to long span crossings. Their aesthetic form and the fast mode of construction are some reasons behind their increasing popularity. Cable stayed bridges have greater flexibility which may occur deflection can't be expected and the experience shortage in design of this types of structures. This type of bridges consist of three main parts, the pylons, the deck and the stayed cables, each differing in inertial and stiffness properties. These three sub-systems are designed to interact to produce an efficient system of load transfer within the bridge. Smith [12] proposed a mixed displacement method to analyses single-plane cable-stayed bridge. The simultaneous equations were developed by initially assuming the girder to be supported by rigid cables then the flexibility coefficients are modified to take the extensibility of the cables, rotation and shortening of the tower into consideration. Smith extended this method to double-plan system. Linear and nonlinear analysis was studied by Lazar [10] using the stiffness matrix method. Lazar considered the nonlinear effect due to large deformations and due to catenary effect of cables. Kajita and Cheung [8] applied the finite element method on a three- dimensional cable-stayed bridge system. Baron and Lien [1] studied the linear and nonlinear behavior of the southern crossing bay bridge, in the United States as a three-dimensional structure. The cable was considered as a catenary supporting its own weight. The analysis was carried out using the nonlinear theory based on the tangent stiffness matrix which had been developed by Baron and Venkasten [2]. Brown [3] presented a method for the analysis of cable-stayed bridges using the potential energy method of a stayed girder with an iterative procedure and the girder deflection was calculated using Fourier series, neglecting the axial deformation of the girder. A combined incremental and iterative approach for the nonlinear analysis of plane cable-stayed bridges was presented by Fleming [6]. Krishna and Agarwal [9] studied the linear behavior of the three-span radiating cable-stayed bridge with different middle span arrangements. Troistsky [15] reported that cable nonlinearity can be ignored by linearization of the cable stiffness using the method of equivalent modulus of elasticity. A lot of research were done on cable-stayed bridge by researches. Some of these research are given in references [7,11]. In this paper, a cable stayed bridge has a different longitudinal layout as harp, fan and radiating is analyzed using the energy method [4] based on the minimization of the total potential energy of structure elements, via conjugate gradient technique [13,14] in order to study the influence of cable arrangement on the deformations of bridge. In the harp pattern, the cables are parallel and cross each other at a constant angle in the eye of the views gives the structure a most acceptable appearance. In the radiating type, all the stays together to the top of pylons. An intermediate solution, between the extremes of harp and fan patterns by spreading the stays in the upper part of the pylon called fan system. Axial stiffness of a cable stay in cable stayed bridges is affected by the cable sag associated with tension in cable, also this paper presents a proposed equation to calculate the relation between the sag and the other main parameters of cable for harp and fan patterns and finally, presents the behavior of the bridge model with different arrangement of cables in the case of free vibration.

2. Description of the Bridge

The bridge with the total length of 800 m comprises three spans: a 400 m central span between two pylons, and two side spans each with a length of 200 m. The bridge deck is a steel box-girder 20.8 m wide and 3.2 m high.

The vertical moment of inertia (I_x), the transverse moment of inertia (I_y), the cross-section area (A_d), and the modulus of elasticity (E_d) of the deck are 2.199 m^4, 48.95 m^4, 1.325 m^2, and 2.1×10^8 kN/m^2, respectively. The self-weight(w_d) and the equivalent live load which transferred for each plane of cables are 87.32 kN/m and 46 kN/m, respectively. The pylons consist of two parts: The lower part under the deck level consists of two legs 7.3 m×5 m with thickness 0.7 m and 40 m height, the upper part 6.7 m×3 m with thickness 0.5 m and 80 m height above the deck level. The modulus of elasticity for the concrete *(E_c)* is 3×10^7 kN/m^2. The cables have constant cross-sectional areas(A_c) of 0.01105 m^2. Numbering of cables are illustrated in Fig.1. The modulus of elasticity (E_{cs}), the breaking load (T_{ult}), and the weight per unit length (w_{cs}) of the cables are 14720 kN/cm^2, 15608.2 kN, and 0.891 kN/m, respectively. Spacing between cables along the upper part of pylon are 7 m, 2 m for harp and fan system, respectively.

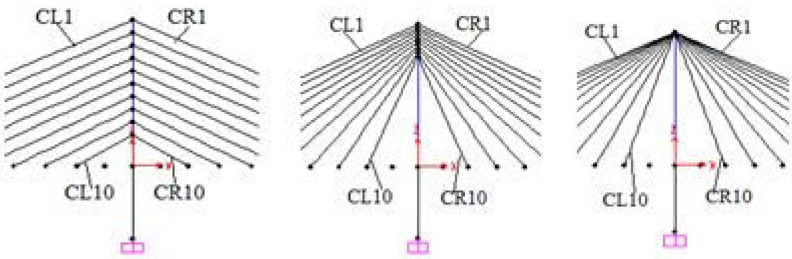


Figure 1. Numbering of cable for three types of cable arrangement (harp, fan and radiating).

3. Methodology

Energy method is a unifying approach to the analysis of both linear and non-linear structures. It is an indirect method of analysis and valid for both small and large structures. The energy method is applied to the analysis of general pin-ended truss and cable structures. The solution of a problem is by determining the structure equilibrium as an iterative process of minimizing the total potential energy (TPE); the equilibrium achieved when the TPE is minimum. The total potential energy is obtained by summing the energy contributions from each member and formulated in terms of the displacement of the nodes or joints of the structure knowing that the stable equilibrium configuration is the global minimum of the TPE function. Minimization techniques are employed to numerically locate the minimum.

The total potential energy of the bridge model is written as:

$$W=\sum_{n=1}^{f}\sum_{s=1}^{12}\sum_{r=1}^{12}\left(\frac{1}{2}x_s k_{sr} x_r\right)_n+\sum_{n=1}^{p}\left(U_\circ+T_\circ e+\frac{EA}{2L_\circ^{(1)}}e^2\right)_n-\sum_{n=1}^{N}F_n x_n .$$

Where; f — the number of flexure members, x_s or x_r — the element of displacement vector of a flexural member including effect of the pretension in the cables, K_{sr} — stiffness matrix in global co-ordinates of a flexural member, p — the number of pin-jointed members and cable links, U_o —initial strain energy in a pin-jointed member or cable link due to pretension, T_o — initial force in a pin-jointed member or cable link due to pretension, e — elongation of pin-jointed members or cable links due to applied load only, E — modulus of elasticity, A — area of the pin-jointed member or cable link, F_n— element in applied load vector , x_n — Element in displacement vector due to applied load only, N — total number of degrees of freedom of all joints.

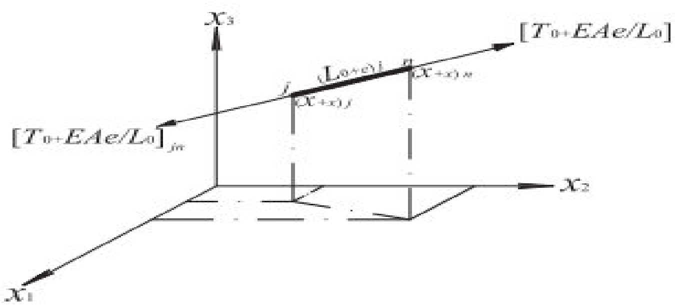


Figure 2. Coordinates and force for link (*jn*), Pretensioned and loaded cable assembly at stage of the iterative process where the extension of the link (*jn*) is e_{jn}.

Using fig.2, we can get the expression of (e) in the following equation ;

$$e_{jn} = e_\circ + e = \frac{1}{L_\circ}\left\{\sum_{i=1}^{3}\left[\left(X_{ni} - X_{ji}\right)\left(x_{ni} - x_{ji}\right) + \frac{1}{2}\left(x_{ni} - x_{ji}\right)^2\right] + \frac{L_\circ^2 T_\circ}{EA}\right\}. \tag{2}$$

The above equation gives the total elongation due to applied load and due to pretension for the cable segment jn. The minimization can take place along defined descent vector using conjugate gradient method. The point of minimum W defines the equilibrium position of the loaded structure. Mathematically, the equilibrium condition in the *i* direction at joint *j* may be expressed by:

$$\frac{\partial W}{\partial x_{ji}} = \left[g_{ji}\right] = 0 \qquad i = 1,2,3 \tag{3}$$

Where; x_{ji} —the displacement of joint j corresponding to a particular ji degree of freedom, direction, i , g_{ji}— the corresponding gradient of the energy surface. The location of minimum W is achieved by moving down the energy surface along a descent vector (v) a distance Sv until W is a minimum in that direction. The displacement vector at the $(k+1)th$ iteration as:

$$\left[x\right]_{k+1} = \left[x\right]_k + S_k v_k \tag{4}$$

Where; v_k — the descent vector at the *k-th* iteration from x_k in displacement space, and S_k — the step length determining the distance along v_k to the point of W_{min}.

4. Results and discussion

The bridge is analyzed using the energy method, based on the minimization of the total potential energy of structural elements, theoretically the minimization can take place along defined descent vector using conjugate gradient method. Calculation of the gradient vector of the total potential energy with respect to displacement is carried out at the beginning of each iteration. Initial tension in cables depending on an iterative scheme was discussed in details in literature [17]. The vertical deflection of the deck and lateral deflections of the pylons' tops are illustrated in Fig. 3,4.

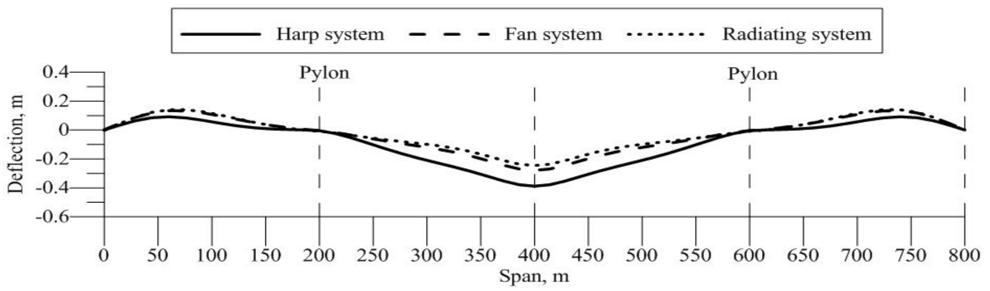


Figure 3. Vertical deflection of bridge deck.

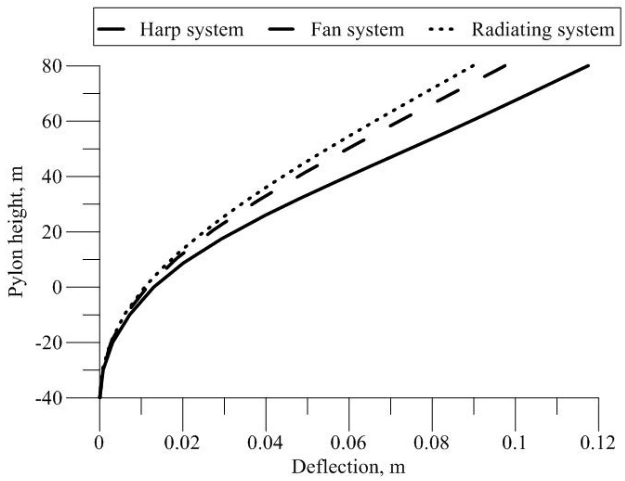


Figure 4. Lateral deflections of the pylons' tops.

As shown in Fig.2, the deflection values in the middle deck for harp, fan and radiating are 0.388 m, 0.281 m, 0.233 m, respectively. In fig.3, the lateral displacement values for harp, fan and radiating are 0.117 m, 0.097 m, 0.089 m, respectively. The cables are the most important part of cable stayed bridges, which brings a considerable difference between the cable stayed and other types of the bridges. Cables contribute a significant part to the overall stiffness of a cable-stayed bridge. More importantly, a cable exhibits strong nonlinear behavior due to the pronounced sagging effect under its own gravity weight. For cables, under the action of its own weight and axial tensile force, a cable supported at its end will sag into a catenary shape. The axial stiffness of a cable will change nonlinearly with cable tension and sagging. The equivalent modulus approach developed by Ernst [5] is the most adopted method for cable modeling in cable-stayed bridges. The following equation obtained from the analysis using different coefficients to calculate the cable sag for harp and fan system.

$$\delta = A \cdot \ln\left(\frac{E_{cs}}{1+\left(\frac{(w_{cs}\cdot l)^2 A_c \cdot E_{cs}}{12T_c^3}\cdot\right)}\cdot\frac{A_c}{l}\right) - \text{б}. \qquad (5)$$

Coefficients for harp and fan type are A=0.274, б=2.341, A=0.533,б=4.152, respectively. Fig.5 shows the variation of cable sag with the different lengths of cables up to 500 m depends on the final stresses as a percentage from 20% to 50% of the breaking stress of cables. Cable forces obtained at the end of non-linear static analysis on this study's models are given in Table.1.

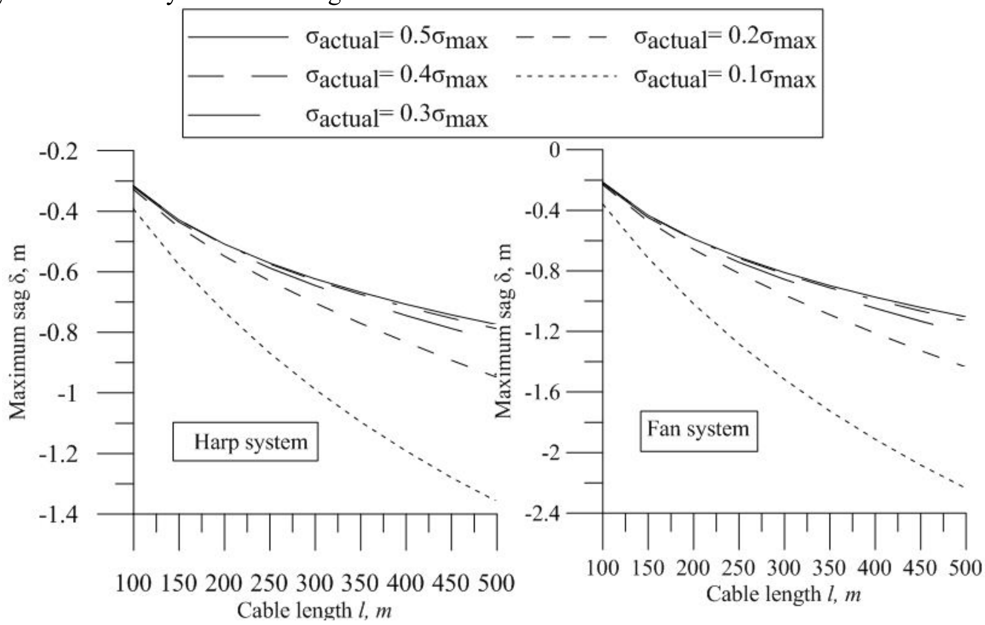


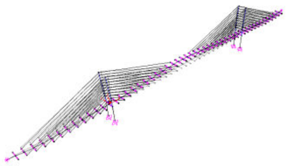
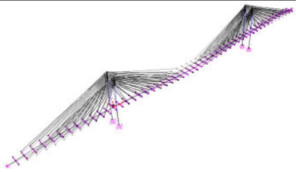
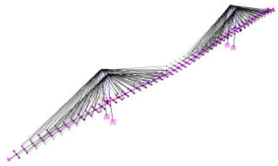
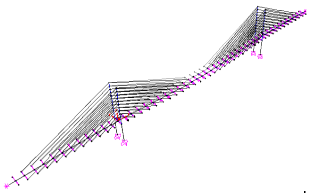
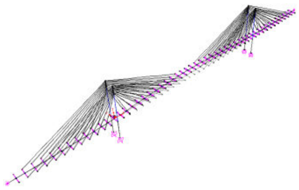
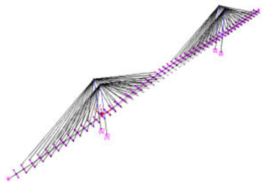
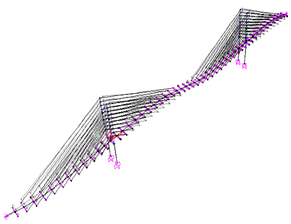
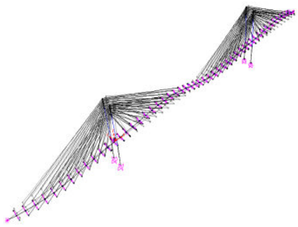
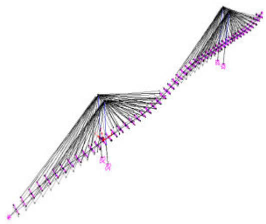
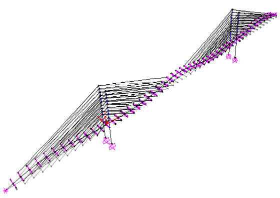
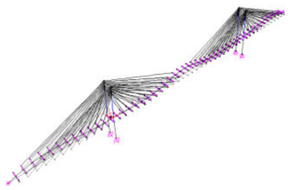
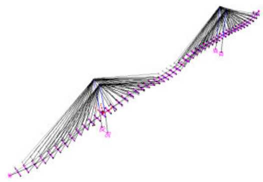
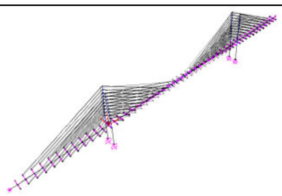
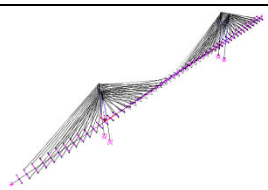
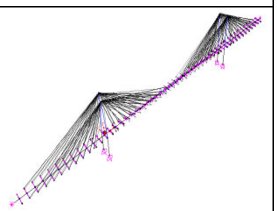
Figure 5. Sag of cables for harp and fan type.

Table 1. Cables forces obtained at the end of non-linear static analysis (kN)

Cable.No (left side)	Harp system	Fan system	Radiating system	Cable.No (right side)	Harp system	Fan system	Radiating system
CL1	6097.9	5890.0	5814.9	CR1	5983.6	5745.3	5666.1
CL2	5246.3	5000.9	4906.8	CR2	5242.7	4976.7	4876.4
CL3	4436.5	4129.6	4022.7	CR3	4504.6	4191.2	4078.6
CL4	3720.8	3354.7	3229.9	CR4	3826.7	3467.2	3337.7
CL5	3144.9	2727.1	2590.2	CR5	3261.5	2857.2	2715.1
CL6	2724.7	2269.8	2129.8	CR6	2830.8	2386.6	2239.3
CL7	2457.3	1983.3	1850.0	CR7	2535.6	2057.8	1914.7
CL8	2351.3	1901.5	1802.6	CR8	2371.0	1866.1	1736.3
CL9	2336.8	1863.2	1743.3	CR9	2320.6	1805.4	1705.1
CL10	2074.6	1679.0	1608.2	CR10	1949.9	1489.7	1440.9

According to Table 1, between the least forced cables, radiating system were less than the others. In free vibration analysis, the bridge is discredited as 3-D finite element model, using the SAP2000 program. including two types of element: catenary cable and frame elements, the deck is modeled using spine passing through its shear center. The translational and rotational stiffness of the deck are calculated and assigned to the frame elements of the spine. The cable anchorages and deck spine are connected by massless, horizontal rigid links to achieve the proper offset of the cables from the centerline of the deck [16]. Mode shapes and corresponding frequencies are illustrated in Table. 2.

Table 2. The first 5 mode shapes of three types of cable stayed bridge model

Harp system	Fan system	Radiating system
 Mode 1 (Vertical) F= 0.36046 cyc/sec	 Mode 1 (Vertical) F= 0.35786 cyc/sec	 Mode 1 (Vertical) F= 0.35458 cyc/sec)
 Mode 2 (Horizontal) F= 0.39259 cyc/sec	 Mode 2 (Horizontal) F= 0.39066 cyc/sec	 Mode 2 (Horizontal) F= 0.39037 cyc/sec
 Mode 3 (Vertical) F= 0.49243 cyc/sec	 Mode 3 (Vertical) F= 0.52552 cyc/sec	 Mode 3 (Vertical) F= 0.52791 cyc/sec
 Mode 4 (Vertical) F= 0.8148 cyc/sec	 Mode 4 (Vertical) F= 0.89009 cyc/sec	 Mode 4 (Vertical) F= 0.90335 cyc/sec
 Mode 5 (Torsion) F= 0.92013 cyc/sec	 Mode5 (Torsion) F= 0.94497 cyc/sec	 Mode 5 (Torsion) F= 0.94196 cyc/sec

5. Conclusion

The result of non-linear static analysis using energy method, showed that the smallest deflection value of deck was in radiating type and the biggest one was in harp type model. For pylons' tops lateral displacement, the smallest value was in radiating type and the biggest one was in harp. It was observed that the most forced cables were the long ones in the middle span and edge spans. The biggest cable forces were in harp type, which contributes to larger normal axial force in the deck and smallest forces were in radiating type. The finding suggests that the harp cable arrangement appears less suitable than the fan and radiating cable arrangement especially for long-span bridges. An equation is proposed for calculating the relationship between the sag and the other main parameters of cable for harp and fan type using coefficients A=0.274, б=2.341 and A=0.533, б=4.152, respectively. The result of free vibration analysis of the first twenty (20) modes examined shows that the fundamental mode is pure vertical motion of the deck for the three types of bridge. The first torsional mode occurs at the fifth mode for all types. For lower modes from 3 to 5, the natural frequencies in harp type are lower than in fan and radiating type, the harp type appears to have more torsional modes for lower frequencies. Finally, vibration of cable-stayed bridges become harmful when a predominant excitation frequency is in the vicinity of the natural frequency of the structural system, therefore, the frequency estimated can be used to determine the frequency range to be covered in a detailed dynamic analysis for wind engineering.

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