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# Certain Simultaneous Triple Series Equations Involving Laguerre Polynomials

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**Abstract**

In this paper, an exact solution has been obtained for the simultaneous triple series equations involving Laguerre polynomials by multiplying factor method.

**Keywords:** Integral Equations , Series Equations , Laguerre Polynomials

**1. Introduction**

In the present paper, an exact solution of the simultaneous triple series equations has been given

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(\alpha+ni+p+1)}L_{ni+p}^{(\alpha)}(x)=f_i(x), 0\le x<y \tag{1.1}$$

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(\alpha+ni+p+1)}L_{ni+p}^{(\gamma)}(x)=h_i(x), y<x<z \tag{1.2}$$

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} c_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{(\sigma)}(x)=g_i(x), z<x<\infty \tag{1.3}$$

i = 1,2,3 ……………,s

where, $\alpha + \beta + 1 > \beta > 1 - m$, $\sigma + 1 > \alpha + \beta > 0$, m is a positive integer, p is an arbitrary non-negative integer, $a_{ij}$, $b_{ij}$ are known constants; $f_i(x)$, $g_i(x)$, $h_i(x)$ are prescribed functions and

$$L_n^{(\alpha)}(x)=\sum_{K=0}^{n}\binom{n+\alpha}{n-k}\frac{(-x)^k}{k}, n=0,1,2,...... \tag{1.4}$$

is the Laguerre polynomial of order $\alpha$ and degree n in x.

**2. Priliminary Results**

The following results are required in our investigation :

(i) The orthogonality property of the Laguerre polynomials is given by Erdelyi (1953-54)

$$\int_0^{\infty} e^{-x}.x^{\alpha}L_m^{(\alpha)}(x)L_n^{(\alpha)}(x)dx=\frac{\Gamma(\alpha+n+1)}{n!}\delta_{m,n}, \alpha>-1; \tag{2.1}$$

where $\delta_{m,n}$ is the kronecker delta.

(ii) Formula (27), p. 190 of Erdelyi (1953-54) in the forms:

$$\frac{d^m}{dx^m}\left\{x^{\alpha+m}L_n^{(\alpha+m)}(x)\right\}=\frac{\Gamma(\alpha+m+n+1)}{\Gamma(\alpha+n+1)}x^{\alpha}L_n^{(\alpha)}(x) \qquad (2.2)$$

(iii) The following forms of the known results Erdelyi ( 1953-54 )

$$\int_0^{\xi} x^{\alpha}(\xi-x)^{\beta-1}L_n^{(\alpha)}(x)dx=\frac{\Gamma(\alpha+n+1)\Gamma\beta}{\Gamma(\alpha+\beta+n+1)}.\xi^{\alpha+\beta}.L_n^{(\alpha+\beta)}(\xi) \qquad (2.3)$$

where, $\alpha>-1,\beta>0 and$

$$\int_{\xi}^{\infty} e^{-x}(x-\xi)^{\beta-1}L_n^{(\alpha)}(x)dx=\Gamma(\beta)e^{-\xi}L_n^{(\alpha-\beta)}(\xi) \qquad (2.4)$$

where $\alpha+1>\beta>0$.

**3. Solution of Triple Series Equations**

Multiplying equation (1.1) by $x^{\alpha}(\xi-x)^{\beta+m-2}$, where m is a positive integer, equation (1.3) by $e^{-x}(x-\xi)^{\sigma-\alpha-\beta}$, integrating them with respect to x over the intervals $(0,\xi) and (\xi,\infty)$ respectively, we find, an using (2.3) and (2.4), that

$$\sum_{n=0}^{\infty}\sum_{J=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+m+ni+p)}L_{ni+p}^{(\alpha+\beta+ni-1)}(\xi) =\frac{\xi^{-\alpha-\beta-m+1}}{\Gamma(\beta+m-1)}\int_0^{\xi} x^{\alpha}(\xi-x)^{\beta+m-2}.f_i(x)dx \qquad (3.1)$$

where, $0<\xi<y,\alpha>-1,\beta+m>1,i=1,2,3,..........,s;$ and

$$\sum_{n=0}^{\infty}\sum_{J=1}^{s} bij\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{(\alpha+\beta-1)}(\xi) =\frac{e^{\xi}}{\Gamma(\sigma-\alpha-\beta+1)}\int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\alpha-\beta}.g_i(x)dx \qquad (3.2)$$

where, $y<\xi<\infty,\sigma+1>\alpha+\beta>0,i=1,2,3,.......,s.$

Now multiply equation (3.1) by $\xi^{\alpha+\beta+m-1}$, differentiate both sides m times with respect to $\xi$, and use the formula (2.2); we thus find

 

$$\sum_{n=0}^{\infty} \sum_{J=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{(\alpha+\beta-1)}(\xi)$$

$$= \sum_{J=1}^{s} e_{ij} \cdot \frac{\xi^{-\alpha-\beta+1}}{\Gamma(\beta+m-1)} \cdot \frac{d^m}{d\xi^m} \int_0^{\xi} x^{\alpha} (\xi - x)^{\beta+m-2} . f_i(\xi) dx \tag{3.3}$$

where, $e_{ij}$ , are the element of the matrix $[b_{ij}]\ [a_{ij}]^{-1}$ and $0<\xi<y$, $\alpha>-1$, $\beta+m>1$, i=1,2,3,….,s.

Now, the left hand sides of the equations (3.2), (3.3) and (1.2) are identical and hence on using the orthogonality relation (2.1) , we obtain the solution of equation (1.1), (1.2) and (1.3) in the form.

$$A_{nj} = \sum_{i=1}^{s} d_{ij} \left[ \sum_{J=1}^{s} e_{ij} \frac{\Gamma(ni+p)}{\Gamma(\beta+m-1)} \int_0^y e^{-\xi} . L_{ni+p}^{(\alpha+\beta-1)}(\xi) . F_1(\xi) d\xi + \int_y^z \xi^{\alpha+\beta-1} e^{-\xi} . L_{ni+p}^{(\alpha+\beta-1)}(\xi) h_i(\xi) d\xi \right.$$

$$\left. + \frac{(ni+p)!}{\Gamma(\sigma-\alpha-\beta+1)} \int_z^{\infty} \xi^{\alpha+\beta-1} L_{ni+p}^{(\alpha+\beta-1)}(\xi) G_1(\xi) d\xi \ \right] \tag{3.4}$$

where, n, p = { 0,1,2,………}, j=1,2,3………..,s; $d_{ij}$ are the element of the matrix $[b_{ij}]^{-1}$ and

$$F_1(\xi) = \frac{d^m}{d\xi^m} \int_0^{\xi} x^{\alpha} (\xi - x)^{\beta+m-2} f_i(x) dx \tag{3.5}$$

$$G_1(\xi) = \int_{\xi}^{\infty} e^{-x} (x-\xi)^{\sigma-\alpha-\beta} g_i(x) dx \tag{3.6}$$

provided that $\alpha+\beta+1>\beta>1-m$ and $\sigma+1>\alpha+\beta>0, m$ being a positive integer

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