

Simultaneous Triple Series Equations

Associated With Laguerre Polynomials With Matrix Argument

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Abstract

Integral and Series equations are very useful in the theory of elasticity, elastostatics, diffraction theory and acoustics. Particularly these equations are very much useful in finding the solution of crack problems of fracture mechanics. In this paper solution of simultaneous triple series equations associated with Laguerre polynomials with matrix argument has been obtained, which arises in the Crack problems of Fracture Mechanics.

Keywords: Integral equations, Series equations, Laguerre polynomials, Matrix argument.

1. Introduction

In the present paper, we have considered the following simultaneous triple series equations of the form

$$\sum_{n=0}^{\infty} \sum_{j=1}^s a_{ij} c_{nj} \Gamma_m(\alpha + \beta + ni) L_{ni}(\alpha; x) = f_i(x), 0 \leq x \leq D, \quad (1.1)$$

$$\sum_{n=0}^{\infty} \sum_{j=1}^s b_{ij} c_{nj} \cdot \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Pi_m(\alpha + \beta + ni)}$$

$$L_{ni}(\alpha + \beta - \frac{m+1}{2}, y) = g_i(x), D \leq x \leq E, \quad (1.2)$$

$$\sum_{n=0}^{\infty} \sum_{j=1}^s c_{nj} \Gamma_m(\alpha + \beta - \frac{m+1}{2}) = L_{ni}(\alpha, x) = h_i(x), E \leq x \leq \infty, \quad (1.3)$$

for $\alpha + \beta > \frac{m+1}{2} - 1, 0 \leq \beta \leq 1$

where,

$$L_n(\alpha, x) = \frac{\Pi_m(\alpha + n)}{\Pi_m(\alpha)} {}_1F_1(-n, \alpha + \frac{m+1}{2}, x)$$

is Laguerre polynomial of matrix argument ,

for $R(\alpha) > -1$,

$$R(n+\alpha) > 1, \text{ and } H_m(a) = \Gamma_m\left(a + \frac{m+1}{2}\right)$$

$$\Gamma_m(a) = \pi^{m(m-1)/4} \prod_{i=1}^m \left(a - \frac{i-1}{2}\right)$$

$n = 0, 1, 2, \dots, J = 1, 2, 3, \dots, s$,

$f(x), g(x), h(x)$ are known functions of non-singular matrix x of order m ; a_{ij}, b_{ij} and c_{ij} are known constant. By using multiplying factor technique [Srivastava], the unknown function C_{nj} is determined.

2. Some Useful Results

(i) The following integrals are required from Erdelyi et al with matrix argument

$$\int_0^y |y|^\alpha |y-x|^{B-(m+1)/2} L_n(\alpha, x) dx$$

$$= \frac{\Gamma_m(\beta) \Gamma_m\left(\alpha + n + \frac{m+1}{2}\right)}{\Gamma_m\left(\alpha + \beta + n + \frac{m+1}{2}\right)} |y|^{\alpha+\beta} L_n(\alpha + \beta; y) \quad (2.1)$$

for $\alpha > -1, \beta > \frac{m+1}{2} - 1$ and

$$\int_y^\infty |x-y|^{-\beta} \text{etr}(-x) L_n(\alpha, x) dx$$

$$= \Gamma_m\left(\frac{m+1}{2} - \beta\right) \text{etr}(-y) L_n\left(\alpha + \beta - \frac{m+1}{2}; y\right) \quad (2.2)$$

for $\beta < \frac{m+1}{2}, \alpha + \beta > \frac{m+1}{2} - 1$.

(ii) The orthogonality relation for Laguerre polynomial with matrix argument

$$\int_{x>0} |x|^\alpha \text{etr}(-x) L_p(\alpha; x) L_q(\alpha; x)$$

$$= \frac{\Gamma_m(\alpha + \beta + \frac{m+1}{2})}{\Gamma_m(\alpha + \beta + \frac{m+1}{2})} \delta_{pq} \quad (2.3)$$

for $\alpha > -1$ and δ_{pq} being the kronecker delta .

(iii) The differential formula with matrix argument due to Erdelyi et al

$$Dx[|x|^\alpha L_n(\alpha; x)] = |x|^{\alpha - \frac{m+1}{2}} L_n(\alpha - \frac{m+1}{2}; x) \quad (2.4)$$

3. Solution

Multiply eq. (1.1) by $|x|^\alpha |y-x|^{\beta - \frac{(m+1)}{2}}$ and eq. (1.2) by $|x-y|^{-\beta} \text{etr}(-x)$ and then integrating w.r.t. x over the range $(0, y)$ and (y, ∞) respectively, on using the result (2.1) and (2.2), we obtain

$$\sum_{n=0}^{\infty} \sum_{j=1}^s a_{ij} c_{nj} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Gamma_m(\alpha + \beta + ni)} L_{ni}(\alpha + \beta; y) =$$

$$\frac{|y|^{-\alpha - \beta}}{\Gamma_m(\beta)} \int_0^y |x|^\alpha |y-x|^{\beta - \frac{(m+1)}{2}} f_i(x) dx, \quad (3.1)$$

for $\beta > \frac{m+1}{2}, \alpha > -1, 0 < y < D$

and

$$\sum_{n=0}^{\infty} \sum_{j=1}^s b_{ij} c_{nj} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Gamma_m(\alpha + \beta + ni)} L_{ni}(\alpha + \beta - \frac{m+1}{2}; y)$$

$$= \frac{\text{etr}(y)}{\Gamma_m(\frac{m+1}{2} - \beta)} \int_y^{\infty} \text{etr}(-x) |x-y|^{-\beta} g_i(x) dx, \quad (3.2)$$

for $\beta < \frac{m+1}{2}, \alpha + \beta > \frac{m+1}{2} - 1, D < y < \infty$.

If we new multiply eq.(3.1) by by $|y|^{\alpha+\beta}$, differentiating w.r.t. Y and using the formula (2.4), we find

$$\sum_{n=0}^{\infty} \sum_{j=1}^s b_{ij} c_{nj} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Gamma_m(\alpha + \beta + ni)} \cdot L_{ni}(\alpha + \beta - \frac{m+1}{2}; y) = \sum_{j=1}^s e_{ij} \frac{|y|^{\frac{(m+1)}{2} - \alpha - \beta}}{\Gamma_m(\beta)}$$

$$Dy \int_0^y |x|^{\alpha} |y-x|^{\beta-(m+1)/2} f_i(x) dx, \tag{3.3}$$

for $0 < y < D$, $\beta > \frac{m+1}{2} - 1$, $\alpha > -1$, and e_{ij} are the element of the matrix $[b_{ij}][a_{ij}]^{-1}$ and $i = 1, 2, \dots, s$.

The left- hand side of eq.. (3.2), (3.3) and (1.2) are identical and hence on using the orthogonality relation (2.3), we find the solution of eq. (1.1), (1.2) and (1.3) for

$$\alpha + \beta > \frac{m+1}{2} - 1, 0 < \beta < 1$$

$$C_{nj} = \sum_{j=1}^s d_{ij} \frac{\Gamma_m(ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Gamma_m(\alpha + ni + \frac{m+1}{2}) [\Gamma_m(\alpha + \beta + ni)]^2} B_{ni}(\alpha + \beta; D) \tag{3.4}$$

Where , $n = 0, 1, 2, \dots, \infty; j = 1, 2, 3, \dots, s$ and d_{ij} are the element of the matrix $[b_{ij}]^{-1}$ and

$$B_{ni}(\alpha, \beta; D) = \sum_{j=1}^s e_{ij} \frac{1}{\Gamma_m(\beta)} \int_0^D \text{etr}(-y) L_{ni}(\alpha + \beta - \frac{m+1}{2}; y) \cdot F_i(y) dy + \int_D^E \text{etr}(-y) |y|^{\alpha+\beta-(m+1)/2} L_n(\alpha + \beta - \frac{m+1}{2}, y) G_i(y) dy + \frac{1}{\Gamma_m(\frac{m+1}{2} - \beta)} \int_E^{\infty} |y|^{\alpha+\beta-(m+1)/2} L_n(\alpha + \beta - \frac{m+1}{2}; y) H(y) dy \tag{3.5}$$

$$F_i(y) = Dy \int_0^y |x|^{\alpha} |y-x|^{\beta-(m+1)/2} f_i(x) dx \tag{3.6}$$

$$G_i(y) = g_i(y)$$

$$H_i(y) = \int_y^{\infty} |x-y|^{-\beta} \text{etr}(-x) h_i(x) dx$$

References

- T.W. Anderson ; (1958) , An Introduction to Multivariate statistical Analysis, John Wiley and Sons, New York .
- A. Erdelyi , et al ; (1953) , Higher Transcendental functions, Vol, II, Mc Graw Hill Book co, Inc; New York.
- A. Erdelyi , et al ; (1954) ,Tables of Integral Transforms , Vol, II, Mc Graw Hill Book co, Inc; New York .
- A.M. Mathoi and R.K. Saxena; (1978) , The H-function with Applications in Statistics and other Disciplines, Wiley Eastern Limited, New Delhi , India, 96-132 .
- H.M. Srivastava,; (1969) , Notices Am. Math. Soc. 16 , 568, (See also p. 517) .
- H. M. Srivastava ; (1969) , Pacific J. Math , 30 (1969) , 525 -27 .
- H.M. Srivastava ; (1970) , J. Math . Anal . Appl. 31, 587-94 (see also p.587) .

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