Fibonacci Random Number Generator using Lehmer's Algorithm

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Abstract

The uniqueness of Fibonacci sequence is been discussed with particular emphasis on its application to random number generation. The Lehmer's algorithm was employed using fibonacci prime. For multiplier a=912, initial seed $x_0 = 415$, modulus m = 28657 and multiplier a = 518, initial seed $x_0 = 211$, modulus m = 514229, we generate random numbers with full period (m-1). This suggest that higher values of Fibonacci primes with appropriate choice of a full multiplier a, modulus m (fibonacci prime) and a starting seed x_0 will produce a full period with finite countable many random numbers. A run test also indicates that the random numbers generated using modulus m as fibonacci prime are truly random

Keywords: Fibonacci sequence, Fibonacci prime, Random numbers generator, Lehmer's Algorithm, run test

1. Introduction

The design of nature has been discovered to have underlying mathematical formulation and numerical representations. One such numerical representation found in nature is Fibonacci numbers (Adam, 2006). The Fibonacci numbers are sequence of numbers generated by summing the first two numbers in the sequence to get the next. It is a deceptively simple series of numbers but it ramifications and applications are nearly limitless (Livio, 2002; Conway and Guy, 1996). The Fibonacci sequence is of interest to non-mathematicians primarily because of the possibility of using them to investigate a wide variety of problems. These numbers are researched in the area of number theory, games theory and sequence and it has continued to attract interest among mathematicians to the extent that a quarterly journal is dedicated to Fibonacci series (Hilton & Pedersen, 1994; Matthew & Fink, 2004).

A random number is a number generated by a process which outcome is unpredictable and which cannot be subsequently reliably reproduced. This definition works fine provided that one has some kind of black box, such a black box is usually called random number generator that fulfil the required task. (von Neumann, 1951) Consequently, a random number can also be defined as a number chosen by chance from some specified distribution such that selection of a large set of these numbers produces the underlying statistical distribution. Almost always such numbers are also required to be independent, so that there are no correlations between successive members. The output can be converted to random variate via mathematical transformations

Historically there are two types of random numbers generators: computer generators (also called True random number generator (TRNG) and algorithmic generators (also called Pseudo-random numbers generator (PRNG)). Pseudo random number generators are algorithm that uses mathematical formulae or simply pre-calculated table to

produce sequence of number that appear random. A good deal of research has gone into pseudo random number theory and modern algorithms for generating pseudo random number and it makes it so good that the number look exactly like they were really random (Knuth, 1997). A good example of pseudo random number generators is the linear congruential method. A linear congruential generator is a method of generating a sequence of numbers that are not actually random but share many properties with complete random numbers. (Neave, 1973 ;Ferguson, 1960).

Pseudo-random numbers generators are widely accepted because they meet the following criteria: **randomness:** It produces output passes all reasonable statistical tests of randomness; **controllability:** able to reproduce random stream of output, if desired; **portability:** able to produce the same output on a wide variety of computer systems **efficiency:** fast, minimal computer resource requirements and **documentation:** theoretically analysed and extensively tested. When used without qualification the word random usually means random with a uniform distribution, other distribution are of course possible. For example the box-miller transformation allows pairs of uniform random numbers to be transformed to the corresponding random numbers having a two dimensional distribution. It is impossible to produce an arbitrary long string of digits and prove that it is random. When generating random numbers over some specified boundary, it is often necessary to normalize the distribution so that all differential areas are equally computed (Bassein, 1996).

True random number generators (TRNG) extract randomness from physical phenomenon and introduce it into the computer. The physical phenomenon can be very simple like the little variations in the movement of a mouse or in the amount of time between key strokes. Regardless of which physical phenomenon that is used, the process of generating true random number involves identifying little unpredictable changes in the real life data.

2. An Overview of Lehmer's Algorithms

Using the note of Leemis and Park, (2006) and Shorey and Stewart, (1981) we present some basic concepts on Lehmer's algorithm. Lehmer's algorithm for random number generation is defined in terms of two fixed parameters: **modulus** m, a fixed large prime integer and **multiplier** a, a fixed integer in X_m

The integer sequence x_0, x_1, \cdots is defined by the iterative equation $x_{i+1} = g(x_i)$ with $g(x_i) = ax_i \mod m$ $x_0 \in X_m$ is called the initial seed We have that $0 \le g(x_i) < m$ because of the mod operator. However, 0 must not occur since g(0) = 0Since m is prime, $g(x) \ne 0$ if $x \in X_m$. If $x_0 \in X_m$, then $x_i \in X_m$ for all $i \ge 0$.

Note: The quality of Pseudo-Random numbers generated depends on a good choice of a (multiplier) and m (modulus). The following observations are important:

- *a* is a fixed (constant) integer in X_m also known as **multiplier**
- *m* is a large fixed prime integer also known as the **modulus**
- x_0 is the initial starting seed in X_m

- The **Mod** function ensures a value less than *m* is always generated,
- m (Modulus) is chosen to be a prime number so that a non-zero remainder always exist, that is x_i is never 0. If x_i becomes 0, then all subsequent x_i will be zero

2.1 The Modulus and Multiplier Selection

Here we discuss how to select a suitable modulus and multiplier that can generate the desired random numbers. When selecting a modulus or multiplier, the following outlined rules must be noted:

(i). The modulus m should be very large as possible $(2^{31} - 1)$ is a good value for modulus m).

(ii). The modulus must be a prime number in other to avoid the occurrence of zero which subsequently causes x_i to be zero.

(iii). The multiplier *a* should be chosen to guarantee a full period multiplier.

Theorem 1

If the sequence x_0, x_1, x_2, \cdots is a produce by Lehmer's generator with multiplier *a* and modulus *m* then $x_i = a^i x_0 \mod m$

Proof

We know that $b \mod a = b - [b/a]a$, then there exist a non-negative integer $c_i = [ax_i/m]$ such that $x_{i+1} = g(x_i) = ax_i \mod m = ax_i - mc_i$ Therefore (by induction), we have that $x_1 = ax_0 - mc_0$ $x_2 = ax_1 - mc_1 = a^2x_0 - m(ac_0 + c_1)$ $x_3 = ax_2 - mc_2 = a^3x_0 - m(a^2c_0 + ac_1 + c_2)$ $\vdots \qquad \vdots \qquad \vdots$ $x_i = ax_{i-1} - mc_{i-1} = a^ix_0 - m(a^{i-1}c_0 + a^{i-2}c_1 + \dots + c_{i+1})$ since $x_i \in \chi_m$, we have that $x_i = x_i \mod m$ Therefore letting $c = a^{i-1}c_0 + a^{i-2}c_1 + \dots + c_{i+1}$, we have that $x_i = a^ix_0 - mc = (a^ix_0 - mc) \mod m = a^ix_0 \mod m$ Hence $x_i = a^ix_0 \mod m$ Note: We do not compute x_i by first computing a^i , this is a wrong approach.

The result of Theorem 1 has a significant theoretical value.

2.2 The Period of the Sequence

Consider sequence produced by $x_{i+1} = a \cdot x_i \mod m$, once a value is repeated, all the sequence is then repeated. That is the sequence: $x_0, x_1, x_2, \dots, x_i, \dots, x_{i+p}$ where $x_i = x_{i+p}$. *p* is the period, that is the number of elements before the first repeat. Clearly we see that $p \le m-1$

It can be shown, that if we pick *any* initial seed x_0 , we are guaranteed this initial seed will reappear.

Theorem 2

If $x_0 \in \chi_m$ and the sequence x_0, x_1, x_2, \cdots is produced by the Lehmer's generator $x_{i+1} = a \cdot x_i \mod m$ with multiplier a and (prime) modulus m, then there exist a positive integer p with $p \le m - 1$ such that: (i). $x_0, x_1, \cdots, x_{p-1}$ are all different and (ii). $x_{i+p} = x_i$, $\forall i = 0, 1, 2, \cdots$

... (*.1)

Proof

We know from modulo arithmetic that

 $(b_1, b_2 \dots b_n) \mod a = (b_1, \mod a)(b_2, \mod a) \dots (b_n, \mod a)$

Therefore $x_i = a^i \cdot x_0 \mod m = (a^i \mod m) x_0 \mod m$

From Fermat's Little theorem, which states that if p is a prime which does not divides a, then

 $a^{p-1} \mod p = 1$

Then $x_{m-1} = (a^{m-1} \mod m) x_0 \mod m = x_0 \qquad \dots \qquad \dots \qquad \dots \qquad (*.*)$

From (*.*), we have a more defined generalization, thus $x_{i+p} = (a^{i+p} \mod p) x_i \mod m = x_i$

 $\Rightarrow x_{i+p} = x_i$ Hence the proof

Note:

- 1. Ideally, the generator cycles through all values in χ_m to maximize the number of possible values that are generated, and guarantee any number can be produced.
- 2. The sequence containing all possible numbers is called a **full-period sequence** (p = m 1).
- 3. Non-full period sequences effectively partition χ_m into disjoint sets, each set has a particular period (not full period).

2.3 Determining if **a** is a full period Multiplier

We present the following Algorithm for finding if p is a full period.

p = 1;

x = a; // assume, initial seed is $x_0 = 1$, thus $x_1 = a$

Do

 $x=(a * x) \mod m$ {// cycle through numbers until repeat//}

p = p + 1 {careful: overflow possible}

Until $x = x_0$

If p = m - 1

Writeln (*a* is a full period multiplier)

Else

Writeln (a is not a full period multiplier)

End if

3. Numerical Experiments

The following numerical experiments show how random numbers are generated using Lehmer's algorithm of the formula $x_{i+1}=a * x_i \mod m$, considering the multiplier a, the modulus m and the initial seed x_0 . In each experiment we generate values for $x_0, x_1, x_2, \cdots, x_i, x_{i+1}$ after making a choice of fibonacci prime as our values for m

Experiment 1 $(m = 28657, a = 912, x_0 = 415)$ – Five digit fibonacci prime (m)

Experiment 2 (m = 514229, a = 518, $x_0 = 211$) – Six digit fibonacci prime (m)

The two numerical experiment above produce a full period sequence since p = 28657 (first experiment) and p = 514229 (second experiment) therefore a = 912 and a = 518 are full period multiplier respectively.

3.1 Tests for Randomness

We apply the run test to test the null hypothesis that randomness does not exist in the number generated. Consider a sequence of numbers made up of two set, c and d, where c represent the corresponding random numbers generated when it is less than the average and d represent the corresponding random numbers generated when it is greater than the average.

Suppose we form all possible sequences consisting of N_1c 's and N_2d 's, for $N_1 + N_2 = N$ and V is the total number of runs, then by using the formula

$$\mu_{\nu} = \frac{2N_1N_2}{N_1 + N_2} + 1 \qquad \dots \qquad \dots \qquad (1)$$

$$\delta_{\nu}^{2} = \frac{2N_{1}N_{2}(2N_{1}N_{2}-N_{1}-N_{2})}{(N_{1}+N_{2})^{2}(N_{1}+N_{2}-1)} \qquad \dots \qquad \dots \qquad (2)$$

When N is relatively large (>20) the distribution of V is approximately normal and thus

$$Z = \frac{V - \mu_v}{\delta_v} \sim N(0, 1)$$
 ... (3)

We can test the null hypothesis at the appropriate level of significance using equation (3)

We have that for the first experiment

$$N_1 = 14329$$
, $N_2 = 14328$, $N = 28657$, $V = 14448.9$

We have that $\mu_v = 14329.5$ and $\delta_v = 84.6404$ and $Z_{cal} = 1.41076$, $Z_{Table} = 1.96$, for $\alpha = 0.05$ level of significance.

Therefore we reject the null hypothesis and conclude that randomness exist in the random numbers generated since Z_{cal} (Test statistics) $< Z_{Table}$ (Critical value)

Using the same approach above, for the second experiment we test for the null hypothesis and conclude that randomness exist in the sets of random number generated.

4 Results and Discussion

From the result of this work, we have shown that a five and six digit fibonacci prime with appropriate choice of full multiplier a, modulus m (fibonacci prime) and a starting seed x_0 will produce a full period. Table 1 shows the first 600 random numbers generated from the 28, 657 that was generated and Table 2 shows the first 570 random numbers generated from the 514, 229 that was generated. A full period guarantees randomness and a longer length of random numbers sets. The longer the digit of the fibonacci prime, the better and more random the numbers generated will be. Further research should be able to show clearly that not all fibonacci prime will generates a full period no matter the choice of full multiplier a, and a starting seed x_0 .

415	27806	9479	4869	2238	2247	27571	984	8133	25721	7885	12488	8159	6540	13865	14036	8023	18696	11242	1530
5939	26284	19091	27350	6409	14617	12563	9041	23790	16126	26870	12227	18845	3824	7143	19810	9441	28494	22155	19824
195	13756	16193	11610	27637	5199	23313	20833	3131	5871	3705	3451	21097	19991	9277	12810	13092	23286	2175	25578
5898	22363	9661	13887	15441	13083	26619	105	18429	24150	26091	23699	11617	5940	6809	19321	18592	1995	6267	338
20117	19929	13133	27207	11605	10384	4049	9789	14246	16224	9682	6110	20271	1107	19896	25354	19617	14049	12761	21686
6224	6710	27327	24479	9327	13398	24592	15241	10731	9276	3628	12862	3387	6589	5271	25306	8736	3009	3290	4302
2202	15579	19291	1045	23752	11094	18130	1147	14635	5897	13181	9431	22645	19855	21433	10187	586	21793	20152	26072
2234	22833	26651	7359	25789	1807	28128	14412	21615	19205	13789	3972	19200	25193	2822	5676	18606	15915	9487	21011
2761	18714	4576	5670	20828	14535	4721	18838	25521	5533	23802	11682	973	21759	23191	18252	3728	14038	26387	19156
24873	16253	18047	12780	24202	16386	7002	14713	5668	2464	14075	22237	27666	13564	1326	24764	18410	21634	21721	18159
16489	7067	9746	20618	6334	13735	23970	6780	10956	11922	26721	19645	13232	19201	5718	3052	25575	14192	7565	25919
21700	25936	4682	4624	16551	3211	24006	22105	19236	11861	11102	5615	2987	1885	27899	3695	26259	18797	21600	24760
17070	11607	91	4509	20930	5418	28181	13889	5148	13543	9103	19934	1729	28357	25129	16971	19613	5978	11841	28061
7089	11151	25678	14257	2598	12212	24400	374	23885	49	20063	11270	713	12970	20705	2772	5088	7106	23960	931
17343	25134	5567	20763	19502	18428	14968	25861	3800	16031	14290	19034	19802	21956	26654	6248	26479	4190	14886	18019
26809	25265	4815	22236	18484	13334	10084	521	26760	5202	22202	21523	5514	21286	7312	24090	19654	9899	21271	12867
5387	1452	6759	18733	7092	10040	26368	16640	18013	15819	16382	27588	13793	12043	20120	18818	13823	933	27020	13991
12597	6002	2953	4924	20079	14897	4393	16127	7395	12457	10087	28067	27450	7585	8960	25130	26153	19843	25877	7427
25664	337	28035	20196	225	2646	23093	6783	9845	12612	447	6403	16839	11183	4275	21617	8912	14249	15113	10372
21456	20774	5876	20958	4601	5964	26578	24841	8999	10687	6466	22165	25673	25661	1448	27345	17813	13467	27696	2454
23798	3611	53	28134	12190	22995	23971	15962	11186	3164	22307	11295	1007	18720	2354	7050	25594	16708	11935	2802
10427	26334	19679	10193	27021	23173	24918	28245	28397	19868	26171	13177	1360	21725	26230	10432	14930	20829	23717	4951
23957	2042	7966	11148	26789	13567	215	25454	20793	8392	25328	10141	8069	11213	21822	28517	4085	25114	22526	16163
12150	28256	14771	22398	15804	21937	24138	1878	20939	2085	1594	21038	22736	24364	13706	15605	110	7025	25300	10958
19198	6829	2362	23192	27434	3958	5280	21973	10806	10158	20878	15123	16221	10793	5420	17888	14349	16289	4715	21060
6530	23361	13081	8560	12016	11618	21183	4078	22383	9512	20530	10339	1015	8656	13597	20640	24688	19711	8493	8226
22635	10080	22720	1629	24141	8016	3057	8255	20426	1462	15122	7247	18154	21359	21305	714	20714	6205	13531	17762
7739	8346	17447	7029	19937	14006	21107	20737	27181	767	11736	14171	28302	20124	12608	7039	400	20916	18487	9828
22152	28096	4194	13547	3697	18795	4154	5724	4714	618	19133	25840	10026	2129	21629	9632	15342	7288	26889	21033
10563	4704	20155	12223	28460	20935	7158	22957	17174	15966	3236	28218	830	11878	390	11796	11577	12448	4404	4468

Table 1: Showing the first 600 random numbers from the 28, 657 that was generated $(m = 28657, a = 912, x_n = 415)$

Table 2: Showing the first 570 random numbers from the 514, 229 that was generated $(m = 514229, a = 518, x_0 = 211)$

109295 226403 226495 52318 28642 201021 411417 129705 442295 404213 339413 17271 11804 118425 58809 15832 200476 330023 6381 51174 122025 501981 549615 173404 23728 24526 337420 276905 231711 22064 507861 45149 10599 48113 24256 455265 217212 24692 772665 36714 269618 192774 107189 15387 29671 54449 347365 337463 35651 24746 444068 30506 45331 86467 46531 69077 465295 27736 6971 137564 415360 172176 496651 13506 13210 192674 422335 43518 13735 13201 192674 422335 43518 13735 13201 192674 422335 43518 13735 13201 192674 422335 435181 451715 137392																-			
51174 122052 80198 360816 17240 227781 245826 337420 216905 251711 2006 507861 458490 100899 45133 24236 486339 300306 17243 28243 456998 40424 237561 334968 317783 323305 459929 451128 286361 112923 300979 37132 218243 460274 355338 160330 202613 340685 9141 269418 329274 071189 153847 293571 17126 49326 53653 224746 440033 335051 453529 77132 47732 17952 164235 16205 17764 16295 277582 40733 19135 30714 34037 458075 278760 172176 49555 197960 51416 458733 93231 31375 15323 131375 15323 131343 440433 343103 449833 317385 15323 141611 141611 141611<	211	392581	398518	101	119679	475901	202328	54850	369153	355182	418629	23166	496588	261314	503384	488725	266436	513912	316801
282453 486998 404244 237561 354968 317793 323305 459929 44112 286361 112925 300979 37152 2874 248366 212752 34992 272665 367441 269618 192754 107189 155867 293671 63494 347555 155195 337468 237046 365237 95735 218263 460274 355338 160330 203261 340688 9144 19818 330714 39007 98109 68131 43007 488078 273700 147361 181846 457706 173764 182992 277352 40733 19844 17325 450501 172176 496551 17980 52316 513210 192674 42233 438191 469979 1704 109896 89798 102451 340695 133445 441031 339765 413506 153210 192674 42233 438191 469979 1704 109996 89728	109298	236503	226495	52318	286242	201027	417417	129705	442295	404523	359413	172721	118084	118425	38809	158882	200476	350023	63867
269618 292754 107189 15567 29371 61494 347365 155195 337468 237046 386237 95735 218265 460274 35538 160530 20261 340688 9144 306065 463246 501399 5153 424023 493465 58520 171286 484793 403526 32645 224746 444083 333905 453331 363671 384452 95837 6921 15684 71995 164235 426020 324286 19179 31785 11329 413500 171216 499563 197960 82416 485783 92210 32139 19877 40483 317385 15333 511299 61722 23457 74119 340453 350739 25851 154353 35433 13445 25858 144002 50616 44705 222302 207949 218505 37444 422709 234754 104031 99981 56317 147281 159965	51174	122052	80198	360816	175404	257728	245826	337420	276905	251711	25036	507861	488490	150899	48131	24236	486539	303306	172450
106065 463246 5135 424025 493465 5520 171286 484799 403326 35652 224746 444053 333905 45333 363671 386482 93537 6921 159138 330714 39037 98109 68131 43097 485073 278760 179022 143894 486815 202674 174531 181846 457706 173764 162395 277582 40733 131239 61722 233487 74119 341494 43525 220010 304816 223551 110071 10481 113506 313210 19274 422335 438191 469979 17044 422709 23754 10401 99981 56317 147281 139962 297476 107053 432535 14355 24856 147022 100616 44762 14704 42913 24371 210610 9817 419937 24452 406242 36725 375382 18566 71001 338097	282453	486998	404244	237561	354968	317793	323305	459929	481128	286361	112923	300979	37152	2874	248866	212752	54992	272663	367483
159135 330714 39037 95106 68131 43037 458075 278760 179022 145894 458515 202074 174631 151846 457706 173764 162392 277582 40733 156544 71595 166235 426020 324286 191679 337965 413560 172176 495635 197960 \$2416 468783 92321 32139 19677 404883 317385 15320 511239 61722 23487 74119 341494 43235 223501 304316 222551 150373 211709 10481 113306 513210 192674 422335 438191 469979 1704 420709 234754 104031 99981 56317 147281 159965 297176 107056 405302 24315 499672 142961 147672 17403 42313 443173 74432 44313 143873 7432 443173 74432 443131 143897 7456 3221	269618	292754	107189	155867	293671	63494	347565	155195	337468	237046	386237	95735	218263	460274	355338	160530	203261	340688	91464
15844 7159 166235 426000 334286 19177 33765 413560 171176 495655 197960 82416 465783 92321 32139 19677 404883 317385 15320 511239 61722 233487 74119 341494 43525 225010 304816 225551 150573 211709 10481 113506 513210 192674 422335 438191 469979 17047 506096 89798 100431 340696 51314 131717 107054 405205 24754 44706 222205 207949 218505 37444 41597 44524 406242 367255 375382 18566 10101 330074 422035 49527 442261 145752 44512 45173 74432 44122 415970 21274 459145 141632 287706 270900 46628 486223 35144 166940 258402 178942 40916 262065 26521	306065	463246	501399	5153	424023	493465	58520	171286	484793	403326	35685	224746	444083	333905	485331	363671	386482	95837	69284
511239 61722 233457 74119 34494 43525 22801 304816 22551 13075 211709 10481 113306 513210 192674 422335 438191 469979 17047 208096 89798 102451 340696 513345 434103 350739 26385 105435 348235 134485 286868 174002 500616 44706 222205 207949 218805 37444 422709 234754 104031 99981 56317 147281 159965 297476 107056 405580 242315 499672 142961 147672 17403 429123 243721 210610 9817 415937 244525 403242 36715 159614 49133 14165 265229 29638 34797 50421 329529 91016 440580 26133 44522 178942 40916 262652 2622 491533 245715 156019 81746 495493 142921 9158 419533 13516 303670 54445 152696 130736	159138	330714	39037	98109	68131	43057	488078	278760	179022	145894	486815	202674	174631	181846	457706	173764	162595	277582	407311
508096 89798 102451 346966 513345 434103 550739 26835 105435 344235 134455 258686 174002 500616 44706 222205 207949 218803 37444 422709 234754 104031 99981 56317 147281 19965 297476 107036 405380 242315 499672 142961 147672 17403 429122 243721 210610 9817 415937 244528 408242 367258 37382 185666 71001 338097 432505 285008 47294 172909 4822 388204 272861 138786 261373 79432 46124 507644 165170 121237 489143 69314 14165 268259 296386 347973 50421 329529 91016 440850 261373 443252 413317 148987 7456 322121 185373 196045 64528 37546 469498 149211 3136	156844	71595	166235	426020	324286	191679	337965	413560	172176	495658	197960	82416	468783	92321	32139	19677	404883	317585	153208
422709 234754 104031 99981 56317 147281 19965 297476 107056 405580 242315 499672 142961 147672 17403 429123 244571 210610 9817 415937 244528 406242 367258 375382 185666 71001 338097 432205 285008 47294 172909 4822 385204 272861 138786 261373 79452 46124 307644 165170 121237 489143 69314 14165 268259 296386 347975 50421 329529 91016 440830 26133 443252 413317 148987 7456 32211 185573 196045 64828 375406 422219 95183 419527 459123 313643 405533 13516 305670 84448 152696 130736 111099 272934 649585 481066 47373 132543 78269 239266 459938 405458 50672	511239	61722	233487	74119	341494	43525	228010	304816	225551	150573	211709	10481	113506	513210	192674	422335	438191	469979	170478
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5.

Conclusion

The uniqueness of Fibonacci sequence is been discussed with particular emphasis on its application to random number generation. The Lehmer's algorithm was employed using Fibonacci prime on a 32 bit machine. Two set of numerical experiments were carried out using a five and six digits fibonacci prime. Both experiments produce large sets of random numbers with full periods. Higher digits fibonacci primes could be studies for randomness and implementation. We suggest that further research be made to devise algorithms that help in finding the appropriate choice of full multiplier a, modulus m (fibonacci prime) and a starting seed x_0 that will produce a full period.

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