# On The Performance of the Logistic-Growth Population Projection Models

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**Abstract**
In this paper, we proposed two mathematical Models for population projection, the exponential growth and the Logistic Model. Thereafter, both Models were transformed and since Logistic Model is not linear in its parameters hence a surgery was performed on the data values separated by a fixed time say τ. This enables the Logistic parameters to be sometimes estimated by Least-Squares. It was shown that the estimates $\hat{z}$, $\dot{z}_1$ and $\tilde{z}_1$ were quite close to the data set from the National Population Commission of Nigeria.

**Keywords:** Carrying capacity, Logistic Model, Exponential growth, Polyfit, Least- Squares, Population Projection.

## 1.1 Introduction

One important area where data collection for human population is so vital is basically for national planning or budgeting. This is while government of nations placed emphasis on Census to determine the population count. Though, this exercise is quite expensive, time consuming and yet is still one of the major ground for population count. This explains the seriousness attached to Census by government for developmental purposes.

Population data gives an insight about the Social- economic conditions of a country; hence the need for projection is usually based on mathematical models which are significant. These models use vital assumptions about its parameters values that reflect the true nature of the population for which these assumptions were made.

Choi (2010) proposed an application of the component model to development of local population projections derived from the housing unit method. Haque et al. (2012) analyzed fourth order Rung-Kutta Scheme for the numerical of the non-autonomous and non-linear model to incorporate the growth rate as a function of time. Wali et al. (2012) focuses on the application of logistic equation to model the population growth of Uganda using data from 1980 to 2010. Earterling et al. (2000) introduced the integral projection model which eliminates the need for division into discrete classes, without requiring any additional biological assumptions. National Population Commission of Nigeria (2005), National Policy on Population for Sustainable Development. Neverova and Frisman (2012) investigate a two-component model of population dynamics with seasonal reproduction. Akamine and Suda (2011) analyzed the population projection matrix models in random walk models and the growth rate of the mean population size, which is equal to the maximum eigenvalue of the mean matrix, is better than the average of the intrinsic rates of natural increase calculated by computer simulation. Simpson reviews the mechanisms which can incorporate into a cohort component population projection incomplete and estimated data since the base year, as well as targets for population, housing and employment.

## 2.1 The Growth Model

Consider the differential equation,

$\frac{dy}{dt} = ry$……………………………………………………………….(1)

$\frac{dy}{y} = rdt$……………………………………………………………...(2)

$y(t) = e^{rt+c}, where\ e^c = A$ ………………………………………….(3)

$At\ t = 0, y(0) = Ae^{r(0)}$………………………………………… …...(4)

$A = y(0) = y_0$ is the initial population

$\therefore\ y(t) = y_0 e^{rt}$……………………………………………………..(5)

From equation (5)

$\log y = rt + \log A$ ……………………………………………………..(5a)

$=> \log y = rt + b, where \log A = b$ , $\log y = \frac{1}{z}$

$\log y = mt + b$ ……………………………………………….........(5b)

Equation (5) gives the solution to the differential equation (1) and equation (5b) is Linear in its parameters. Equation (5) gives the population size accurately after some time t, thereafter the population goes unbounded resulting to explosion in the population. Hence, we now consider the Logistic Model.

**2.2 Logistic Model**

Consider the model by Verhulst in 1845, Yeargers et al. (1996),

$\frac{dy}{dt} = ry(1 - \frac{y}{k})$ ………………………………………………………..(6)

Where, y = population size

$\frac{dy}{dt} = per\ capita\ growth\ rate$

K = demographic indicator (Carrying Capacity)

r = growth Rate.

It is easy to see that $\frac{dy}{dt} = 0$, when y = 0 or y = k. These are the stationary points of the equation (6). The stationary point y = k, at which the per capita growth rate becomes zero is called the carrying capacity of the environment.

**2.3 Solution to the Model**

$\frac{dy}{dt} = ry(1 - \frac{y}{k})$ ……………………………………………………….(6)

$\frac{dy}{y(1-\frac{y}{k})} = rdt$……………………………………………………………(7)

Decomposing the L.H.S of equation (7) by partial fraction

$\left(\frac{1}{y} + \frac{\frac{1}{k}}{\left(1-\frac{y}{k}\right)}\right) dy = rdt$…………………………………………........(8)

$\frac{y}{1-\frac{y}{k}} = Ae^{rt}$…………………………………………………….....……( 9)

Reciprocating both sides,

$\frac{1}{y} - \frac{1}{k} = \frac{1}{Ae^{rt}}$…………………………………………………………(10)

Isolating y,

$y = \frac{Ae^{rt}}{1+\frac{Ae^{rt}}{k}}$ ………………………………………………………….....(11)

Equation (12) is the solution to the differential equation (6).But, equation (6) with initial population $y_0 = y(0)$ can be obtained.

$y\ (t) = \frac{k}{1+\frac{e^{-rt}(k-y_0)}{y_0}}$ ………………………………………………….(12)

$\therefore\ y(t) = \frac{ke^{rt}y_0}{k+y_0\,(e^{rt}-1)}$ ………………………………………………….(13)

Logistic parameters can sometimes be estimated by Least Squares. Equation (12) is not Linear in its parameters A, r and K. If the data values are separated by fixed time $\tau$, then Least Squares can be applicable.

Suppose the data points are $(t_1, y_1), (t_2, y_2), \ldots, (t_n, y_n)$ $with\ t_i = t_{i-1} + \tau, i = 2, \ldots, n. Then\ t_i = t_1 + (i-1)\tau$ and the predicted value of $\frac{1}{y_i}$ from equation (10)

$\frac{1}{y_i} = \frac{1}{Ae^{rt_1}e^{(i-1)rt}} + \frac{1}{k}$ ……………………………………………………...(14)

$= \frac{1}{e^{rt}}(\frac{1}{Ae^{rt_1}e^{(i-2)rt}} + \frac{e^{r\tau}}{k})$ ………………………………………………(15)

$but, \frac{e^{rt}}{k} = \frac{e^{rt}-1}{k} + \frac{1}{k}$ ……………………………………….………(16)

$where, \frac{1}{y_{i-1}} = \frac{1}{Ae^{rt_1(i-2)r\tau}} + \frac{1}{k} => \frac{1}{Ae^{rt_1(i-2)r\tau}} = \frac{1}{y_{i-1}} - \frac{1}{k}$ ……….……...(17)

Substitute equation (17) into equation (15)

$\frac{1}{y_i} = \frac{1}{e^{rt}}(\frac{1}{Ae^{rt_1}e^{(i-2)rt}} + \frac{e^{r\tau}}{k})$

$\frac{1}{y_i} = \frac{1}{e^{rt}}(\frac{1}{y_{i-1}} - \frac{1}{k} + \frac{e^{r\tau}}{k})$ ……………………………………………..(18)

$\frac{1}{y_i} = \frac{1}{e^{rt}}(\frac{1}{y_{i-1}} + \frac{e^{r\tau}-1}{k})$ …………………………………………….........(19)

$\frac{1}{y_i} = \frac{1}{e^{rt}}.\frac{1}{y_{i-1}} + \frac{1-e^{-r\tau}}{k}$ ……………………………………...……….(20)

Set $z = \frac{1}{y}$ in equation (20)

$z_i = \frac{1}{e^{rt}}.z_{i-1} + \frac{1-e^{-r\tau}}{k}$ ……………………………………..………..(21)

$\therefore\ z_i = e^{-r\tau}.z_{i-1} + \frac{1-e^{-r\tau}}{k}$ ………………………………..….........(22)

Therefore, a Least-Square is performed on the points, $(z_1, z_2), (z_2, z_3), \ldots, (z_{i-1}, z_i)$ to determine r and k. With r and k known, A can be determined from equation (10) to obtain an estimate of z.

From equation (22), set $\rho = e^{-r\tau}$ $and$ $\beta = \frac{1-e^{-r\tau}}{k}$,

$=> z_i = \rho z_{i-1} + \beta$…………………………………………………(23)

We shall perform Least-Squares on equation (23) with the set of data $(z_0, z_1), (z_1, z_2), \ldots, (z_{i-1}, z_i)$ using MATLAB inbuilt function.

**3.1 Analysis of Model for Data I**

**CASE 1:** Consider the solution from equation (5b)

$\log y = mt + b$ and applying MATLAB inbuilt function on the set of data for the population projection system of Male in KANO STATE, NIGERIA.

Applying MATLAB inbuilt function to estimate equation (5b)

```
>> t=[0 1 2 3 4 5 6 7 8 9 10 11];
>> z=[4947952 5113958 5285534 5462867 5646149 5835580 6031367 6233723     6442867 6659029
6882443 7113353];
>> logy=(z);
>> p=polyfit(t,logy,1)
p =
  1.0e+006 *
    0.1965    4.8903
```

$where\ m = 196500\ and\ b = 4890300$

$=> \hat{z} = mt + b$

$\therefore\ \hat{z} = 196500t + 4890300$ …………………………………………...(24)

**CASE 2:** Consider the solution from equation (23)

$z_i = \rho z_{i-1} + \beta$ ……………………………………………………….…...(23)

applying Least-Squares on $(z_0, z_1), (z_1, z_2), \ldots, (z_{i-1}, z_i)$ and using MATLAB inbuilt function on the set of data for the population projection system of Male in KANO STATE, NIGERIA.As in Table 1.

$set\ z_{i-1} = x\ and\ z_i = y\ (dummy\ variables)$

```
>> x=[4947952 5113958 5285534 5462867 5646149 5835580 6031367 6233723 6442867 6659029 6882443];
>> y=[5113958 5285534 5462867 5646149 5835580 6031367 6233723 6442867 6659029 6882443 7113353];
>> p=polyfit(x,y,1)
p =
   1.0336   -0.8600
>> xp=4947952:10000:6882443;
>> zp=polyval(p,xp)
>> plot(x,z,'o',xp,zp)
```

From equation (23)

$\dot{z}_i = 1.0336\dot{z}_{i-1} + (-0.8600)$ ……………………………………….(25)

Recall, $\rho = e^{-r\tau}$ $and$ $\beta = \frac{1-e^{-r\tau}}{k}$

$r = -0.0330, k = 0.0390$ with r and k known A can be determined from equation (10).

**CASE 3:** Consider equation (10)

$at\ t\ \frac{1}{y} = \frac{1}{Ae^{rt}} + \frac{1}{k} => z_i = \frac{e^{-rt}}{A} + k^{-1}$ ……………………………….(10)

$= 0, z_0 = 4947952, k^{-1} = 25.63297102$ $and$ $r = -0.0330$

$\therefore \frac{1}{A} = 4947926.367$

From equation (10), we have

$\tilde{z}_i = 4947926.367e^{-(-0.0330)t} + 25.63297102$ ………………..…….(26)

From equation (24), (25) and (26) we have,

$\hat{z} = 196500t + 4890300$ …………………………………………......(24)

$\dot{z}_i = 1.0336\dot{z}_{i-1} + (-0.8600)$ ……………………………………..(25)

$\tilde{z}_i = 4947926.367e^{-(-0.0330)t} + 25.63297102$ …………………......(26)

A projection can be obtained from the three equations above to estimate the given data z.

**3.2 Analysis of Model for Data II**

**CASE 1:** Consider the solution from equation (5b)

$\log y = mt + b$ and applying MATLAB inbuilt function on the set of data for the population projection system for Male and Female in KANO STATE, NIGERIA.

Applying MATLAB inbuilt function to estimate equation (5b)

```
>> t=[0 1 2 3 4 5 6 7 8 9 10 11];
>> z=[9401288 9716706 10042707 10379645 10727887 11087814 11459816
       11844299 12241682 12652397 13076891 13515628];
>> logy=(z);
>> p=polyfit(t,logy,1)
p =
  1.0e+006 *
   0.3734    9.2917
```

$where\ m = 373400$ $and$ $b = 9291700$

$=> \hat{z} = mt + b$

$\therefore \hat{z} = 373400t + 9291700$ …………………………………..…….(27)

**CASE 2:** Consider the solution from equation (23)

$z_i = \rho z_{i-1} + \beta$ ………………………………………………….......(23)

 

applying Least-Squares on $(z_0, z_1), (z_1, z_2), \ldots, (z_{i-1}, z_i)$ and using MATLAB inbuilt function on the set of data for the population projection system for both Male and Female in KANO STATE, NIGERIA.As in Table 3.

$set\ z_{i-1} = x\ and\ z_i = y\ (dummy\ variables)$

```
>> x=[9401288 9716706 10042707 10379645 10727887 11087814 11459816   11844299 12241682 12652397
13076891];
>> y=[9716706 10042707 10379645 10727887 11087814 11459816 11844299  12241682 12652397 13076891
13515628];
>> p=polyfit(x,y,1)
p =
   1.0336   -0.2401
>> xp=9401288:10000:13076891;
>> yp=polyval(p,xp)
>> plot(x,y,'o',xp,yp)
```

From equation (25),

$\dot{z}_i = \rho z_{i-1} + \beta$

$\dot{z}_i = 1.0336 z_{i-1} - 0.2401$……………………………………………….(28)

Recall, $\rho = e^{-r\tau}\ and\ \beta = \frac{1-e^{-r\tau}}{k}$

$r = -0.0330,\ k = 0.13973569$ with r and k known A can be determined from equation (10).

**CASE 3:** Consider equation (10)

$\frac{1}{y} = \frac{1}{Ae^{rt}} + \frac{1}{k} => z_i = \frac{e^{-rt}}{A} + k^{-1}$ ……………………………………….(10)

$at\ t = 0, z_0 = 9401288, k^{-1} = 7.156367839\ and\ r = -0.0330$

$\therefore\ \frac{1}{A} = 9401295.156$

From equation (10), we have

$\tilde{z}_i = 940295.156 e^{-(-0.0330)t} + 7.156367839$ …………………………..(29)

Therefore,

$\hat{z} = 373400t + 9291700$ ………………………………………………….(27)

$\dot{z}_i = 1.0336 z_{i-1} - 0.2401$…………………………………………………(28)

$\tilde{z}_i = 940295.156 e^{-(-0.0330)t} + 7.156367839$ …………………………...(29)

### 3.3 Analysis of Model for Data III

**CASE 1:** Consider the solution from equation (5b)

$\log y = mt + b$ and applying MATLAB inbuilt function on the set of data for the population projection system for both Male and Female in AKWA IBOM STATE, NIGERIA.

Applying MATLAB inbuilt function to estimate equation (5b)

```
>> t=[0 1 2 3 4 5 6 7 8 9 10 11 12];
>> z=[3902051 4037001 4176620 4321066 4470509 4625119 4785077 4950567
      5121781 5298916 5482177 5671776 5867932];
>> logy=(z);
>> p=polyfit(t,logy,1)
p =
  1.0e+006 *
   0.1635    3.8430
```

$where\ m = 163500\ and\ b = 3843000$

$=> \hat{z} = mt + b$

$\therefore\ \hat{z} = 163500t + 3843000$ …………………………………………..(30)

 

**CASE 2:** Consider the solution from equation (23)
$z_i = \rho z_{i-1} + \beta$ ……………………………………………………….(23) applying Least-Squares on $(z_0, z_1), (z_1, z_2), \ldots, (z_{i-1}, z_i)$ and using MATLAB inbuilt function on the set of data for the population projection system for both Male and Female in AKWA-IBOM STATE, NIGERIA.As in Table 5.
$set\ z_{i-1} = x\ and\ z_i = y\ (dummy\ variables)$

```
>> x=[3902051 4037001 4176620 4321066 4470509 4625119 4785077 4950567
5121781 5298916 5482177 5671776];
>> y=[4037001 4176620 4321066 4470509 4625119 4785077 4950567 5121781
5298916 5482177 5671776 5867932];
>> p=polyfit(x,y,1)
p =
   1.0346   -0.8325
>> xp=3902051:10000:5671776;
>> yp=polyval(p,xp)
>> plot(x,y,'o',xp,yp)
```

$\dot{z}_i = 1.0346\dot{z}_{i-1} - 0.8325$ ……………………………………………(31)
Recall, $\rho = e^{-r\tau}\ and\ \beta = \frac{1-e^{-r\tau}}{k}$
$r = -0.0340, k = 0.041543071$ with r and k known A can be determined from equation (10).

**CASE 3:** Consider equation (10)
$\frac{1}{y} = \frac{1}{Ae^{rt}} + \frac{1}{k} \Rightarrow z_i = \frac{e^{-rt}}{A} + k^{-1}$ ……………………………………..(10)
$at\ t = 0, z_0 = 3902051, k^{-1} = \frac{1}{k} = 24.7140282$
$\therefore\ \frac{1}{A} = 3902026.929$ and from equation (10) we have,
$\tilde{z}_i = 3902026.929e^{-(-0.0340)t} + 24.07140282$ ……………………….(32)

Therefore,
$\hat{z} = 163500t + 3843000$ ………………………………………………...(30)
$\dot{z}_i = 1.0346\dot{z}_{i-1} - 0.8325$ ……………………………………………....(31)
$\tilde{z}_i = 3902026.929e^{-(-0.0340)t} + 24.07140282$ ………………………...(32)

## 4. Conclusion

In this paper a mathematical estimation for the population projection of three states in Nigeria was analyzed based on two ordinary differential equations model which are the growth model and the logistic model. Firstly, the growth model was transformed to linear model whose parameters were estimated. Secondly, it was established that the solution to the logistic model is not linear in its parameters A, r and k. Therefore a surgery was performed on the data values which are separated by fixed time $\tau$, and then Least Squares can be applicable. Thereafter, three equations were obtained to estimate the data from the National Population commission of Nigeria. The data values from the National population Commission of Nigeria compares well with the estimated population model $\hat{z}$, $\dot{z}_i$ and $\tilde{z}_i$ in each of the states considered. We also consider the MATLB inbuilt function to polyfit the data set, and obtained a linear plot for the data set. It was discovered that $\tilde{z}_i$ has the best approximation to the population data.

**Appendix**

**Table 1: Population Data**

| YEAR (t) | MALE IN KANO STATE(DATA I) | MALE & FEMALE IN KANO STATE(DATA II) | MALE & FEMALE IN AKWA-IBOM STATE (DATA III) |
|---|---|---|---|
| 2006 | 4947952 | 9401288 | 3902051 |
| 2007 | 5113958 | 9716706 | 4037001 |
| 2008 | 5285534 | 10042707 | 4176620 |
| 2009 | 5462867 | 10379645 | 4321066 |
| 2010 | 5646149 | 10727887 | 4470509 |
| 2011 | 5835580 | 11087814 | 4625119 |
| 2012 | 6031367 | 11459816 | 4785077 |
| 2013 | 6233723 | 11844299 | 4950567 |
| 2014 | 6442867 | 12241682 | 5121781 |
| 2015 | 6659029 | 12652397 | 5298916 |
| 2016 | 6882443 | 13076891 | 5482177 |
| 2017 | 7113353 | 13515628 | 5671776 |
| 2018 | --------- | ----------- | 5867932 |

**Source:** National Population Commission of Nigeria, **2005**.

**Table 2: Population of Male in KANO STATE, NIGERIA.**

| YEAR (t) | POPULATION (z) |
|---|---|
| 2006 | 4947952 |
| 2007 | 5113958 |
| 2008 | 5285534 |
| 2009 | 5462867 |
| 2010 | 5646149 |
| 2011 | 5835580 |
| 2012 | 6031367 |
| 2013 | 6233723 |
| 2014 | 6442867 |
| 2015 | 6659029 |
| 2016 | 6882443 |
| 2017 | 7113353 |

**Table 3 : Population projection of Male in KANO STATE, NIGERIA.**

| YEAR (t) | POPULATION (z) | $\hat{z}$ | $\dot{z}_t$ | $\tilde{z}_t$ |
|---|---|---|---|---|
| 2006 | 4947952 | 4890300 | 4947952 | 4947952 |
| 2007 | 5113958 | 5086800 | 5114203 | 5113958 |
| 2008 | 5285534 | 5283300 | 5286039 | 5285533 |
| 2009 | 5462867 | 5479800 | 5463649 | 5462864 |
| 2010 | 5646149 | 5676300 | 5647227 | 5646146 |
| 2011 | 5835580 | 5872800 | 5836973 | 5835580 |
| 2012 | 6031367 | 6069300 | 6033094 | 6031362 |
| 2013 | 6233723 | 6265800 | 6235805 | 6233716 |
| 2014 | 6442867 | 6462300 | 6445328 | 6442860 |
| 2015 | 6659029 | 6658800 | 6661890 | 6659021 |
| 2016 | 6882443 | 6855300 | 6885728 | 6882434 |
| 2017 | 7113353 | 7051800 | 7117088 | 7113342 |

**Table 4: Population for both Male and Female in KANO STATE, NIGERIA.**

| YEAR (t) | POPULATION (z) |
|---|---|
| 2006 | 9401288 |
| 2007 | 9716706 |
| 2008 | 10042707 |
| 2009 | 10379645 |
| 2010 | 10727887 |
| 2011 | 11087814 |
| 2012 | 11459816 |
| 2013 | 11844299 |
| 2014 | 12241682 |
| 2015 | 12652397 |
| 2016 | 13076891 |
| 2017 | 13515628 |

**Table 5: Population projection for both Male and Female in KANO STATE, NIGERIA.**

| YEAR (t) | POPULATION (z) | $\hat{z}$ | $\dot{z}_t$ | $\tilde{z}_t$ |
|---|---|---|---|---|
| 2006 | 9401288 | 9291700 | 9401288 | 9401302 |
| 2007 | 9716706 | 9665100 | 9603900 | 9716720 |
| 2008 | 10042707 | 10038500 | 9926591 | 10042721 |
| 2009 | 10379645 | 10411900 | 10260124 | 10379660 |
| 2010 | 10727887 | 10785300 | 10604864 | 10727903 |
| 2011 | 11087814 | 11158700 | 10961188 | 11087829 |
| 2012 | 11459816 | 11532100 | 11329483 | 11459832 |
| 2013 | 11844299 | 11905500 | 11710154 | 11844315 |
| 2014 | 12241682 | 12278900 | 12103615 | 12241698 |
| 2015 | 12652397 | 12652300 | 12510296 | 12652414 |
| 2016 | 13076891 | 13025700 | 12930642 | 13076909 |
| 2017 | 13515628 | 13399100 | 13365111 | 13515646 |

**Table 6: Population for both Male and Female in AKWA-IBOM STATE, NIGERIA.**

| YEAR (t) | POPULATION (z) |
|---|---|
| 2006 | 3902051 |
| 2007 | 4037001 |
| 2008 | 4176620 |
| 2009 | 4321066 |
| 2010 | 4470509 |
| 2011 | 4625119 |
| 2012 | 4785077 |
| 2013 | 4950567 |
| 2014 | 5121781 |
| 2015 | 5298916 |
| 2016 | 5482177 |
| 2017 | 5671776 |
| 2018 | 5867932 |

**Table 7: Population projection for both Male and Female in AKWA-IBOM STATE, NIGERIA.**

| YEAR (t) | POPULATION (z) | $\hat{z}$ | $\dot{z}_\iota$ | $\tilde{z}_\iota$ |
|---|---|---|---|---|
| 2006 | 3902051 | 3840300 | 4037061 | 3902051 |
| 2007 | 4037001 | 4006500 | 4176742 | 4037001 |
| 2008 | 4176620 | 4170000 | 4321257 | 4176618 |
| 2009 | 4321066 | 4333500 | 4470771 | 4321065 |
| 2010 | 4470509 | 4497000 | 4625459 | 4470506 |
| 2011 | 4625119 | 4660500 | 4758499 | 4625116 |
| 2012 | 4785077 | 4824000 | 4951077 | 4785073 |
| 2013 | 4950567 | 4987500 | 5122383 | 4950562 |
| 2014 | 5121781 | 5151000 | 5299617 | 5121774 |
| 2015 | 5298916 | 5314500 | 5482983 | 5298908 |
| 2016 | 5482177 | 5478000 | 5672693 | 5482167 |
| 2017 | 5671776 | 5641500 | 5868967 | 5671765 |
| 2018 | 5867932 | 5805000 | 6072033 | 5867920 |

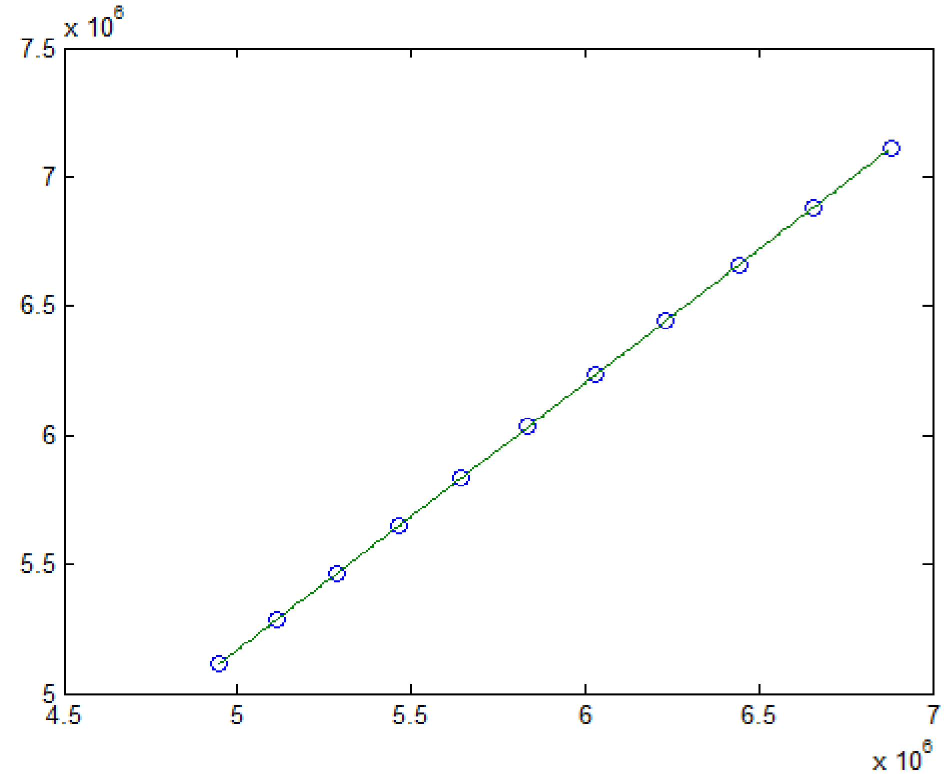


Figure 1: Linear plot for $\dot{z}_\iota$ in data I.

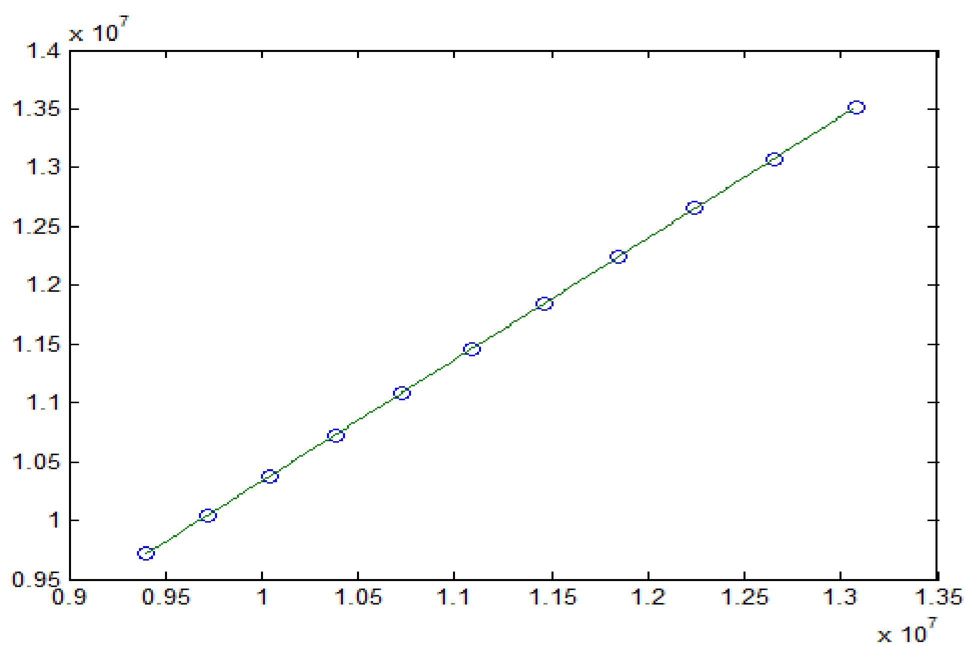


Figure 2: Linear plot for $\dot{z}_\iota$ in data II.

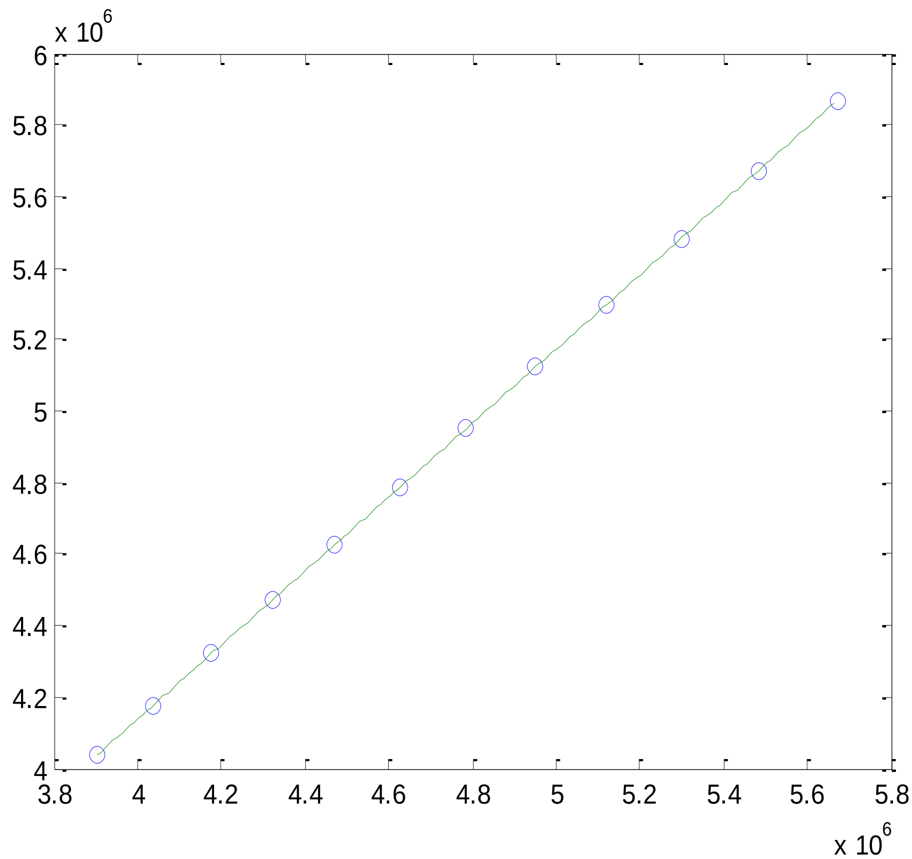


Figure 3: Linear plot for $\dot{z}_t$ in data III.

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