A Note on the Multiplicity of Solutions of a Boundary Value Problem Arising From the Theory of Microwave Heating of Cancerous Tumor.

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Abstract

This paper investigates the *multiplicity of solutions of a boundary value problem arising from the theory of microwave heating of cancerous tumor. It is shown that the problem has two solutions. However, prescribing an initial gradient results in the existence of a unique solution.*

Keywords: Cancer, Heating, Hyperthermia, Multiplicity, Tumor

1. Introduction

It is obvious that cancer is currently a leading cause of death in the world. It is characterized by uncontrolled growth and spread of abnormal cells and tumor (Popoola et. al, 2012). Research findings have shown that physical exercise couple with maintenance of a healthy body weight may directly or indirectly reduce the risk of certain cancers.

Meanwhile, studies have led to the discovery of both chemotherapies and radiotherapies. In this paper, we shall investigate microwave heating (Hyperthermia) as a typical example of radiotherapies. This hyperthermia is therefore a type of medical therapy in which body tissue is exposed to slightly higher temperatures to damage and kill cancer cells. It could be used alone or in conjunction with anti cancer drugs. According to (Erinle, 2005 Erinle et.al, 2006 and Guajardo-Cuella, 2006), microwave heating requires directing a carefully controlled measure of heat to the cancerous tumor and surrounding body tissue. Cancerous tissues can be destroyed at a moderate exposure to a temperature of about 108 °F for an hour. However, if too much heat misses the tumor target, the skin or other healthy parts of the tissues could be damaged. The leading Microwave energy is very effective in the cancerous tumours, because tumours typically have high- water content (Erinle, 2005). Such tissue heats very rapidly when exposed to high- power microwaves. Furthermore, microwaves can be delivered to tissue by special purpose antennas that are located adjacent to the patients' body, depending on the tumour size and location in the body one or more microwave antennas can be used to treat the tumour. When a microwave thermotherapy antenna is turned on, the body tissue with high- water content that is irradiated with significant amounts of microwave energy is heated (Erinle, 2005). The tumour cells are heated within the body to 41^{o}c-43^{o}c (Guajardo-Cuella, 2006).

In the paper of (Ayeni et. al, 2005), we showed the effects of the mode of heating on the temperature rise. Moreover, in this paper, we show that there exist two solutions θ_1, θ_2 for a Dirichlet problem and that within the body $\theta_1 < \theta_2$. Thus, the choice $\theta_1 or \theta_2$ will depend on the gradient at the boundaries.

2. Mathematical Equation of the problem

Following (Ayeni et. al, 2005, Erinle, 2005 and Guajardo-Cuella, 2006), we take the steady equation as

$$k\frac{d^2T}{dx^2} + \rho_b w_b (T_b - T) + Q(T - T_0)^n = 0 \quad (1)$$

where $T = temperature$

$T_b = temperature\ of\ arterial\ blood$

$T_0 = reference\ temperature\ (T_b > T_0)$

$\rho_b w_b = blood\ perfusion$

$k = thermal\ conductivity$

$Q = electromagnetic\ energy\ per\ unit\ temperature$

$n = real\ \ number$

As boundary conditions we take

$$T(0) = T_0,\ T(L) = T_0, \tag{2}$$

3. Non dimensionalization

We may non-dimensionalize the equations by writing

$$\theta = \frac{T - T_0}{T_b - T_0}, \quad y = \frac{x}{L} \tag{3}$$

to obtain

$$\frac{d^2\theta}{dy^2} - \varepsilon(\theta - 1) + a\theta^n = 0 \tag{4}$$

$$\theta(0) = \theta(1) = 0 \tag{5}$$

where $\varepsilon = \dfrac{\rho_b w_b}{k}$ $and\ \ a = \dfrac{Q}{k}$ (6)

4 Multiplicity of solutions

Equation (4) which satisfies (5) could be solved by a finite difference scheme or by a shooting method. We are interested only in positive solutions.

4.0.1 Finite difference method

The problem (4)which satisfies the boundary conditions (5) has the finite difference scheme

$\theta_{i+1} = 2\theta_i - \theta_{i-1} + h^2\{\varepsilon(\theta_i - 1)\} - a\theta_i^n$, together with the boundary conditions $\theta_0 = \theta_2 = 0$. The numerical values generated from the above scheme for various values of n and when the step length h is taken as 0.5, is presented in the table 2.

4.0.2 *Shooting method*

In shooting method technique, the gradient at y =0 must be prescribed in addition to the boundary values at y=0 and y= 1. It is therefore not surprising that it gives only one real positive solution (as presented in table 1 and figure 1) which coincides with one of the two solutions obtained by finite difference method. The observation could be explained through the following theoretical basis.

Theorem 1 (Descartes' rule of signs). The number of positive zeros of a polynomial with real coefficients is either equal to number of variations in sign of the polynomial or is less than this by an even number.

Proof: see (Labarre, 1961)

Theorem 2: For each given gradient $\theta'(0)$ equation (4) has three positive roots or one positive root.

Proof: $\theta(y) = \theta(-y)$ so $\theta'\left(\frac{1}{2}\right) = 0$ and $\theta\left(\frac{1}{2}\right)$ is maximum.

Integrating equation (4) we obtain

$$\frac{1}{2}\left(\frac{d\theta}{dy}\right)^2 - \varepsilon\left(\frac{\theta^2}{2} - \theta\right) + \frac{a}{n+1}\theta^{n+1} = k \tag{7}$$

Where k is a constant of integration.

At $y = \frac{1}{2}$,

$$\frac{1}{2}\theta'\left(\frac{1}{2}\right) - \varepsilon\left(\frac{1}{2}\theta^2\left(\frac{1}{2}\right) - \theta\left(\frac{1}{2}\right)\right) + \frac{a}{n+1}\theta^{n+1}\left(\frac{1}{2}\right) = k$$

$$\Rightarrow \qquad -\varepsilon\left(\frac{1}{2}\theta^2\left(\frac{1}{2}\right) - \theta\left(\frac{1}{2}\right)\right) + \frac{a}{n+1}\theta^{n+1}\left(\frac{1}{2}\right) = k \qquad (8)$$

In particular, if $\theta'(0) = \beta$, equation (7) gives $k = \frac{\beta^2}{2}$

Then equation (8) becomes

$$\frac{a}{n+1}\theta^{n+1}\left(\frac{1}{2}\right) \quad -\frac{\varepsilon}{2}\theta^2\left(\frac{1}{2}\right) + \varepsilon\theta\left(\frac{1}{2}\right) - \frac{\beta^2}{2} = 0 \qquad (9)$$

By theorem, equation (9) has three positive roots or one positive root.

Lemma: As $\varepsilon \to 0$ equation (9) has only one positive root.

Proof: The number of changes in sign is one. Hence the result.

Table 1. The numerical values obtained by shooting method for some values of m > 1.

y	n= 2	n=3	n=4
0	0.0000	0.0000	0.0000
0.25	8.3229	2.2728	1.3804
0.50	12.8492	3.9021	2.5619
0.75	8.3229	2.2728	1.3804
1.00	0.0000	0.0000	0.0000

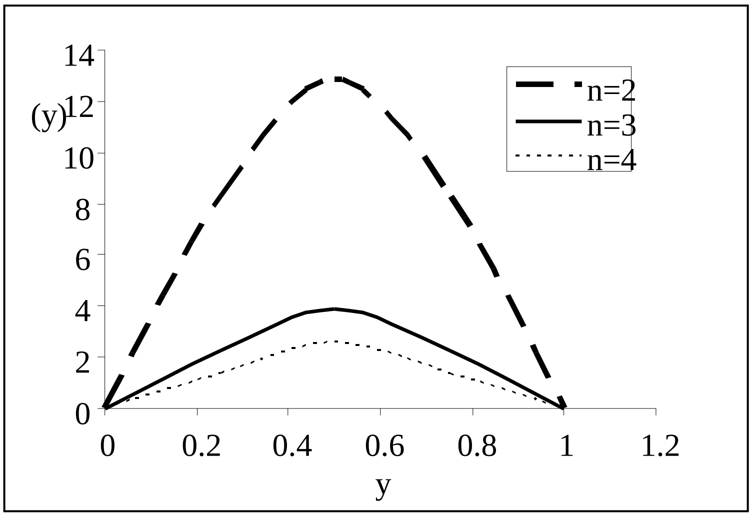

Figure 1. The graph shows the unique solution of temperature by shooting method.

Description for the above table 1

Table 2. The numerical values obtained by finite difference method

y	n = 2		n = 3		n = 4	
	Solution 1	Solution 2	Solution 1	Solution 2	Solution 1	Solution 2
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.0009	8.3229	0.0009	2.2728	0.0009	1.3804
0.50	0.0012	12.8492	0.0012	3.9021	0.0012	2.5619
0.75	0.0009	8.3229	0.0009	2.2728	0.0009	1.3804
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

5. Conclusion

Our results have shown that the problem has two solutions. However, prescribing an initial gradient results in the existence of a unique solution.

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