Network and Complex Systems
ISSN 2224-610X (Paper) ISSN 2225-0603 (Online)
Vol.5, No.2, 2015

Application of Idle/ Waiting Time Operator $O_{i,W}$ on Two Stage Flow Shop Scheduling Problem with Arbitrary Lags

Deepak Gupta Harminder Singh* Renuka
Department of Mathematics, M.M. University, Mullana, Ambala, India.
*E-mail of the corresponding author: harminder.cheema85@gmail.com

Abstract
In this paper an attempt is made to study n jobs over two machines where processing times are associated with respective probabilities and also Idle/Waiting Time Operator $O_{i,w}$ is used for equivalent job block as well as transportation time in addition with arbitrary lags i.e. start lag or stop lag. The objective of the study is to propose an algorithm by which we can minimize the total elapsed time in two stage flow shop scheduling with considering various system parameters. A numerical illustration is also given to clarify the proposed concept.
Keywords: Flow shop, Start Lag, Stop Lag, Transportation Time, idle waiting operator.
Subject Classification Number: 90B35, 90B36.

1. Introduction

Sequencing simply refers to the determination of order of jobs over time in which the jobs are to be processed on various machines. The scheduling /sequencing problems are common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, waiting air craft for landing clearance, programs to be run in a sequence at a computer center etc. The selection of an appropriate order or sequence in which to receive waiting customer is called sequencing. Johnson [1] gave procedure for finding the optimal schedule for n-jobs, two machine flow-shop problem with minimization of the make span (i.e. total elapsed time) as the objective. Also Mitten [2] discussed the n job, 2 machines flowshop Scheduling problem in which despite of processing times some additional tags are introduced. Maggu and Das [4] introduced the equivalent job-block concept in the theory of scheduling which has many applications in the production concern, hospital management etc. where priority of one job over other becomes significant it may arise the additional cost for providing this facility. Bagga [3], Maggu and Das [6],[7], Szwarch [8], Yoshida & Hitomi [5], Singh, Anup [12], etc. derived the optimal algorithm for two/ three or multistage flow shop problems taking into account the various constraints and criteria. Kern and Nawjin [10], Riezebos and Goalman[11] continues with dealing different scheduling problems including time lags. Singh, T.P. and Gupta, D. [9],[13] associated probabilities with processing time and set up time, transportation time as well as concept of breakdown interval in their studies. Later, Singh, T.P, Gupta, D[14],[15] studied two /multiple flow shop problem to minimize rental cost under a pre-defined rental policy in which the probabilities have been associated with processing time on each machine. The present paper addresses the flowshop scheduling problem in which processing times are associated with probabilities with arbitrary lags, transportation time and idle/waiting time operator for job block for effective scheduling.

2. Theorem

Let n jobs 1, 2, 3, …………n are processed through two machines A & B in order AB with processing time a_i & b_i (I = 1, 2, 3, ……….n) on machine A and B respectively. If $(a_p, b_p)\ O_{i,w}\ (a_q, b_q) = (\alpha_\beta, b_\beta)$ then $a_\beta = a_p + \max(a_q - b_p, 0)$ and $b_\beta = b_q + \max(b_q - a_q, 0)$
where β is the equivalent job for job block (p, q) and p, q ϵ {1, 2, 3, …………..n}.

Proof

Starting by the equivalent job block criteria theorem for β = (p, q) given by Maggu & Das (5)
we have:

$a_\beta = a_p + a_q - \min(b_p, a_q)$ …(1)
$b_\beta = b_p + b_q \min(b_p, a_q)$ …(2)

Now, we prove the above said theorem by a simple logic:

Case I: When $a_q > b_p$

$a_q > b_p > 0$
$\max\{a_q > b_p, 0\} = a_q > b_p$ and $b_p > a_q < 0$ …(3)
$\max\{b_p > a_q, 0\} = 0$ …(4)
(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$
$= a_p + a_q - b_p$ as $a_q > b_p$
$= a_p + \max\{a_q - b_p, 0\}$
using (3)
(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$
$= b_p + b_q - b_p$ as $a_q > b_p$

$= b_q + (b_p - b_p)$
$= b_q + 0$
$= b_q + \max(b_p - a_q, 0)$
(using (4))

Case II: When $a_q < b_p$

$a_q - b_p < 0$
$\max(a_q - a_q, 0) = b_p - a_q$ and $b_p - a_q > 0$
$\max(b_p - a_q, 0) = b_p - a_q$
(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$
$= a_p + a_q - a_q$ as $a_q > b_p$
$= a_p + a_q - a_q$ as $a_q > b_p$
$= a_p + 0$
$= a_p + \max(a_q - b_p, 0)$
using (7)…(9)
(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$
$= b_p + b_q - a_q$ as $a_q < b_p$
$= b_p + (b_p - a_q)$
$= b_p + \max(b_p - a_q, 0)$
using (8)-(10)

Case III: When $a_q = b_p$

$a_q - b_p = 0$
$\max(a_q - b_p, 0) = 0$ …(11)
also $b_p - a_q = 0$
$\max(b_p - a_q, 0) = 0$
(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$ …(12)
$= b_p + a_q - a_p$ as $b_q = a_p$
$= a_p + 0$
$= a_p + \max(a_q - b_p, 0)$ …(13)
(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$ …(14)
$= b_p + b_q - b_p$
$= b_q + (b_p - b_p)$
$= b_q + 0$
$= b_q + \max(b_p - a_q, 0)$
using (12)-(14)
by (5), (6), (9), (10), (13) and (14) for all possible three cases we conclude

$a_\beta = a_p + a_q - \max(a_q, b_p, 0)$
$b_\beta = b_p + \max(b_p, a_q, 0)$

The theorem can be generalized for more number of job blocks as stated:
Let n jobs 1, 2, 3, ………..n are processed through two machines A & B in order AB with processing time a_i & b_i (i = 1, 2, 3, ……..n) on machine A & B respectively.
If $(a_{io}, b_{io})\ O_{i,w}\ (ai_1, bi_1)\ O_{i,w}\ (ai_2, bi_2)\ O_{i,w} \ldots\ O_{i,w}\ (a_{ip}, b_{ip}) = (a_\beta, b_\beta)$

Then $a_\beta = a_{io} + \sum_{J=1}^{p} \max\{a_{ij} - b_{i(j-1)}\ 0\}$

and $b_\beta = b_{ip} + \sum_{J=1}^{p} \max\{b_{i(j-1)} - a_{ij}, 0\}$

where $i_0, i_1, i_2, i_3, \ldots ip \in \{1, 2, 3 \ldots n\}$ and β is the equivalent job for job block $(i_0, i_1, i_2, i_3, \ldots\ldots ip)$. The proof can be made using Mathematical induction technique on the lines of Maggu & Das (7).

3. Definitions

Idle/Waiting Time Operator: Let R_+ be the set of non negative real numbers. Let $G = R_+ \times R_+$. Then $O_{i,w}$ is defined as a map from G x G → G given by:
$O_{i,w}[x_1, y_1), (x_2, y_2)] = (x_1, y_1)\ O_{i,w}\ (x_2, y_2)$
$= [x_1 + \max(x_2 - y_1, 0), y_2 + \max(y_1 - x_2, 0)]$ where $x_1, x_2, y_1, y_2 \in R_+$

Total Elapsed Time: It is the time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of the machines.

Processing Time: It is the time required to process job j. It includes both actual time as well as set-up time.

Effective Transportation Time: The effective transportation time of job i is denoted by t'_i and defined as t'_i = max (D_i-G_i, E_i-H_i, t_i) where t_i is the transportation time of job i, $G_i = A_{i1} + A_{i2} + A_{i3+......}A_{i(m-1)}$ and $H_i = A_{i2} + A_{i3+......}A_{im}$

Concept of Transportation time in flow shop

In many practical situations of scheduling it is seen that machines are distantly situated and therefore, definite finite time is taken in transporting the job from one machine to another if the form of.

i) Loading time of jobs.
ii) Moving time of jobs.
iii) Unloading time of jobs.

4. Assumptions

1. The time intervals for processing are independent of the order in which the jobs are done.
2. Each machine is assumed to be continuously available for the assignment of jobs.
3. No significant division of time scale into shifts or days for the machines is assumed.
4. Each machine can handle at most one operation at a time.
5. Pre-emption is not allowed, that is, once a job is started it is performed to completion.
6. The technological ordering of the machines operating the jobs is known to be predetermined.

5. Notations

S_K : Sequence obtained by applying Johnson's Procedure.
a_i^1 : Processing time of i^{th} job on machine A.
a_i^2 : Processing time of i^{th} job on machine B.
A_α : Expected processing time of i^{th} job on machine A.
B_α : Expected processing time of i^{th} job on machine B.
p_i : Probability associated to a_i^1.
q_i : Probability associated to a_i^2.
β : Equivalent job for job-block.
D_i : Start lag for job i.
E_i : Stop lag for job i.
t_i : Transportation time of i^{th} job.
t'_i : Effective transportation time of i^{th}.
$CT(S_k)$: Completion time of Sequence S_k.

6. Algorithm

Step 1: Calculate expected processing time given by

$$A_\alpha = a_i^1 \times p_i$$
$$B_\alpha = a_i^2 \times q_i$$

Step 2: Calculate effective transportation times given by

$$t_i' = \max (D_i - A_\alpha, E_i - B_\alpha, t_i)$$

Step 3: Define two fictitious machines G & H with processing time G_i & H_i as below:

$$G_i = A_\alpha + t_i' \qquad H_i = B_\alpha + t_i'$$

Step4: Determine equivalent jobs for each job block using operator theorem and concept of the idle/waiting time operator $O_{i,w}$ as per definition.

Step5: Apply Johnson's (1954) technique to obtain the optimal Sequence S_K.

Step6: Obtain In –Out table for Sequence S_K and calculate total elapsed time.

7. Numerical Illustration

Obtain optimal sequence for 5 jobs and 2 machines problem given by the following tableau1:

Jobs	Machine A		Machine B		Transportation Time t_i	Start Lag D_i	Stop Lag E_i
	a_i^1	p_i	a_i^2	q_i			
1	24	.2	13	.3	5	20	12
2	22	.3	14	.2	8	8	20
3	18	.2	9	.2	12	22	8
4	16	.1	28	.1	7	14	9
5	24	.2	13	.2	6	10	10

Tableau 1

Our objective is to minimize the total elapsed time of the jobs when the jobs 3 & 5 are processed as a group job (3, 5).

Solution: As per step 1: Expected processing time on machines A and B are shown in tableau-2

As per step 2: t_1' = max (15.2,8.1,5) = 15.2

t_2' = max (2.6,17.2,8) = 17.2

t_3' = max (18.4,6.2,12) = 18.4

t_4' = max (12.4,6.2,7) = 12.4

t_5' = max (5.2,7.4,6) = 7.4

As per step 3: The processing time on fictitious machines are shown in tableau-3.

As per Step 4: Using equivalent job block criteria β over job (3,5), we have

G_β = 22.0, H_β = 18 and it is represent in tableau – 4.

As per step 5: Sequence obtained by Johnson rule is in tableau -5.

As per step 6: In- Out table as shown in tableau- 6.

Hence CT(S_k) = 31.4 units of time for sequence 4-2-1-3-5.

References

1. Johnson, S. M. (1954), "Optimal two and three stage production schedule with set up times included," Nay Res Log Quart, Vol 1, pp 61-68.
2. Mitten, L.G. (1958), "Sequencing n jobs on machines with arbitrary time lags", management sciences, Vol.29, 477-481.
3. Bagga, P.C. (1969), "Sequencing in a rental situation", Journal of Canadian Operation Research Society, Vol. 7, pp.152-153.
4. Maggu, P.L. and Das, G. (1977), "Equivalent-job block in job scheduling", Operation research, Vol. 14, No.4, pp 277-281.
5. Yoshida and Hitomi (1979), "Optimal two stage production scheduling with set up times separated", AIIE Transactions, Vol. 2, pp 261-263.
6. Maggu, P.L and Dass, G. (1980), "nx2 scheduling with transportation time", PAMS, Vol 2, pp1-6.
7. Maggu, P.L. and Dass G. (1981), "On n x 2 sequencing problem with transportation time of jobs", Pure and Applied Mathematika Sciences, pp 12-16.
8. Szwarc, W. (1983), "Flow shop problems with time lags", Management Science, 29, pp 477-481.
9. Singh, T.P. (1985), "On n x 2 flow shop problem solving job block, Transportation times, Arbitrary time and Break-down machine times", PAMS, Vol., 21, No. 1-2.
10. Kern, W. and Nawjin, W.M. (1991), "Scheduling multi-operation jobs with time lags on a single machine", Working paper University of Twente, Holland.
11. Riezebos, J. and Goalman, G.J.C. (1998), "Time lag size in multiple operations flowshop Scheduling heuristics", European journal of operation research, Vol 105, pp 72-90.
12. Anup (2002), "On two machine flow shop problem in which processing time assume Probabilities and there exists equivalent for an ordered job block", JISSO, Vol 23, No.1-4, pp.41-44.
13. Singh,T.P., Gupta, D. and Kumar, R. (2005), "Optimal three stage production schedule the processing time and set up times associated with probabilities including job block", Proceedings of National Conference FACM, pp 463-470.
14. Singh, T.P. and Gupta, D. (2006), "Minimizing rental cost in two stage flow shop, the processing time associated with probabilities including job block, Reflections de ERA, Vol 1, No. 2, pp 107-120.
15. Singh,T.P.,. Gupta, D. and Kumar, R. (2007), "Optimal Rental Cost in Restrictive 3-Stage scheduling including Job Block concept", Acta Ciencia Indica, Vol. 33, No.3, pp. 761-767.

Dr.Deepak Gupta: The author has been working as a Professor and Head in Maharishi Markandeshwar university, Mullana, Haryana,(India). With a strong desire to work efficiently by putting forth the innovative ideas, more than 100 research papers have been published in reputed journals with national and international eminence. Apart from that he has attended many national and international conferences and participated in Staff Development Programme and has published 10 books.

Harminder Singh: is a research scholar at Maharishi Markandeshwar university, Mullana, Haryana,(India). With appreciable academic potency, twenty research papers have been published in reputed journals with national and international eminence. Moreover, a book named "Flowshop Scheduling Techniques" has been published by Lam Lambert Academic Publishing. The author's major field of study is operation research.

Renuka: is a research scholar at Maharishi Markendeshwar University, Mullana (Haryana, India). The author has published six research papers in national and international journals. The author has published a book and presented many papers in national conferences. Further her chief field of interest is operation research.

Tables

Tableau-2: Expected processing time on machines A & B

Jobs	A_α	B_α	t_i	D_i	E_i
1	4.8	3.9	5	20	12
2	6.6	2.8	8	8	20
3	3.6	1.8	12	22	8
4	1.6	2.8	7	14	9
5	4.8	2.6	6	10	10

Tableau-3: Processing time on fictitious machines G &H

jobs	G_i	H_i
1	20.0	19.1
2	23.8	20.0
3	22.0	20.2
4	14.0	15.2
5	12.2	10.0

Tableau - 4: Processing time of β job

Jobs	G_i	H_i
1	20.0	19.1
2	23.8	20.0
β	22.0	18.0
4	14.0	15.2

Tableau -5: Sequence obtained by Johnson rule

4	2	1	3	5

Tableau- 6: In- Out table for sequence S_k

Jobs	**Machine A**	t'_i	**Machine B**
	In-Out		**In-Out**
4	0.0-1.6	12.4	14.0-16.8
2	1.6-8.2	17.2	25.4-28.2
1	8.2-13.0	15.2	28.2-32.1
3	13.0-16.6	18.4	35.0-36.8.
5	16.6-21.4	7.4	28.8-31.4