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Common Fixed Point of Weakly Compatible Mappings Under a New Property In Fuzzy Metric Spaces

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Abstract

In this paper, we prove some common fixed point theorems for weakly compatible mappings under a new property in fuzzy metric spaces. We prove a new result under (S-B) property defined by Sharma and Bamboria [22].

Keywords: Fixed point, Fuzzy metric space, (S-B) property.

1. Introduction

The foundation of fuzzy mathematics was laid down by Zadeh [25] with the evolution of the concept of fuzzy sets in 1965. The proven result becomes an asset for an applied mathematician due to its enormous applications in various branches of mathematics which includes differential equations, integral equation etc. and other areas of science involving mathematics especially in logic programming and electronic engineering. It was developed extensively by many authors and used in various fields. Especially, Deng [7], Erceg [8], and Kramosil and Michalek [17] have introduced the concepts of fuzzy metric spaces in different ways. To use this concept in topology and analysis, several researchers have studied fixed point theory in fuzzy metric spaces and fuzzy mappings [2],[3],[4],[5], [10],[14], [15] and many others. Recently, George and Veeramani [12],[13] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek and defined the Hausdoff topology of fuzzy metric spaces. They showed also that every metric induces a fuzzy metric.Grabiec [11] extended the well known fixed point theorem of Banach [1] and Edelstein [9] to fuzzy metric spaces in the sense of Kramosil and Michalek [17].Moreover, it appears that the study of Kramosil and Michalek [17] of fuzzy metric spaces paves the way for developing a soothing machinery in the field of fixed point theorems in particular, for the study of contractive type maps.

2 . Preliminaries

Definition 2.1 : [20] A binary operation *: $[0,1]\times[0,1]\rightarrow[0,1]$ is called a continuous t-norm if $([0,1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, d are in $[0,1]$.

Examples of $t-norm$ are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2 : [17] The 3-tuple $(X, M, *)$ is called a fuzzy metric space (shortly FM-space) if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty)$ satisfying the following conditions for all x, y, z in X and $t, s > 0$,

$(FM-1)$ $M(x, y, 0) = 0$,

$(FM-2)$ $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,

$(FM-3)$ $M(x, y, t) = M(y, x, t)$,

$(FM-4)$ $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

$(FM-5)$ $M(x, y, .): [0,1] \rightarrow [0,1]$ is left continuous.

In what follows, $(X, M, *)$ will denote a fuzzy metric space. Note that $M(x, y, t)$ can be thought as the degree of nearness between x and y with respect to t. We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$ and $M(x, y, t) = 0$ with ∞ and we can find some topological properties and examples of fuzzy metric spaces in (George and

Veeramani [12].

Example 2.1 : [12] Let (X, d) be a metric space. Define $a * b = ab$ or $a * b = min\{a, b\}$ and for all x, y in X and $t > 0$,

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

Then $(X, M, *)$ is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Lemma 2.1 : [11] For all $x, y \in X$, $M(x, y, .)$ is non-decreasing.

Definition 2.3 : [11] Let $(X, M, *)$ be a fuzzy metric space :

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ $(denoted$ by lim $x_n = x)$, if

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \quad \text{for all } t > 0.$$

(ii) A sequence $\{x_n\}$ in X called a Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x, t) = 1, \quad \text{for all } t > 0 \text{ and } p > 0.$$

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.1 : Since $*$ is continuous, it follows from $(FM - 4)$ that the limit of the sequence in FM $-$space is uniquely determined.

Let $(X, M, *)$ be a fuzzy metric space with the following condition:

$(FM - 6)$ $\quad \lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$.

Lemma 2.2 : [6], [18] If for all $x, y \in X$, $t > 0$ and for a number k $\in$ (0,1),

$$M(x, y, kt) \geq M(x, y, t) \quad \text{then } x = y.$$

Lemma 2.3 : [18] Let $\{y_n\}$ be a sequence in a fuzzy metric space $(X, M, *)$ with the condition $(FM - 6)$. If there exists a number $k \in (0,1)$ such that

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, \ldots \ldots$ then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.4 : [18] Let S and T be mappings from a fuzzy metric space $(X, M, *)$ into itself. The mappings S and T are said to be compatible if

$$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1,$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \quad for\ some\ z \in X.$$

Definition 2.5 : [6] Let S and T be mappings from a fuzzy metric space $(X, M, *)$ into itself. The mappings S and T are said to be compatible of type (α) if,

$$\lim_{n\to\infty} M(STx_n, TTx_n, t) = 1, \lim_{n\to\infty} M(TSx_n, SSx_n, t) = 1,$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \quad for\ some\ z \in X.$$

Definition 2.6 : [16] A pair of mappings S and T is called weakly compatible pair in fuzzy metric space if they commute at coincidence points; i.e. , if $Tu = Su$ for some $u \in X$, then $TSu = STu$.

Definition 2.7 : Let S and T be two self mappings of a fuzzy metric space $(X, M, *)$. We say that S and T satisfy the property (S-B) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \quad for\ some\ z \in X.$$

Example 2.2 : [22] Let $X = [0, +\infty)$. Define $S, T : X \to X$ by

$$Tx = x / 4 \ and\ Sx = 3x / 4, \ \forall x \in X.$$

Consider the sequence $x_n = 1/n$, clearly $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 0$.

Then S and T satisfy (S-B) property.

Example 2.3 : [22] Let $X = [2, +\infty)$. Define $S, T : X \to X$ by

$$Tx = x + 1 \text{ and } Sx = 2x + 1, \forall x \in X.$$

Suppose property $(S - B)$ holds; then there exists in X a sequence $\{x_n\}$ satisfying

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \quad for\ some\ z \in X.$$

Therefore

$$\lim_{n\to\infty} x_n = z - 1 \text{ and } \lim_{n\to\infty} x_n = \frac{(z-1)}{2}.$$

Then $z = 1$, which is a contradiction since $1 \notin X$. Hence S and T do not satisfy the property (S-B).

Remark 2.2 : It is clear from the definition of Mishra et al. [18] and Sharma and Deshpande [23] that two self mappings S and T of a fuzzy metric space $(X, M, *)$ will be non-compatible if there exists at least one sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \quad for\ some\ z \in X.$$

but $\lim_{n\to\infty} M(STx_n, TSx_n, t)$ is either not equal to 1 or non-existent. Therefore two non-compatible self mappings of a fuzzy metric space $(X, M, *)$ satisfy the property (S-B).

It is easy to see that if S and T are compatible , then they are weakly compatible and the converse is not true in general.

Example 2.3. Let $= R_+$. Define S and T by :

$$Sx = x \text{ and } Tx = 2x - 1$$

$$Sx = Tx \text{ iff } x = 1,$$

As $ST(1) = S(1) = 1$, $TS(1) = T(1) = 1$

Therefore $\{S, T\}$ are weakly compatible.

Turkoglu, Kutukcu and Yildiz **[**24**]** prove the following :

Theorem 2.1. Let $(X, M, *)$ be a complete fuzzy metric space with $t*t \geq t\ for\ all\ t \in [0,1]$ and let $P, S, T\ and\ Q$ be maps from X into itself such that

(2.1) $PT(X) \cup QS(X) \subset ST(X)$,

(2.2) there exists a constant $k \in (0,1)$ such that

$$M^2(Px, Qy, kt) * [M(Sx, Px, kt)M(Ty, Qy, kt)] * M^2(Ty, Qy, kt) \geq [p\, M(Sx, Px, t) + q\, M(Sx, Ty, t)]\, M(Sx, Qy, 2kt)$$

for all $x, y \in X$ and $t > 0$, where $0 < p, q < 1$ such that $p + q = 1$,

(2.3) the pairs $\{P, S\}$ and $\{Q, T\}$ are compatible of type (α),

(2.4) S and T are continuous and $ST = TS$.

Then $P, S, T\ and\ Q$ have a unique common fixed point.

Sharma, Pathak and Tiwari [21] improved Theorem 2.1 and proved the following .

Theorem 2.2. *Let* $(X, M, *)$ be a complete fuzzy metric space with $t^*t \geq t\ for\ all\ t \in [0,1]$ and let P, S, T and Q be maps from X into itself such that

(2.5) $PT(X) \cup QS(X) \subset ST(X)$,

(2.6) there exists a constant $k \in (0,1)$ such that

$$M^2(Px, Qy, kt) * [M(Sx, Px, kt)M(Ty, Qy, kt)] * M^2(Ty, Qy, kt) \geq [p\, M(Sx, Px, t) + q\, M(Sx, Ty, t)]\, M(Sx, Qy, 2kt)$$

for all $x, y \in X\ and\ t > 0$, where $0 < p, q < 1$ such that $p + q = 1$, and

(2.7) the pairs $\{P,S\}$ *and* $\{Q,T\}$ are weak compatible.

Then P, S, T *and* Q have a unique common fixed point.

Rawal [19] proved the following :

Theorem 2.3. Let $(X, M, *)$ be a complete fuzzy metric space with $t^*t \geq t$ *for all* $t \in [0,1]$. Let A, B, S *and* T be mappings of X into itself such that

(2.8) $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$,

(2.9) there exists a constant k ∈ (0, 1) such that

$$M^{2p}(Ax,By,kt) \geq \min\{ M^{2p}(Sx,Ty,t), M^{q}(Sx,Ax,t). M^{q'}(Ty,By,t),$$
$$M^{r}(Sx,By,(2-\alpha)\, t). M^{r'}(Ty,Ax,t), M^{s}(Sx,Ax,t). M^{s'}(Ty,Ax,t),$$
$$M^{l}(Sx,By,(2-\alpha)\, t). M^{l'}(Ty,By,t)\},$$

$for\ all\ x,y \in X, \alpha \geq 0, \alpha \in (0,2), t > 0\ and\ 2p = q + q' = r + r' = s + s' = l + l'$.

(2.10) If the pairs $\{A,S\}$ *and* $\{B,T\}$ are weakly compatible, then

A, B, S *and* T have a unique common fixed point in X.

3. Main Results

We prove Theorem 2.3 under a new property [22] in the following way.

Theorem 3.1. Let $(X, M, *)$ be a fuzzy metric space with $t^*t \geq t$ for all $t \in [0,1]$ and condition $(FM-6)$. Let A, B, S *and* T be mappings of X into itself such that

(3. 1) $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$,
(3. 2) $\{A,S\}$ *or* $\{B,T\}$ satisfies the property $(S-B)$,
(3. 3) there exists a constant $k \in (0,1)$ such that

$$M^{2p}(Ax,By,kt) \geq min\{ M^{2p}(Sx,Ty,t), M^{q}(Sx,Ax,t). M^{q'}(Ty,By,t),$$
$$M^{r}(Sx,By,t). M^{r'}(Ty,Ax,(2-\alpha)t),$$
$$M^{s}(Sx,Ax,t). M^{s'}(Ty,Ax,,(2-\alpha)t),$$
$$M^{l}(Sx,By,t). M^{l'}(Ty,By,t)\},$$

$for\ all\ x,y \in X, \alpha \geq 0, \alpha \in (0,2), t > 0\ and\ 0 < p,q,q',r,r',s,s',l,l' \leq 1\ such\ that$

$2p = q + q' = r + r' = s + s' = l + l'$.

(3.4) if the pairs $\{A,S\}$ *and* $\{B,T\}$ are weakly compatible,

(3.5) one of $A(X), B(X), S(X)$ *or* $T(X)$ is closed subset of X, then A, B, S *and* T have a unique common fixed point in X.

Proof : Suppose that (B,T) satisfies the property (S-B). Then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Since $BX \subset SX$, there exists in X a sequence $\{y_n\}$ such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = z$. Let us show that $\lim_{n\to\infty} Ay_n = z$. Indeed, in view of (3.3) for $\alpha = 1 - a, a \in (0,1)$, we have

$$M^{2p}(Ay_n,Bx_n,kt) \geq min\{ M^{2p}(Sy_n,Tx_n,t), M^{q}(Sy_n,Ay_n,t). M^{q'}(Tx_n,Bx_n,t),$$
$$M^{r}(Sy_n,Bx_n,t). M^{r'}(Tx_n,Ay_n,(2-\alpha)\, t),$$
$$M^{s}(Sy_n,Ay_n,t). M^{s'}(Tx_n,Ay_n,(2-\alpha)t),$$
$$M^{l}(Sy_n,Bx_n,t). M^{l'}(Tx_n,Bx_n,t)\},$$

$$M^{2p}(Ay_n,Bx_n,kt) \geq min\{ M^{2p}(Sy_n,Tx_n,t), M^{q}(Sy_n,Ay_n,t). M^{q'}(Tx_n,Bx_n,t),$$
$$M^{r}(Sy_n,Bx_n,t). M^{r'}(Tx_n,Ay_n,(1+a)t),$$
$$M^{s}(Sy_n,Ay_n,t). M^{s'}(Tx_n,Ay_n,(1+a)t),$$
$$M^{l}(Sy_n,Bx_n,t). M^{l'}(Tx_n,Bx_n,t)\},$$

$$M^{2p}(Ay_n, Bx_n, kt) \;\geq\; min\,\{\, M^{2p}(Bx_n, Tx_n, t),$$
$$M^{q}(Bx_n, Ay_n, t).\, M^{q'}(Tx_n, Bx_n, t),$$
$$M^{r}(Bx_n, Bx_n, t).\, M^{r'}(Ay_n, Bx_n, t) * M^{r'}(Bx_n, Tx_n, at),$$
$$M^{s}(Bx_n, Ay_n, t).\, M^{s'}(Ay_n, Bx_n, t) * M^{s'}(Bx_n, Tx_n, at),$$
$$M^{l}(Bx_n, Bx_n, t).\, M^{l'}(Tx_n, Bx_n, t)\,\}.$$

Thus it follows that $M^{2p}(Ay_n, Bx_n, kt) \geq M^{s}(Bx_n, Ay_n, t).\, M^{s'}(Ay_n, Bx_n, t) * M^{s'}(Bx_n, Tx_n, at)$,

Since the t-norm * is continuous and $M\,(x, y, \cdot)$ is continuous, letting $a \to 1$, we have

$$M^{2p}(Ay_n, Bx_n, kt) \geq M^{s+s'}(Bx_n, Ay_n, t).$$

It follows that

$$\lim_{n\to\infty} M(Ay_n, Bx_n, kt) \;\geq\; \lim_{n\to\infty} M(Bx_n, Ay_n, t)\;,$$

and we deduce that $\lim_{n\to\infty} Ay_n = z.$

Suppose S(X) is a closed subset of X. Then z = Su for some u $\in$ X. Subsequently, we have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = Su.$$

By (3.3) with $\alpha = 1$, we have

$$M^{2p}(Au, Bx_n, kt) \;\geq\; min\,\{\; M^{2p}(Su, Tx_n, t), M^{q}(Su, Au, t).\, M^{q'}(Tx_n, Bx_n, t)\,,$$
$$M^{r}(Su, Bx_n, t).\, M^{r'}(Tx_n, Au, t),$$
$$M^{s}(Su, Au, t).\, M^{s'}(Tx_n, Au, t),$$
$$M^{l}(Su\,, Bx_n, t).\, M^{l'}(Tx_n, Bx_n, t)\,\}.$$

Taking the $\lim_{n\to\infty}$, we have

$$M^{2p}(Au, Su, kt) \;\geq\; min\,\{\; M^{2p}(Su, Su, t), M^{q}(Su, Au, t).\, M^{q'}(Su, Su, t)\,,$$
$$M^{r}(Su, Su, t).\, M^{r'}(Su, Au, t),$$
$$M^{s}(Su, Au, t).\, M^{s'}(Su, Au, t),$$
$$M^{l}(Su\,, Su, t).\, M^{l'}(Su, Su, t)\,\}.$$

Thus $M^{2p}(Au, Su, kt) \;\geq\; M^{s+s'}(Su, Au, t).$

This gives $M(Au, Su, kt) \;\geq\; M(Su, Au, t).$

Therefore by Lemma 2.2, we have $Au = Su$. The weak compatibility of A and S implies that $ASu = SAu$ *and then* $AAu = ASu = SAu = SSu.$ On the other hand, since $A(X) \subset T(X)$, there exists a point $v \in X$ such that $Au = Tv$. We claim that $Tv = Bv$ using (3.3) with $\alpha = 1$, we have

$$M^{2p}(Au, Bv, kt) \;\geq\; min\,\{\; M^{2p}(Su, Tv, t), M^{q}(Su, Au, t).\, M^{q'}(Tv, Bv, t)\,,$$
$$M^{r}(Su, Bv, t).\, M^{r'}(Tv, Au, t),$$
$$M^{s}(Su, Au, t).\, M^{s'}(Tv, Au, t),$$
$$M^{l}(Su\,, Bv, t).\, M^{l'}(Tv, Bv, t)\,\},$$

$$M^{2p}(Au, Bv, kt) \;\geq\; min\,\{\; M^{2p}(Au, Au, t), M^{q}(Au, Au, t).\, M^{q'}(Au, Bv, t)\,,$$
$$M^{r}(Au, Bv, t).\, M^{r'}(Au, Au, t),$$
$$M^{s}(Au, Au, t).\, M^{s'}(Au, Au, t),$$
$$M^{l}(Au\,, Bv, t).\, M^{l'}(Au, Bv, t)\,\}.$$

Thus

$$M^{2p}(Au, Bv, kt) \geq M^{l+l'}(Au\,, Bv, t).$$

It follows that

$$M(Au, Bv, kt) \geq M(Au\ , Bv, t).$$

Therefore by Lemma 2.2, we have $Au = Bv$.

Thus $Au = Su = Tv = Bv$. The weak compatibility of B and T implies that $BTv = TBv$ and $TTv = TBv = BTv = BBv$. Let us show that Au is a common fixed point of A, B, S *and* T. In view of (3.3) with $\alpha = 1$, we have

$$M^{2p}(AAu, Bv, kt) \geq min\{ M^{2p}(SAu, Tv, t), M^q(SAu, AAu, t). M^{q'}(Tv, Bv, t),$$
$$M^r(SAu, Bv, t). M^{r'}(Tv, AAu, t),$$
$$M^s(SAu, AAu, t). M^{s'}(Tv, AAu, t),$$
$$M^l(SAu\ , Bv, t). M^{l'}(Tv, Bv, t)\},$$

$$M^{2p}(AAu, Au, kt) \geq min\{ M^{2p}(AAu, Au, t), M^q(AAu, AAu, t). M^{q'}(Au, Au, t),$$
$$M^r(AAu, Au, t). M^{r'}(Au, AAu, t),$$
$$M^s(AAu, AAu, t). M^{s'}(Au, AAu, t),$$
$$M^l(AAu\ , Au, t). M^{l'}(Au, Au, t)\},$$

$$M^{2p}(AAu, Au, kt) \geq M^{2p}(AAu, Au, t).$$

This gives $M(AAu, Au, kt) \geq M(AAu, Au, t)$.

Therefore by Lemma 2.2, we have $Au = AAu = SAu$ and Au is a common fixed point of A *and* S.

Similarly, we can prove that Bv is a common fixed point of B and T. Since $Au = Bv$, we conclude that Au is a common fixed point of A, B, S *and* T.

If $Au = Bu = Su = Tu = u$ *and* $Av = Bv = Sv = Tv = v$, then by (3.3) with $\alpha = 1$, we have

$$M^{2p}(Au, Bv, kt) \geq min\{ M^{2p}(Su, Tv, t), M^q(Su, Au, t). M^{q'}(Tv, Bv, t),$$
$$M^r(Su, Bv, t). M^{r'}(Tv, Au, t),$$
$$M^s(Su, Au, t). M^{s'}(Tv, Au, t),$$
$$M^l(Su\ , Bv, t). M^{l'}(Tv, Bv, t)\},$$

$$M^{2p}(u, v, kt) \geq min\{ M^{2p}(u, v, t), M^q(u, u, t). M^{q'}(v, v, t),$$
$$M^r(u, v, t). M^{r'}(v, u, t),$$
$$M^s(u, u, t). M^{s'}(v, u, t),$$
$$M^l(u\ , v, t). M^{l'}(v, v, t)\},$$

This gives $M^{2p}(u, v, kt) \geq M^{2p}(u, v, t)$.

Thus $M(u, v, kt) \geq M(u, v, t)$.

By Lemma 2.2, we have $u = v$ and the common fixed point is a unique. This completes the proof of the theorem.

If we take $S = T$ in Theorem 3.1, we have the following.

Corollary 3.1. Let $(X, M, *)$ be a fuzzy metric space with $t^*t \geq t$ *for all* $t \in [0,1]$ and condition $(FM-6)$. Let A, B *and* S be mappings of X into itself such that

(3.6) $A(X) \subset S(X)$ *and* $B(X) \subset S(X)$,

(3.7) $\{A, S\}$ *or* $\{B, S\}$ satisfies the property $(S-B)$,

(3.8) there exists a constant $k \in (0,1)$ such that

$$M^{2p}(Ax, By, kt) \geq min\{M^{2p}(Sx, Ax, t)\ , M^r(Sx, By, t). M^{r'}(Sx, Ax, t),$$
$$M^s(Sx, Ax, t). M^{s'}(Sx, By\ t)\},$$

for all $x, y \in X\ , t > 0$ *and* $0 < p, r, r', s, s' \leq 1$, such that $2p = r + r' = s + s'$.

(3.9) if the pairs $\{A, S\}$ and $\{B, S\}$ are weakly compatible, (3.10) one of $A(X), B(X)$ or $S(X)$ is closed subset of X, then A, B and S have a unique common fixed point in X.

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