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Common Fixed Point Theorem for Two Mapping in Fuzzy Metric Space

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Abstract
In this present paper we wil find Common fixed theorem for two mapping in fuzzy metric space taking the concept of integral type mappings.
Keywords: Fixed point, Common fixed point, S-fuzzy metric space, Integral type mapping, Lebesgue integral.
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1. Introduction and Preliminaries

Ever since the notion of fuzzy set was introduced by Zadeh[7]in 1965, the concept of fuzzy metric space was introduced by various authors in different directions.Especially,Deng[1], Erceg[2], Kaleva and Seikkala[4],Karmosil and Michalek[5] have introduced the concept of fuzzy metric space in different ways. George and Veeramani[3]modified the concept of fuzzy metric spaces in the sense of Karmosil and Michalek[5] and defined the Hausdorff topology of fuzzy metric spaces.Consequently they showed every metric induces a fuzzy metric. Mishra, Sharma and Singh[6]also proved some fixed point theorem in fuzzy metric spaces. Sushil Sharma [8] also proved common fixed point theorems for six mappings.

In fuzzy metric space Banach's contraction mapping principle [10] is one of the pivotal results of nonlinear analysis. It has been the source of metric fixed point and its significance rests in its applicability in different branches of mathematics.

Theorem 2.1[10] Let (X,d) be a complete metric space, c Є [0,1) and f:X→X be a mapping such that for each x,y Є X,

$$d(fx,fy) \leq cd(x,y),$$

then f has a unique fixed point a Є X , such that for each x Є X, $\text{Lim}_{n\to+\infty} f_n x = a$.

In 2002, Branciari [11]obtained a fixed point theorem for a single mapping satisfying an analogue of a Banach contraction principle for integral type inequality. After the paper of Branciari , a lot of research works have been carried out on generalizing contractive conditions of integral type for different contractive mappings satisfying various known properties.

Theorem 2.2(Branciari)[11] Let (X,d) be a complete metric space , cϵ(0,1) and let f:X → X be a mapping such that for each x,yϵX,

$$\int_0^{d(fx,fy)} \varphi(t)dt \leq c\int_0^{d(x,y)} \varphi(t)dt$$

Where $\varphi: [0,+\infty) \to [0,+\infty)$ is a Lesbesgue – integrable mapping which is summable on each compact subset of $[0,+\infty)$, *nonnegative, and such that for each* $\epsilon > 0$,
$\int_0^{\epsilon} \varphi(t) > 0$, *then f has a unique fixed point a* ϵ *X* such that for each xϵX, $\lim_{n\to\infty} f^n x = a$

Theorem 2.3[12] Let (X,d) be a complete metric space and f: X →X such that

$$\int_0^{d(fx,fy)} u(t)dt \leq \alpha \int_0^{d(x,fx)+d(y,fy)} u(t)dt + \beta \int_0^{d(x,y)} u(t)dt + \gamma \int_0^{\max\{d(x,fy),d(y,fy)\}} u(t)dt,$$

For each x,y ϵ X with non-negative reals α,β,γ such that $2\alpha+\beta+2\gamma<1$, Where $u: [0,+\infty) \to [0,+\infty)$ is a Lesbesgue – integrable mapping which is summable,non-negative and such that for each $\epsilon > 0$,

$$\int_0^{\varepsilon} u(t)dt > 0.$$

Then f has a unique fixed point in X.

There is a gap in the proof of Theorem 2.3 . In fact , the authors [12]

Used the inequality $\leq \int_0^a u(t)dt + \int_0^b u(t)dt$ for 0 ≤a<b, which is not true in general. The aim of the paper is to present in the presence of this inequality an extension of Theorem 2.3 using altering diatance functions. On taking the concept of Branciari we establish some common fixed point theorem for two mapping in S-fuzzy metric space.

Definition- [2.4] The 3-tuple (X,S.*) is said to be a S-Fuzzy Metric *Space if X is an arbitrary Set, * is a continuous t-norm and S is a Fuzzy* set on X^3 x (0,∞) satisfying the following conditions.

(i) S(x,y,z,t) >0
(ii) S(x,y,z,t)=1 if and only if x=y=z (Coincidence)

(iii) $S(x,y,z,t)= S(y,z,x,t)= S(z,y,x,,t)$ (Symmetry)
(iv) $S(x,y,z,r+s+t)\geq S(x,y,w,r)* S(x,w,z,s)*S(w,y,z,t)$ (Tetrahedral inequality)
(v) $S(x,y,z,.); (o, \infty)\rightarrow[0,1]$ is continuous for all x,y,z,w Є X and r,s,t >0

Geometrically S(x,y,z,t)represents the Fuzzy Perimeter of the triangle whose vertices are the points x,yand z with respect to t >0.

Definition- [2.5] A Sequence $\{x_n\}$ in a S-Fuzzy Metric Space (X,S,*) is called A Cauchy Sequence if and only if for each $\varepsilon >0$, t>0 there exists n_0Є N such that $S(x_n,x_m,x_p,t) > 1- \varepsilon$ for all $n,m,p \geq n_0$

Definition- [2.6] A binary operation *:[o,1] X [o,1] →[0,1] is a (continuous) t-norm if ([0,1],*) is an abelian (topological) monoid with unit 1 such that a*b≤ c*d whenever a≤ c and b≤d (a,b,c,d ϵ [0,1])

Definition- [2.7] The 3-tuple (X,S,*) is said to be a Fuzzy Metric Space if X is an arbitrary Set,* is continuous t-norm and S is Fuzzy set on X^2 x(o, ∞).

(i) $S(x,y,t) >0$
(ii) $S(x,y,t)=1$ for all t>0 if and only if x=y
(iii) $S(x,y,t)= S(y,x,t)$
(iv) $S(x,y,t)* S(y,z,s) \leq S(x,z,t+s)$
(v) $S(x,y,.); (o, \infty)\rightarrow[0,1]$ is a continuous for all x,y,z ϵ X and t,s>0

Definition- [2.8] A Sequence $\{x_n\}$ in a Fuzzy Metric Space (X,S,*) converges to x in X if and only if $S(x_n,x,t) \rightarrow 1$ as $n\rightarrow\infty$

Definition- [2.9] A Sequence $\{x_n\}$ in a Fuzzy Metric Space (X,S,*) is said to be a Cauchy Sequence if and only if for each $\varepsilon >0$, t>0 there exists n_0Є N such that $S(x_n,x_m,t) > 1- \varepsilon$ for all $n,m \geq n_0$

Definition- [2.10] A Fuzzy Metric Space (X,S,*) is said to be complete if every Cauchy Sequence in (X,S,*) is a convergent sequence.

Definition- [2.11] A S- Fuzzy Metric Space in which every Cauchy Sequence is a convergent sequence , is called a Complete S- Fuzzy Metric Space.

2. Main Result

Theorem-3.1:- Let T & P be two self mappings of a complete Fuzzy Metric Space (X,S,*) with t-norm * defined by a*b=min{a,b} : a,b Є [0,1]
Satisfying the conditions

$$\int_0^{S(Tx,Py,z,kt)} \varphi(t)dt \geq \int_0^{\min\{S(x,y,z,t),S(x,Tx,z,t),S(y,Py,z,t),\frac{S(x,Tx,z,t)S(y,Py,z,t)}{S(x,y,z,t)},\}} \varphi(t)dt$$

for all x,y,z in X and 0<k<1, t > 0

(2.5) $S(x,y,z,t) \rightarrow 1$ as $t\rightarrow\infty$

Then T & P have a unique common fixed point.

Proof:- Consider an arbitrary point x_0 in X and define a sequence $\{x_n\}$ in X by $x_{2n+1}= Tx_{2n}$, $x_{2n+2} = Px_{2n+1}$ for all n = 0,1,2……….

On using (2.4) for any p Є N, we have

$$\int_0^{S(x_1,x_2,x_p,kt)} \varphi(t)dt = \int_0^{S(Tx_0,Px_1,x_p,kt)} \varphi(t)d\,t$$

$$\geq \int_0^{\min\{S(x_0,x_1,x_p,t),S(x_0,Tx_0,x_p,t),S(x_1,Px_1,x_p,t),\frac{S(x_0,Tx_0,x_p,t)S(x_1,Px_1,x_p,t)}{S(x_0,x_1,x_p,t)}\}} \varphi(t)dt$$

$$\geq \int_0^{\min\{S(x_0,x_1,x_p,t),S(x_0,x_1,x_p,t),S(x_1,x_2,x_p,t),\frac{S(x_0,Tx_1,x_p,t)S(x_1,x_2,x_p,t)}{S(x_0,x_1,x_p,t)}\}} \varphi(t)dt$$

$$\geq \int_0^{\min\{S(x_0,x_1,x_p,t),S(x_1,x_2,x_p,t)\}} \varphi(t)dt$$

This implies that

$$\int_0^{S(x_1,x_2,x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_0,x_1,x_p,t)} \varphi(t)dt$$

Again using (2.4) for any PϵN we have

$$\int_0^{S(x_2,x_3,x_p,kt)} \varphi(t)dt = \int_0^{S(P,Tx_2,x_p,kt)} \varphi(t)dt$$

$$=\int_0^{S(Tx_1,Px_1,x_p,kt)} \varphi(t)dt$$

$$\geq\int_0^{\min\{S(x_2,x_1,x_p,t),S(x_2,Tx_2,x_p,t),S(x_1,Px_1,x_p,t),\frac{S(x_2,Tx_2,x_p,t)S(x_1,Px_1,x_p,t)}{S(x_1,x_2,x_p,t)}\}} \varphi(t)dt$$

$$\geq \int_0^{\min\{S(x_1,x_2,x_p,t),S(x_2,x_3,x_p,t),S(x_1,x_2,x_p,t),\frac{S(x_2,x_3,x_p,t)S(x_1,x_2,x_p,t)}{S(x_1,x_2,x_p,t)}\}} \varphi(t)dt$$

$$\geq \int_0^{\min\{S(x_2,x_3,x_p,t),S(x_1,x_2,x_p,t)\}} \varphi(t)dt$$

This implies that

$$\int_0^{S(x_2,x_3,x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_1,x_2,x_p,t)} \varphi(t)d\,t$$

Inductively we have

$$\int_0^{S(x_n,x_{n+1},x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_{n-1},x_n,x_p,t)} \varphi(t)d\,t$$

$$\geq \int_0^{S(x_{n-2},x_{n-1},x_p,t/k)} \varphi(t)d\,t$$

$\geq$ ……………………

$\geq$ ……………………

$$\geq \int_0^{S(x_0,x_1,x_p,t/k^{n-1})} \varphi(t)d\,t$$

Or

$$\int_0^{S(x_n,x_{n+1},x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_0,x_1,x_p,t/k^n)} \varphi(t)d\,t$$

So for p,q ЄN & t>0 we have for k=3

$$\int_0^{S(x_n,x_{n+p},x_{n+p+q},3t)} \varphi(t)dt$$

$$\geq \int_0^{S(x_n,x_{n+1},x_{n+p+q},t)*S(x_n,x_{n+1},x_{n+p},t)*S(x_{n+1},x_{n+p},x_{n+p+q},t)} \varphi(t)d\,t$$

$$\geq \int_0^{A_1*A_2*A_3*A_4} \varphi(t)d\,t$$

Where $A_1 = S\left(x_0, x_1, x_{n+p+q}, \frac{t}{k^n}\right)$,

$A_2 = S\left(x_0, x_1, x_{n+p}, \frac{t}{k^n}\right)$,

$A_3 = S(x_{n+1}, x_{n+2}, x_{n+p+q}, t/3)$,

$A_4 = S\left(x_{n+2}, x_{n+P}, x_{n+p+q}, \frac{t}{3}\right)$

$$\geq \int_0^{A_1*A_2*A_5*A_6*A_7)} \varphi(t)d\,t$$

Where $A_5 = S(x_0, x_1, x_{n+p+q}, t/3k^{n+1})$,

$A_6 = S(x_0, x_1, x_{n+p}, t/3k^{n+1})$,

$A_7 = S(x_{n+2}, x_{n+p}, x_{n+p+q}, t/3)$,

$$\geq \int_0^{A_1*A_2*A_5*A_6*A_7} \varphi(t)d\,t$$

$$\geq \int_0^{A_1*A_2*A_5*A_6*A_7*\ldots\ldots} \varphi(t)d\,t$$

Where $A_5 = S(x_0, x_1, x_{n+p+q}, t/3k^{n+1})$,

$A_6 = S(x_0, x_1, x_{n+p}, t/3k^{n+1})$,

$A_7 = S(x_{n+2}, x_{n+p}, x_{n+p+q}, t/3)$,

$$\geq \int_0^{A_1*A_2*A_5*A_6*\ldots*A_8*A_9*A_{10}} \varphi(t)d\,t$$

Where $A_8 = S(x_0, x_1, x_{n+p+q}, t/k^{n+P-2}3^{p-2})$,

$A_9 = S(x_0, x_1, x_{n+p}, t/k^{n+p-2}3^{p-2})$,

$A_{10} = S\left(x_{n+p-1}, x_{n+p}, x_{n+p+q}, \frac{t}{3^{p-2}}\right)$,

$$\geq \int_0^{A_1*A_2*A_5*A_6*\ldots*A_8*A_9*A_{11}} \varphi(t)d\,t$$

Where $A_{11} = S(x_0, x_1, x_{n+p+q}, t/3^{P-1}k^{n+p-1})$,

On taking lim n→∞ we have

$\lim_{n\to\infty} S(x_n,x_{n+p},x_{n+p+q},3t) \geq 1*1*1*$……………………..(2p-1) times

Which implies that

$S(x_n,x_{n+p},x_{n+p+q},3t) \to$ as n→∞

i.e. for even ϵ>0,t>0,∃ n_0ЄN such that

$S(x_n,x_{n+p},x_{n+p+q},t) > 1-\epsilon$ for all $n \geq n_0$

Thus $\{x_n\}$ is a Cauchy Sequence. By the completeness of the space , there is a point u in X such that

$\lim_{n\to\infty} x_n = u$

Now we shall prove that u is a fixed point of T

By(2.4) we have

$$\int_0^{S(Tu,x_{2n+2},x_n kt)} \varphi(t)dt \geq \int_0^{S(Tu,Px_{2n+1},x_n,kt)} \varphi(t)dt,$$

$$\geq \int_0^{\min\{S(u,x_{2n+1},x_n,t),S(uTu,x_{n,}t),S(x_{2n+1},Px_{2n+1},X_n,t)\},\frac{S(u,Tu,x_n,t)S(X_{2n+1},Px_{2n+1},x_n,t)}{S(u,x_{2n+1},x_n,t)}} \varphi(t)dt$$

On taking $lim_{n\to\infty}$ we have

$$\int_0^{S(Tu,u,u,kt)} \varphi(t)dt \geq \int_0^{\min\{S(u,Tu,u,t),1\}} \varphi(t)dt$$

$$= \int_0^{S(u,Tu,u,t)} \varphi(t)dt$$

Which yields Tu=u

Similarly we can prove Pu=u

Thus Tu=u=Pu

Hence u is a common fixed point of T&P. For the uniqueness of u, let v be another common fixed point of T&P. Then by (2.4) we have

$$\int_0^{S(U,U,V,Kt)} \varphi(t)dt = \int_0^{S(Tu,Pu,v,kt)} \varphi(t)dt$$

$$\geq \int_0^{S(u,u,v,t)} \varphi(t)dt$$

Which gives us u=v

To prove T&P are continuous at u. Let $\{y_n\}$ be a sequence in X such that

$\lim_{n\to\infty} y_n = u$

On using (2.4) we have

$$\int_0^{S(Ty_n,x_{2n+2},x_m,kt)} \varphi(t)dt = \int_0^{Ty_n,Px_{2n+1},x_m,kt} \varphi(t)dt$$

$$\int_0^{\min\{s(y_n,x_{2n+1},x_m,t),S(\quad,\quad,\quad,),S()\}}$$

$$\geq \min\{S(y_n,x_{2n+1},x_m,t),S(y_n,Ty_n,x_m,t),S(x_{2n+1},Px_{2n+1},x_m,t),\frac{S(yn,Tyn,xm,t)S(x2n+1,Px2n+1,xm,t)}{S(yn,x2n+1,xm,t)}\}$$

On taking $\lim_{n\to\infty}$ or $_{m\to\infty}$ we have

$S(\lim Ty_n,u,u,kt) \geq \min\{1,S(u,\lim Ty_n,u,t)\}$

This implies that

$\lim_{n\to\infty} Ty_n = u = Tu = T \lim_{n\to\infty} y_n$

Hence T is continuous at u, Similarly we can show that Pis continuous at u.

Theorem-3.2:- Let $\{T_n\}$ & $\{P_n\}$ be two self mappings of a complete Fuzzy Metric Spaces (X,S,*) with t-norm * defined by

a*b = min {a,b : a,b Є [0,1]} satisfying the conditions

$$\int_0^{S(T_{ix},P_{jy},z,kt)} \varphi dt \geq \int_0^{\min\{S(x,y,z,t),S(x,T_{ix},z,t),S(y,P_{jy},z,t),\frac{S(x,T_{ix},z,t)S(y,P_{jy}z,t)}{S(x,y,z,t)}\}} \varphi dt$$

For all x,y,z in X: 0<K<1, t>0, i,j Є N

$S(x,y,z,t) \to 1$ as $t \to \infty$

Then $\{T_n\}$ and $\{P_n\}$have a unique common fixed point in X.

Proof:- Consider an arbitrary point x0inX. Define a sequence {Xn} such that $x_{2n+1}=T_{2n+1}x_{2n}$ and $x_{2n+2}=P_{2n+2}x_{2n+1}$ ∀ n=0,1,2,…………

On using for any PЄn we have

$$\int_0^{S(x_1,x_2,x_p,kt)} \varphi dt = \int_0^{S(T_{1x_0},P_{2x_1},x_p,kt)} \varphi dt$$

$$\geq \int_0^{\min\{S(x_0,x_1,x_p,t),S(x_0,T_{1x_0},x_p,t),S(x_1,P_{2x_1},x_p,t),\frac{S(x_0,T_{1x_0},x_p,t)S(x_1,P_2x_1,x_p,t)}{S(x_0,x_1,x_p,t)}\}} \varphi dt$$

$$\geq \int_0^{\min\{S(x_0,x_1,x_p,t),S(x_0,x_1,x_p,t),S(x_1,x_2,x_p,t),\frac{S(x_0,x_1,x_p,t)S(x_1,x_2,x_p,t)}{S(x_0,x_1,x_p,t)}\}} \varphi dt$$

Which implies that

$$\int_0^{S(x_1,x_2,x_p,kt)} \varphi dt \geq \int_0^{S(x_0,x_1,x_p,t)} \varphi dt$$

Again using (2.2.1) for any p∈N we have

$$\int_0^{S(x_2,x_3,x_p,kt)} \varphi dt = \int_0^{S(P_2x_1,T_3x_2,x_p,kt)} \varphi dt$$

$$\geq \int_0^{min\{S(x_2,x_1,x_p,t),S(x_2,T_{3x_2},x_p,t)S(x_1,P_{2x_1},x_p,t)\},\frac{S(x_2,T_{3x_2},x_p,t)S(x_1,P_{2x_1},x_p,t)}{S(x_1,x_2,x_p,t)}\}} \varphi(t)dt$$

$$\geq \int_0^{min\{S(x_1,x_2,x_p,t),S(x_2,x_3,x_p,t)S(x_1,x_2,x_p,t)\},\frac{S(x_2,x_3,x_p,t)S(x_1,x_2,x_p,t)}{S(x_1,x_2,x_p,t)}\}} \varphi(t)dt$$

Which implies that

$$\int_0^{S(x_2,x_3,x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_1,x_2,x_p,t)} \varphi(t)dt$$

Inductively we have

$$\int_0^{S(x_n,x_{n+1},x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_{n-1},x_n,x_p,t)} \varphi(t)dt$$

$$\geq \int_0^{S(x_{n-2},x_{n-1},x_p,t/k)} \varphi(t)dt$$

$$\geq \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\geq \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\geq \int_0^{S(x_0,x_1,x_p,t/k^{n-1})} \varphi(t)dt$$

Or

$$\int_0^{S(x_n,x_{n+1},x_p,kt)} \varphi(t)dt \geq \int_0^{S(x_0,x_1,x_p,t/k^n)} \varphi(t)dt$$

So for p,q ∈N & t>0 we have for k=3

$$\int_0^{S(x_n,x_{n+p},x_{n+p+q},3t)} \varphi(t)dt \geq \int_0^{S(x_n,x_{n+1},x_{n+p+q},t)*S(x_n,x_{n+1},x_{n+p},t)*S(x_{n+1},x_{n+p},x_{n+p+q},t)} \varphi(t)dt$$

$$\geq \int_0^{S(x_0,x_1,x_{n+p+q},t/k^n)*S(x_0,x_1,x_{n+p},t/k^n)*S\left(x_{n+1},x_{n+2},x_{n+p+q},\frac{t}{3}\right)*S(x_{n+2},x_{n+p},x_{n+p+q},t/3)} \varphi(t)dt \geq$$

$$\int_0^{A_1*A_2*A_3*A_4*A_5} \varphi(t)dt$$

Where A_1= S$(x_0, x_1, x_{n+p+q}, t/k^n)$
A_2= S$(x_0, x_1, x_{n+p}, t/k^n)$
A_3= S$(x_0, x_1, x_{n+p+q}, t/3k^{n+1})$
A_4= S$(x_0, x_1, x_{n+p}, t/3k^{n+1})$
A_5= S$(x_{n+2}, x_{n+p}, x_{n+p+q}, t/3)$

$$\geq \int_0^{A_1*A_2*A_3*A_4*A_5\ldots\ldots\ldots\ldots} \varphi(t)dt$$

Where A_1= S$(x_0, x_1, x_{n+p+q}, t/k^n)$
A_2= S$(x_0, x_1, x_{n+p}, t/k^n)$
A_3= S$(x_0, x_1, x_{n+p+q}, t/3k^{n+1})$
A_4= S$(x_0, x_1, x_{n+p}, t/3k^{n+1})$
A_5= S$(x_{n+2}, x_{n+p}, x_{n+p+q}, t/3)$

$$\geq \int_0^{A_1*A_2*A_3*A_4*\ldots\ldots\ldots*A_6*A_7*A_8} \varphi(t)dt$$

Where A_1= S$(x_0, x_1, x_{n+p+q}, t/k^n)$
A_2= S$(x_0, x_1, x_{n+p}, t/k^n)$
A_3= S$(x_0, x_1, x_{n+p+q}, t/3k^{n+1})$
A_4= S$(x_0, x_1, x_{n+p}, t/3k^{n+1})$
A_6= S$(x_0, x_1, x_{n+p+q}, t/k^{n+p-2}3^{p-2})$
A_7= S$(x_0, x_1, x_{n+p}, t/k^{n+p-2}3^{p-2})$
A_8= S$(x_{n+p-1}, x_{n+p}, x_{n+p+q,} t/3^{p-2})$

$$\geq \int_0^{A_1*A_2*A_3*A_4*\ldots\ldots\ldots*A_6*A_7*A_9} \varphi(t)dt$$

Where A_1= S$(x_0, x_1, x_{n+p+q}, t/k^n)$
A_2= S$(x_0, x_1, x_{n+p}, t/k^n)$
A_3= S$(x_0, x_1, x_{n+p+q}, t/3k^{n+1})$
A_4= S$(x_0, x_1, x_{n+p}, t/3k^{n+1})$
A_6= S$(x_0, x_1, x_{n+p+q}, t/k^{n+p-2}3^{p-2})$

$$A_7 = S(x_0, x_1, x_{n+p}, t/k^{n+p-2}3^{p-2})$$
$$A_9 = S(x_0, x_1, x_{n+p+q}, t/3^{p-1}k^{n+p-1})$$

On taking lim n→∞ we have

$\lim_{n\to\infty} S(x_n, x_{n+p}, x_{n+p+q}, 3t) \geq 1*1*1*$……………………..(2p-1) times

Which implies that

$S(x_n, x_{n+p}, x_{n+p+q}, 3t) \rightarrow$ as $n\to\infty$

i.e. for even $\epsilon>0, t>0, \exists\ n_0 \in N$ such that

$S(x_n, x_{n+p}, x_{n+p+q}, t) > 1-\epsilon$ for all $n \geq n_0$

Thus $\{x_n\}$ is a Cauchy Sequence. By the completeness of the space , there is a point u in X such that

$\lim_{n\to\infty} x_n = u$

Now we shall prove that u is a fixed point of T_i

By(2.4) we have

$$\int_0^{S(T_i u, x_{2n+2}, x_n, kt)} \varphi(t)dt \geq \int_0^{S(T_i u, P_{2n+2}x_{2n+1}, x_n, kt)} \varphi(t)dt$$
$$\geq \int_0^{\min\{S(u,x_{2n+1},x_n,t), S(u,T_i u,x_n,t), S(x_{2n+1},P_{2n+2}x_{2n+1},x_n,t), \frac{S(u,T_i u,x_n,t)S(x_{2n+1},P_{2n+2}x_{2n+1},x_n,t)}{S(u,x_{2n+1},x_n,t)}\}} \varphi(t)dt$$

On taking $\lim_{n\to\infty}$ we have

$$\int_0^{S(T_i u,u,u,kt)} \varphi(t)dt \geq \int_0^{\min\{S(u,T_i u,u,t),1\}} \varphi(t)dt$$
$$= \int_0^{S(u,T_i u,u,t)} \varphi(t)dt$$

Which yields $T_i u = u$

Similarly we can prove $P_j u = u$

Thus u is common fixed point of T_i and P_j

To prove uniqueness, let v be another common fixed point of T_i and P_j.

Then by (2.2.1) we have

$$\int_0^{S(u,u,v,kt)} \varphi(t)dt = \int_0^{S(T_i u, P_j u, v, kt)} \varphi(t)dt$$
$$\geq \int_0^{S(u,u,v,t)} \varphi(t)dt$$

Which gives u=v

Thus u is a unique common fixed point of T_i and P_j

Now

To prove that T_i and P_j are continuous at u. Let $\{y_n\}$ be a sequence in X such that

$\lim_{n\to\infty} y_n = u$

On using (2.4) we have

$$\int_0^{S(T_i y_n, x_{2n+2}, x_m, kt)} \varphi(t)dt = \int_0^{S(T_i y_n, P_j x_{2n+1}, x_m, kt)} \varphi(t)dt$$
$$\geq \int_0^{\min\{S(y_n,x_{2n+1}x_m,t), S(y_n,T_i y_n x_m,t), S(x_{2n+1},P_j x_{2n+1},x_m,t), -\}}$$
$$\geq \min\{S(y_n,x_{2n+1},x_m,t), S(y_n,T_i y_n,x_m,t), S(x_{2n+1},P_j x_{2n+1},x_m,t), \frac{S(y_n,T_i y_n,x_m,t)S(x_{2n+1},P_j x_{2n+1},x_m,t)}{S(y_n,x_{2n+1},x_m,t)}\}$$

On taking $\lim_{n\to\infty}$ or ${}_{m\to\infty}$ we have

$S(\lim T_i y_n, u, u, kt) \geq \min\{1, S(u, \lim T_i y_n, u, t)\}$

This implies that

$\lim_{n\to\infty} T_i y_n = u = T_i u = T_i \lim_{n\to\infty} y_n$

Hence T_i is continuous at u, Similarly we can show that P_j is continuous at u.

Therefore T_i and P_j are continuous at u.

This completes the proof.

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