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Distribution of Sibship Sizes and Correlation between Numbers of Male and Female Children in A Family: Evidence from Sub-Saharan Africa

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Abstract
This paper is concerned with the re-investigation of the distribution of sibship sizes as negative binomial distribution, truncated below one and computation of correlation between the numbers of boys and girls in a family. The distribution involves two parameters m and p and these are estimated by the method given in Brass (1958c) which has asymptotic efficiency more than 90 % in comparison to maximum likelihood. It is also found that the scoring method provides the similar estimates as given by Brass. The suitability of the distribution and correlation is tested using 10% sample of new DHS data gathered recently on 26 sub-Saharan African countries dividing in Western-Central, Eastern-Southern, implying a mean sibship size of 4.22 and 3.83 respectively which is significant at 1% level of probability. The observed and expected correlation coefficients are found to be the same in each country. Through the values of χ^2 at 1% and 5% level, we deem the fit to be very good except in few cases. The values of m and p will be useful to generate the underlying sibship sizes for the simulation study.
Keywords: Sibship sizes,Negative binomial distribution,Maximum likelihood,Simulation,Correlation.

1. Introduction

Considerable evidence has accumulated by human biologists, geneticists, demographers and social scientists during last few decades about the distribution of completed family size (sibship size) based on different sets of data in sub-Saharan Africa (Frank, 1983; Menken *et al*., 1986; Menken and Larsen, 1994; Larsen and Menken, 1989, 1991; Larsen, 1994, 1995, 2004; Sunil and Pillai, 2002) but few has been devoted to the description of sibship sizes in the form of probability distribution. Distributions frame to represent approximately events or process which may lead to a better understanding of underlying factors. Often, they suggest useful way of describing and analyzing the data. Perhaps the main reasons for fewer attempts to develop analytical distribution for the numbers of sibship sizes in sub-Saharan Africa are a belief that the behavior of this process can be adequately represented by simple methods such as rates, parity progression ratios and percentages etc. However, empirical statistics seems to show that the sibship size fits better with a negative binomial distribution (Kojima and Kelleher, 1962). This distribution has been discussed by many researchers in the past and is extensively used for the description of the data too heterogeneous to be fitted by a Poisson distribution.

Brass (1958a) has suggested the negative binomial for the distribution of births in human populations using the data from four African countries. Further, Waller *et al*. (1973) have noticed that the number of childless families is much larger than the expected number of childless families when they fit a negative binomial distribution to the observed frequencies of completed family size from various sources although the fit is good for the rest of the distribution. This led them to suggest that the childless family is a mixture of two types of families. The first type is biologically fertile and could have children, but by chance they did not. This type of families should be a part of the general negative binomial distribution of family size. The second type is either biologically or electively not fertile (sterile) thus has no children. This should not be a part of the general negative binomial distribution of family size. The proportion of this type of families is expected to vary among population studies, due to socio-cultural factors influencing the deliberate choice to have no children.

Rao *et al*. (1973) have also suggested the completed family size (more precisely, sibship size) (N) distribution to be a random variable following a negative binomial distribution. They divided the frequency of childless families into a sterile group that did not have any children and a fertile group. They have not included the frequency of former group in the general distribution of family size.

Hamdan (1975) has also considered the distribution of family size a zero truncated negative binomial when the number of childless families (N=0) is not available or cannot be estimated.

Further, Sharma (1991) has considered the distribution of family size as negative binomial and Poisson distribution with zeroes deriving an estimator of proportion sterile. He has also determined the correlation coefficient between the numbers of male and female offspring. Sharma tested the suitability of his distribution using the data of Reed and Reed (1965) and Brass (1958). The distribution consists of three parameters in each and these are estimated by method of moments and maximum likelihood. The distributions fit the data well.

Iwunor (2002) has also estimated the proportion of sterile couples as 0.0935 for the same data of Reed and Reed (1965), while Sharma's estimate was 0.10.

Recently, Mohammad Fraiwan and Fatima Khalid (2003) have estimated the proportion of sterile couples using the negative binomial distribution in Jordan indirectly based on the data of 1994 population census, by separating the two types of childless couples into sterile and fertile couples. They estimated the proportion of sterile couples to be 6.1%.

Sharma and Vishwakarma (2004) have proposed two inflated zero truncated probability distributions of the sibship sizes being truncated at the point zero, and inflated at the point one with an estimator of secondary sterility. They have also suggested the distribution for male and female offspring having correlation coefficient between these on the basis of same data of Reed and Reed (1965).

The objective of the present paper is to re-investigate negative binomial distribution, truncated below one, for describing the inherent variability for sibship sizes and to find out correlation between the numbers of two types of children in a family in sub-Saharan Africa. Observed samples, however, may be truncated, in the sense that the number of individuals falling into the zero class can not be determined or estimated. Section 2 deals with sources of data, distribution and its estimation procedure while Section 3 devotes its application. The discussion and conclusions are given at the end.

2. Data and Methods

The data for this paper come from the Demographic and Health Surveys conducted recently on 26 sub-Saharan African countries divided into two equal parts that is in first part, the countries are Benin, Burkina, Cameroon, Chad, Congo- Dem Republic, Congo- Brazilian, Ghana, Guinea, Liberia, Mali, Niger, Nigeria and Senegal, known as Western and Central African countries. In second part, the countries are Ethiopia, Kenya, Lesotho, Madagascar, Malawi, Mozambique, Namibia, Rwanda, Swaziland, Tanzania, Uganda, Zambia and Zimbabwe, known as Eastern and Southern African countries.

The DHS is an international data collection and analysis project coordinated by ORC Macro, in cooperation with national governments and organizations, and funded primarily by the US Agency for International Development. The surveys are typically nationally representative household surveys of women aged 15-49 and men aged 15-59 with large samples. They usually cover a wide range of indicators in the areas of reproductive behavior and health. Only data from the women surveys are used in this paper. All women interviewed were asked about the number of children they have had in their lifetime. In addition they were asked to indicate the number of boys and the number of girls. For the purpose of this paper women reported number of boys and number of girls ever had been cross tabulated.

Brass (1958b) has shown that the negative binomial distribution, truncated below one fits the observed distribution of sibship sizes very well in a wide variety of human populations, for appropriate choice of the parameters m and p in the probability density.

$$\Pr(x \text{ offspring}) = (x+m-1)!\, p^m (1-p)^x / x!\,(m-1)!\,(1-p^m) \text{ for } x= 1, 2, 3,\dots,\ q=1-p$$

where m indicates the shape parameter of the distribution and p is the probability of the success.

This distribution consists of two parameters m and p and these are estimated with the help of following estimators given by Brass (1958c).

$p^* = (\bar{x}/s^2)(1- n_1/n)$ and $m^* = (p^*x - n_1/n)/(1-p^*)$ which have the asymptotic efficiency to be above 90%, where $\bar{x}$ is the mean of the n non zero observations, n_1 is the number of ones in the sample, and $s^2= \Sigma^{R}_{x=1} n_x(x-\bar{x})^2/(n-1)$. For the asymptotic variances and co - variances of the maximum likelihood estimates for each country, we set up the information matrix with estimates substituted for parameters and with the partials given by Sampford (1955) equation (14). In order to compute the correlation between the number of boys and girls, we have used the equation number (4) and (5) of Rao *et al.* (1973). The 95% confidence limits for the correlation coefficients have also been computed with usual formula of standard error of correlation and given in the respective tables. For finding out the average sibship size of Western and Central Africa, Eastern and Southern Africa with their sample variances, the combined mean and combined variance formula have been used and significant difference between these two sibship sizes has been tested with the help of z-test.

3. Results

The suitability of the distribution is investigated using 10% sample data of sibship size of 26 countries in sub-Saharan Africa gathered recently divided into two parts, 13 belong to Western and Central Africa and rest, Eastern and Southern. The expected frequencies corresponding to observed ones and their graphical representations are given in Appendix -1 and Appendix -2 respectively.

It is important to understand that $m_1' = mq/p\,(1-p^m)$ provides the average number of sibship sizes in a family. It is a fact that mean in the distribution is dependent on higher values of m and q.

Table 3.1 reveals about the estimates of m and p along with correlation coefficient between the numbers of boys and girls in a family according to descending order of mean sibship sizes in 13 countries of Western and Central Africa.

Table 3.1 Estimated values of m and p along with correlation between boys and girls according to descending order of mean sibship sizes in 13 countries of Western and Central Africa.

Country	Mean sibship size	Estimated value		Correlation Coefficient			95% confidence limits	
		m	p	Observed*	Expected if gender proportions are not equal*	Expected if gender proportions are equal *	Lower	Upper
Niger	4.892	4.247	0.479	0.301	0.288	0.289	0.281	0.322
Chad	4.676	4.357	0.498	0.265	0.268	0.268	0.239	0.291
Mali	4.542	3.731	0.470	0.254	0.285	0.285	0.237	0.271
Nigeria	4.487	2.908	0.418	0.304	0.322	0.322	0.280	0.329
Burkina	4.357	4.098	0.504	0.232	0.249	0.250	0.213	0.251
Guinea	4.297	5.200	0.564	0.206	0.198	0.198	0.182	0.230
Congo Dem	4.144	3.099	0.454	0.286	0.283	0.282	0.265	0.307
Senegal	4.137	2.911	0.440	0.268	0.294	0.294	0.249	0.287
Benin	4.089	4.751	0.556	0.204	0.204	0.204	0.188	0.220
Cameroon	3.927	2.344	0.409	0.282	0.310	0.310	0.261	0.303
Liberia	3.820	2.979	0.468	0.260	0.258	0.257	0.236	0.284
Ghana	3.708	3.785	0.532	0.185	0.200	0.200	0.155	0.215
Congo Brazilian	3.278	2.991	0.516	0.211	0.188	0.188	0.185	0.237
Overall	**4.218**	**3.441**	**0.472**	**0.251**	**0.271**	**0.270**	**0.232**	**0.270**

* The correlation coefficients are significant at 1% level of probability.

The maximum and minimum mean sibship size belongs to Niger and Congo -Brazilian respectively. The estimated value of m is higher for Guinea and lower for Cameroon. There is little bit variation in the estimated values of p. The observed and expected correlation coefficients (if proportion of boys and girls are not equal/equal) are found to be significant at 1% level of probability and they are approximately the same. In addition to these, 95% confidence limits have also been given in the last column of the table.

The truncated negative binomial for the Western and Central African countries does not fit well to the Benin, Burkina, Guinea, Mali, Niger, Nigeria and Senegal because of their higher mean sibship sizes. In rest of the countries in Western and Central Africa the χ^2 values are just beyond of the significance level. In these countries, the various programmes of health, family planning, education of women, wage employment, digamous marriages etc. are the important reasons for implying the mean sibship sizes more than four, but where the distribution fits well, all these programmes have been implemented well. In order to describe the inherent variation for such higher mean sibship sizes, a mixture of two distributions may be suggested in future.

Table 3.2 Estimated values of m and p along with correlation between boys and girls according to descending order of mean sibship sizes in 13 countries of Eastern and Southern Africa.

Country	Mean sibship size	Estimated value		Correlation Coefficient			95% confidence limits	
		m	p	Observed*	Expected if gender proportions are not equal*	Expected if gender proportions are equal *	Lower	Upper
Uganda	4.657	3.947	0.476	0.274	0.281	0.281	0.251	0.297
Ethiopia	4.556	4.192	0.497	0.270	0.262	0.262	0.252	0.288
Rwanda	4.283	4.440	0.527	0.224	0.233	0.233	0.202	0.246
Zambia	3.978	3.596	0.499	0.244	0.240	0.240	0.219	0.269
Mozambique	3.902	3.074	0.469	0.255	0.260	0.260	0.237	0.273
Tanzania	3.860	3.024	0.468	0.276	0.265	0.265	0.256	0.297
Kenya	3.832	2.921	0.463	0.271	0.264	0.264	0.247	0.295
Malawi	3.823	3.330	0.493	0.229	0.242	0.242	0.210	0.248
Madagascar	3.795	2.633	0.441	0.292	0.288	0.288	0.269	0.315
Swaziland	3.233	2.014	0.437	0.257	0.247	0.247	0.226	0.288
Zimbabwe	3.062	2.829	0.524	0.194	0.170	0.170	0.170	0.218
Lesotho	3.045	2.483	0.501	0.143	0.175	0.175	0.115	0.171
Namibia	2.883	2.535	0.522	0.159	0.150	0.150	0.135	0.183
Overall	**3.833**	**2.899**	**0.461**	**0.238**	**0.265**	**0.266**	**0.218**	**0.258**

* The correlation coefficients are significant at 1% level of probability.

The maximum and minimum mean sibship size belongs to Uganda and Namibia respectively. The estimated value of m is higher for Rwanda and lower for Swaziland. Further, there is little bit variation in the estimated values of p. The observed and expected correlation coefficients (if proportion of boys and girls are not equal/equal) are found to be significant at 1% level of probability and they are approximately the same. In addition to these, 95% confidence limits have also been given in the last column of the table.

The truncated negative binomial for all the Eastern and Southern African countries fits well except in Ethiopia and Uganda. In these African countries, all the 10 countries do have the mean sibship size below four while in three countries, it is more than four. It reveals that the fertility in Eastern and Southern African countries is found to be smaller rather than that of their counterpart. In this context, Sibanda (1998) have clearly stated that the fertility change in sub-Saharan Africa can be attributed to shifts in starting, timing and/or stopping patterns of child bearing. He has used the parity progression ratios and conditional age parity specific birth probabilities to fertility decline in sub-Saharan Africa. He has utilized DHS data from Zimbabwe and Kenya and pointed out that the decline in fertility has been a pervasive phenomenon. He has also stated that the future decline of fertility in sub-Saharan Africa is being engineered by large declines in teenage fertility which are a result of postponement of first birth and a marked increase in the postponement of middle and higher births among women in the prime child bearing ages and women in their thirties. Samson Waga Wasao (1998) has rightly pointed out that social and economic factors are associated with family planning use and fertility at the district level in Kenya and state their policy implications. He has used the data from the Welfare Monitoring Survey II of 1994 using the multivariate regression procedure to examine the nature of the relationship between family planning use, fertility and selected socio-economic variables. His findings indicate that family planning use and fertility were significantly associated with female literacy, educational attainment and wage employment. Also monogamous marriage as well as smaller land size was significant determinants of family planning use and lower fertility.

4. Discussion

This paper presents a revisit of the truncated negative binomial distribution for describing the inherent variation in the number of sibship sizes per couple in 26 countries of sub-Saharan Africa divided into two parts, 13 belong to Western and Central Africa and rest, Eastern and Southern. The pattern of number of sibship sizes can be summarized by two parameters in the distribution m and p. where m indicates the shape parameter of the distribution and p is the probability of the success. A satisfactory fit is obtained by the distribution when the sample data are taken on the basis of 10 % for the Eastern and Southern African countries where the mean sibship sizes is estimated to be less than four. Further, for the Western and Central African countries the fit is not so adequate where the mean sibship size is more than four. This suggests that there must be another mixture of two distributions which can describe the pattern of sibship sizes where the average is more than four because there is little bit variation of the proportion of the couples at parity 1, 2 and 3. This has been shown by giving the values in each table in Appendix-1. Since the paradox of having a large body of records is that statistical test of fit will differentiate increasingly small deviation as significant departures from the distribution.

Thus, the observed and fitted distribution has almost the same shape where the mean sibship sizes is less than four i.e., that is differences between the two curves are small and show no consistent features in the distribution. This is an acceptable fit of the truncated negative binomial distribution for the number of sibship sizes especially for the Eastern and Southern African countries. It indicates that the distribution under consideration has successfully described the data. Thus, it may be useful to generate the underlined sibship sizes for the simulation study in specified population. The results of this paper are consistent with other researchers mentioned above. For applying the χ^2 test, some last cells are grouped together .The values of χ^2 do not approach significance except in few cases.

Further, research is required to explore the possibility of describing the mean number of sibship sizes more than four by mixing two distributions in different proportions at least at the micro level data, in retrospective/prospective surveys, by cross tabulating the data through other characteristics such as age, education, occupation etc. with sibship sizes. Thus, it may be useful to compare the parameters of the distribution both in developed and developing countries for the mean number of sibship sizes as well as the correlation coefficients between the number of boys and girls in a family.

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Appendix-1

Distribution of the Observed and Expected number of couples according to the number of sibship sizes in 13 countries of Western and Central Africa.

Size of sibship	Niger		Chad		Mali		Nigeria	
	Observed	Expected	Observed	Expected	Observed	Expected	Observed	Expected
1	100	76.9	67	53.3	180	145.6	97	76.6
2	100	105.1	67	71.7	173	182.6	73	87.1
3	90	114.0	65	76.3	150	185.0	64	82.9
4	94	107.5	59	70.5	140	165.1	65	71.2
5	80	92.4	55	59.2	121	135.4	53	57.2
6	73	74.1	43	46.4	106	104.4	44	43.9
7	62	56.5	40	34.5	91	77.0	37	32.5
8	52	41.4	33	24.6	71	54.8	30	23.4
9	43	29.3	26	16.9	48	37.9	25	16.5
10	30	20.2	16	11.4	45	25.6	19	11.4
11	18	13.6	7	7.5	20	16.9	11	7.8
12	8	9.0	5	4.8	11	11.0	4	5.3
13	3	5.9	1	3.0	3	7.1	3	3.5
14	2	3.8	0	1.9	1	11.6	1	6.7
15	0	2.4	1	3.0	0		0	
16	1	3.9	0		0		0	
total	756	756.0	485	485.0	1160	1160.0	526	526.0
χ^2	37.809		21.019		53.198		29.745	
df	10		9		9		9	
Estimated V(p)	0.000089		0.000151		0.000069		0.000159	
Estimated V(m)	0.016181		0.030199		0.010424		0.015356	
Estimated Cov(p,m)	0.000900		0.001638		0.000649		0.001164	

Size of sibship	Burkina		Guinea		Congo Dem		Senegal	
	Observed	Expected	Observed	Expected	Observed	Expected	Observed	Expected
1	153	124.0	97	75.3	136	115.4	181	155.0
2	145	156.9	89	101.7	116	129.1	156	169.7
3	130	158.2	85	106.4	103	119.8	131	155.4
4	117	139.4	83	95.1	81	99.8	108	128.6
5	100	112.0	69	76.3	76	77.4	92	99.5
6	88	84.3	63	56.5	65	57.0	80	73.4
7	81	60.4	53	39.5	50	40.5	67	52.3
8	58	41.6	39	26.2	35	27.9	51	36.3
9	39	27.7	24	16.8	24	18.8	32	24.6
10	24	18.0	11	10.4	17	12.4	22	16.4
11	10	11.5	6	14.7	10	8.1	13	10.8
12	5	7.2	0		5	5.2	0	7.0
13	1	4.4	0		1	3.3	6	4.5
14	1	6.4	0		1	5.3	1	2.9
15	0		0		0		1	4.6
16	0		0		0		0	
total	952	952.0	619	619.0	720	720.0	941	941.0
χ^2	44.778		31.672		23.029		34.797	
df	9		7		9		9	
Estimated V(p)	0.000082		0.000155		0.000122		0.000088	
Estimated V(m)	0.014626		0.014370		0.012903		0.007778	
Estimated Cov(p,m)	0.000830		0.002437		0.000936		0.000593	

Size of sibship	Benin		Cameroon		Liberia	
	Observed	Expected	Observed	Expected	Observed	Expected
1	219	188.9	168	146.7	117	104.9
2	226	241.4	130	144.9	100	110.9
3	209	241.5	104	123.9	91	97.9
4	190	207.9	86	97.8	68	77.8
5	154	161.8	67	73.3	59	56.7
6	125	116.9	61	53.0	41	40.8
7	99	79.8	45	37.3	36	27.8
8	68	52.1	37	25.7	22	18.4
9	41	32.8	24	17.5	15	12.0
10	25	20.0	16	11.7	8	7.6
11	9	11.9	9	7.8	8	4.8
12	5	6.9	4	5.1	3	7.4
13	1	9.1	1	3.3	0	
14	0		1	2.2	0	
15	0		1	3.8	0	
16	0		0		0	
total	1371	1371.0	754	754.0	568	568.0
χ^2	32.360		25.534		8.230	
df	9		9		8	
Estimated V(p)	0.000060		0.000127		0.000127	
Estimated V(m)	0.013819		0.008353		0.014111	
Estimated Cov(p,m)	0.000720		0.000748		0.000927	

Size of sibship	Ghana		Congo Brazilian	
	Observed	Expected	Observed	Expected
1	79	70.0	126	119.4
2	70	78.4	110	115.3
3	64	70.7	88	92.8
4	50	56.1	61	67.3
5	41	40.9	46	45.5
6	33	28.0	41	29.3
7	22	18.3	23	18.2
8	16	11.6	16	11.0
9	10	7.1	7	6.5
10	3	4.2	4	3.8
11	2	2.5	1	2.2
12	1	3.2	1	2.7
13	0		0	
14	0		0	
15	0		0	
16	0		0	
total	391	391.0	514	514.0
χ^2	9.386		5.966	
df	7		7	
Estimated V(p)	0.000212		0.000181	
Estimated V(m)	0.031377		0.016011	
Estimated Cov(p,m)	0.001886		0.001182	

Distribution of the Observed and Expected number of couples according to the number of sibship sizes in 13 countries of Eastern and Southern Africa.

Size of sibship	Uganda		Ethiopia		Rwanda		Zambia	
	Observed	Expected	Observed	Expected	Observed	Expected	Observed	Expected
1	98	75.4	147	115.1	109	91.7	100	87.2
2	87	97.7	135	150.5	110	118.0	92	100.4
3	82	101.4	126	156.3	103	119.9	80	93.8
4	75	92.2	119	141.5	96	105.6	73	77.5
5	71	76.8	106	116.8	76	84.3	59	59.0
6	63	59.9	100	90.1	63	62.8	42	42.3
7	52	44.6	78	66.0	58	44.3	34	29.0
8	43	32.0	66	46.5	37	30.0	26	19.3
9	31	22.2	42	31.7	27	19.6	18	12.4
10	20	15.1	27	21.1	18	12.5	9	7.8
11	14	10.0	14	13.7	6	7.8	5	4.8
12	6	6.5	9	8.7	4	4.7	3	7.5
13	2	4.2	1	5.5	1	6.8	0	
14	1	7.0	1	3.4	0		0	
15	0		0	2.1	0		0	
16	0		1	3.0	0		0	
total	645	645.0	972	972.0	708	708.0	541	541.0
χ^2	31.428		42.381		22.532		12.280	
df	9		9		8		8	
Estimated V(p)	0.000111		0.000070		0.000095		0.000148	
Estimated V(m)	0.017543		0.011848		0.017937		0.017601	
Estimated Cov(p,m)	0.001038		0.000970		0.000948		0.001124	

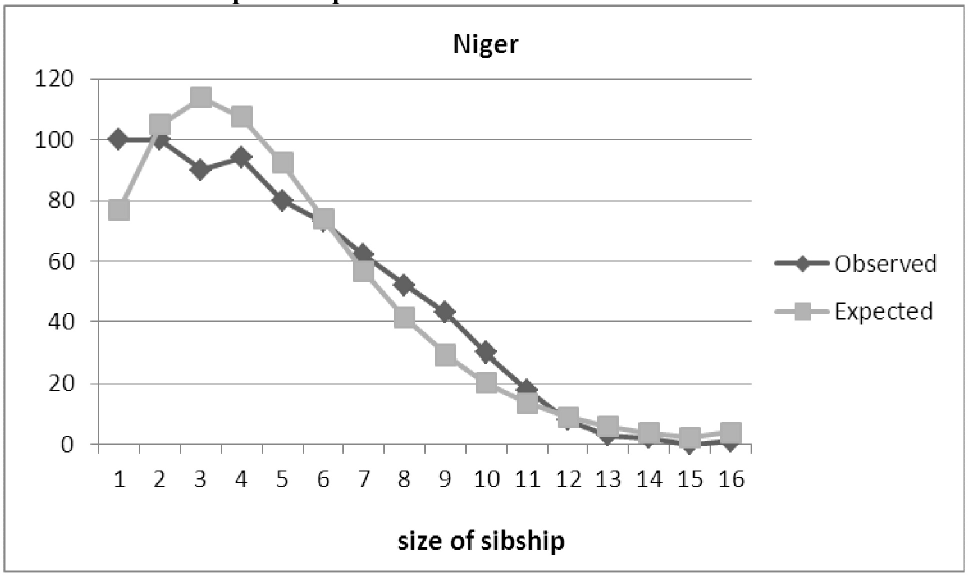
Size of sibship	Mozambique		Tanzania		Kenya		Malawi	
	Observed	Expected	Observed	Expected	Observed	Expected	Observed	Expected
1	200	176.4	152	140.0	121	109.1	179	163.5
2	174	190.7	148	149.9	104	114.8	173	179.6
3	147	171.2	121	133.7	94	101.2	146	161.9
4	121	137.9	95	107.2	74	80.4	119	130.0
5	115	103.5	72	80.2	57	59.8	91	96.7
6	74	73.9	63	57.1	45	42.4	74	68.1
7	60	50.8	45	39.2	33	29.0	49	46.1
8	42	34.0	32	26.1	26	19.3	40	30.2
9	30	22.2	22	17.0	15	12.6	27	19.3
10	18	14.2	17	10.9	12	8.1	17	12.1
11	10	9.0	8	6.9	5	5.1	7	7.4
12	5	5.6	2	4.3	3	3.2	3	4.5
13	2	8.6	1	2.6	0	2.0	1	6.6
14	0		1	3.9	1	3.0	0	
15	0		0		0		0	
16	0		0		0		0	
total	998	998.0	779	779.0	590	590.0	926	926.0
χ^2	22.457		14.007		10.229		16.511	
df	9		8		8		8	
Estimated V(p)	0.000085		0.000109		0.000147		0.000088	
Estimated V(m)	0.008223		0.010038		0.012726		0.009529	
Estimated Cov(p,m)	0.000601		0.000745		0.000970		0.000651	

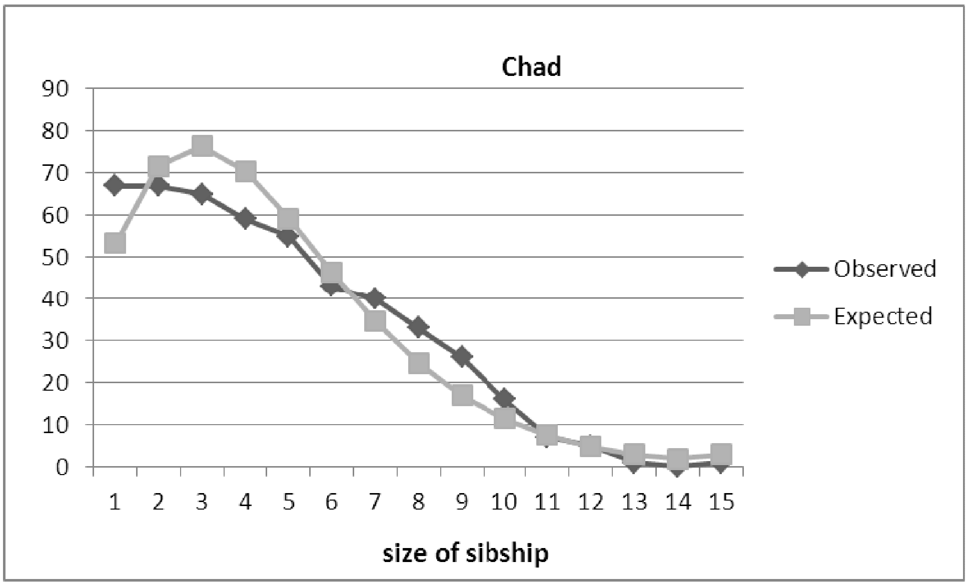
Size of sibship	Madagascar		Swaziland		Zimbabwe	
	Observed	Expected	Observed	Expected	Observed	Expected
1	124	118.6	96	91.4	164	162.7
2	121	120.4	76	77.6	154	148.2
3	99	103.9	53	58.5	106	113.4
4	75	81.8	36	41.3	74	78.6
5	56	60.7	27	28.0	51	51.1
6	48	43.2	22	18.4	32	31.7
7	26	29.8	14	11.9	21	19.0
8	23	20.0	11	7.6	14	11.1
9	17	13.2	6	4.7	7	5.8
10	11	8.6	3	2.9	4	3.3
11	8	5.5	2	1.8	2	1.8
12	5	3.5	1	2.9	1	3.3
13	2	5.8	0		0	
14	0		0		0	
15	0		0		0	
16	0		0		0	
total	615	615.0	347	347.0	630	630.0
χ^2	6.346		4.790		2.442	
df	9		7		7	
Estimated V(p)	0.000152		0.000319		0.000148	
Estimated V(m)	0.011397		0.014831		0.011006	
Estimated Cov(p,m)	0.000945		0.001498		0.000848	

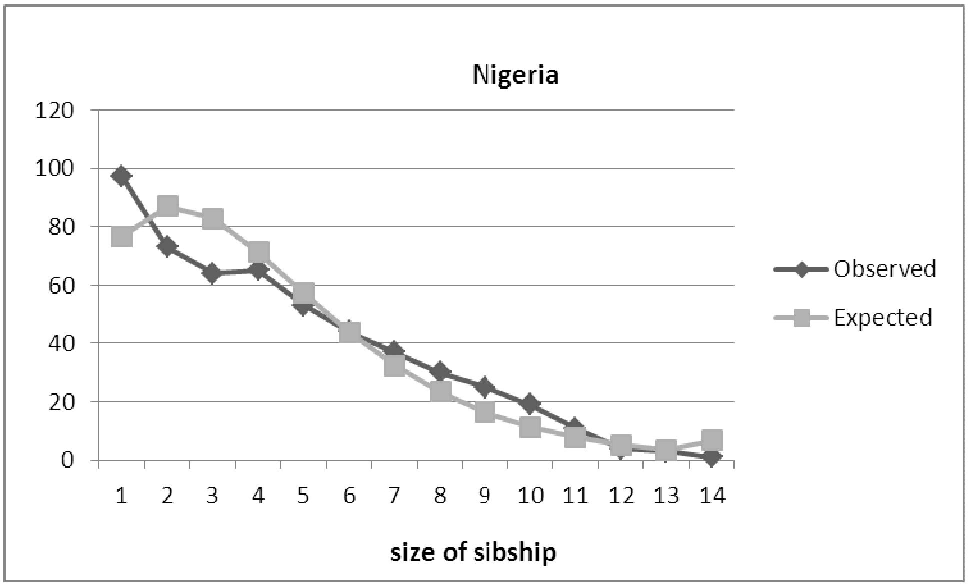
Size of sibship	Lesotho		Namibia	
	Observed	Expected	Observed	Expected
1	139	131.7	190	186.9
2	104	114.4	158	157.9
3	82	85.3	109	114.0
4	54	58.4	73	75.4
5	41	37.8	44	47.1
6	28	23.5	31	28.3
7	17	14.2	20	16.5
8	11	8.4	10	9.4
9	5	4.9	7	5.2
10	2	2.8	4	2.9
11	1	1.6	1	3.4
12	1	2.0	0	
13	0		0	
14	0		0	
15	0		0	
16	0		0	
total	485	485.0	647	647.0
χ^2	5.201		2.481	
df	7		7	
Estimated V(p)	0.000208		0.000158	
Estimated V(m)	0.012478		0.009654	
Estimated Cov(p,m)	0.001069		0.000809	

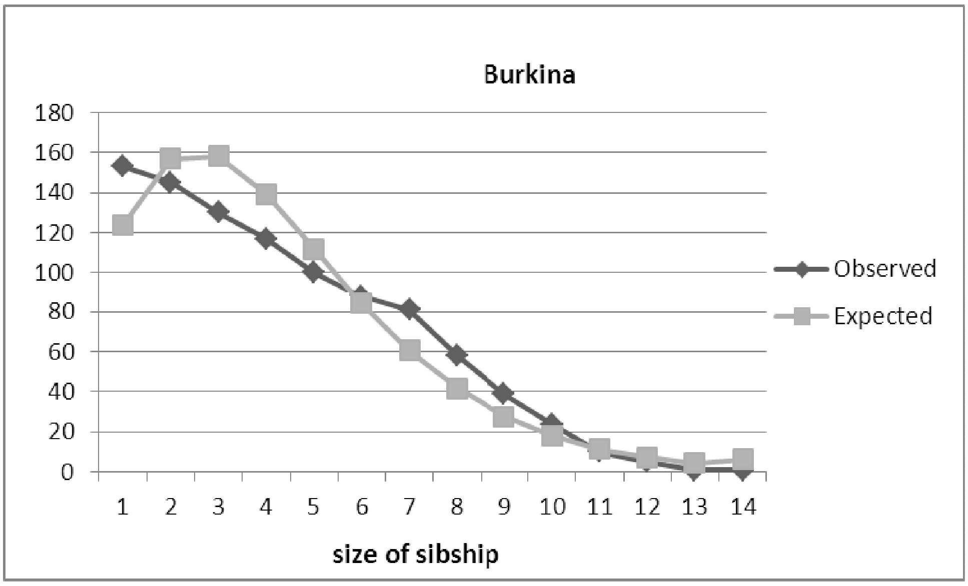
Appendix-2

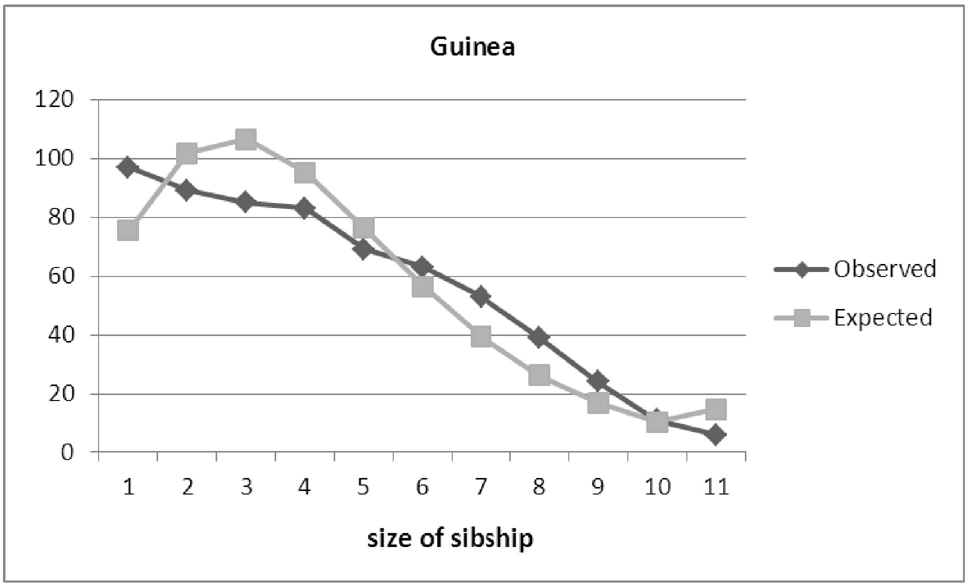
Graphical representation of Western and Central Africa

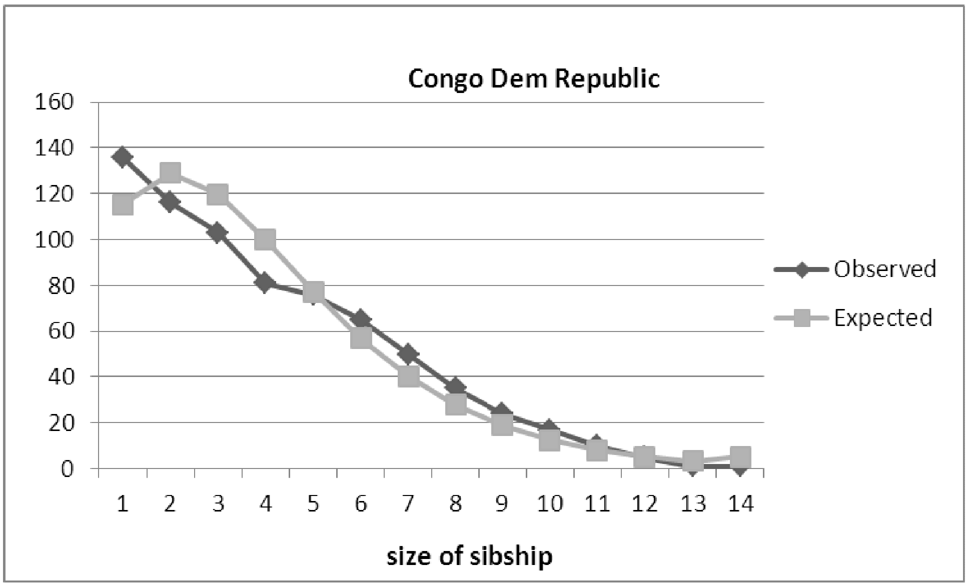


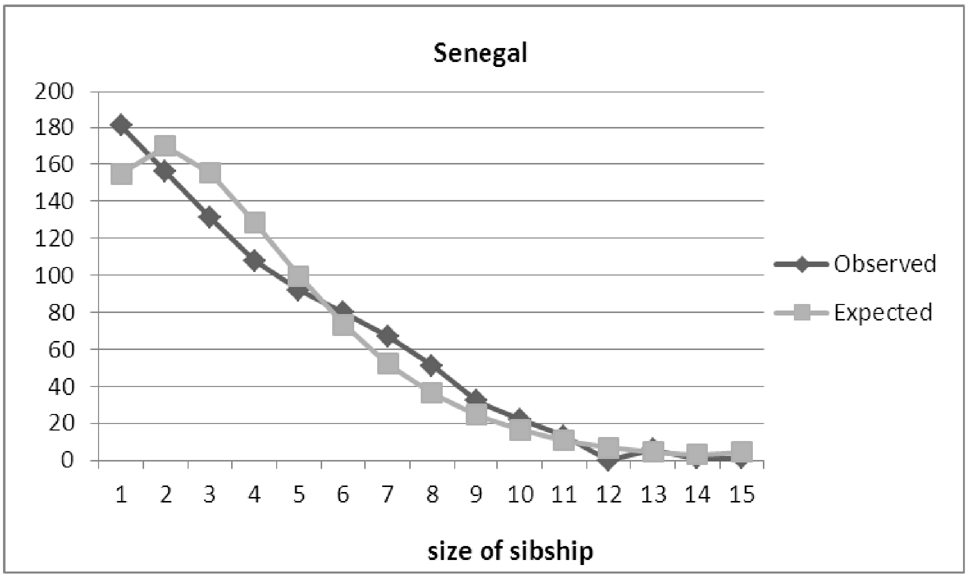


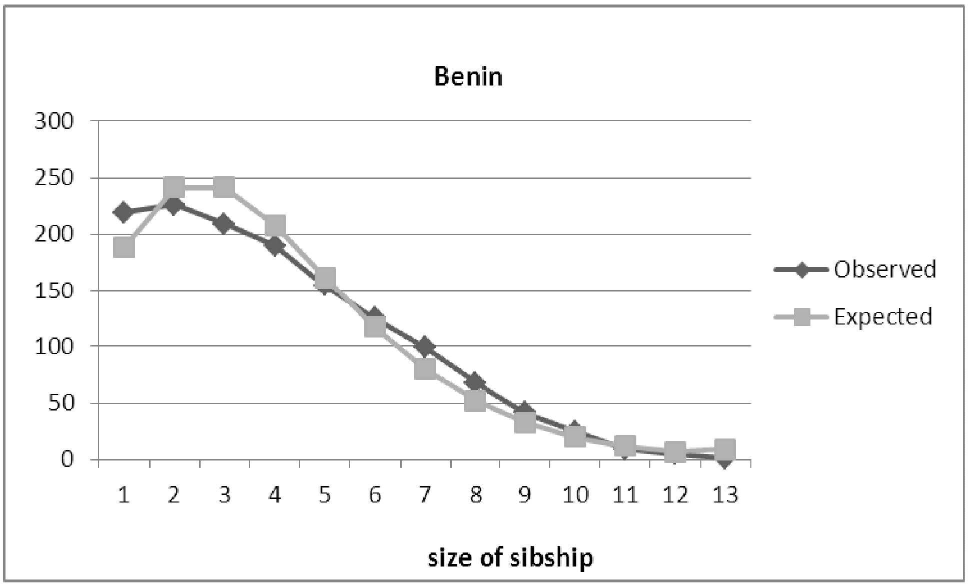

Nigeria
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
size of sibship
Observed
Expected

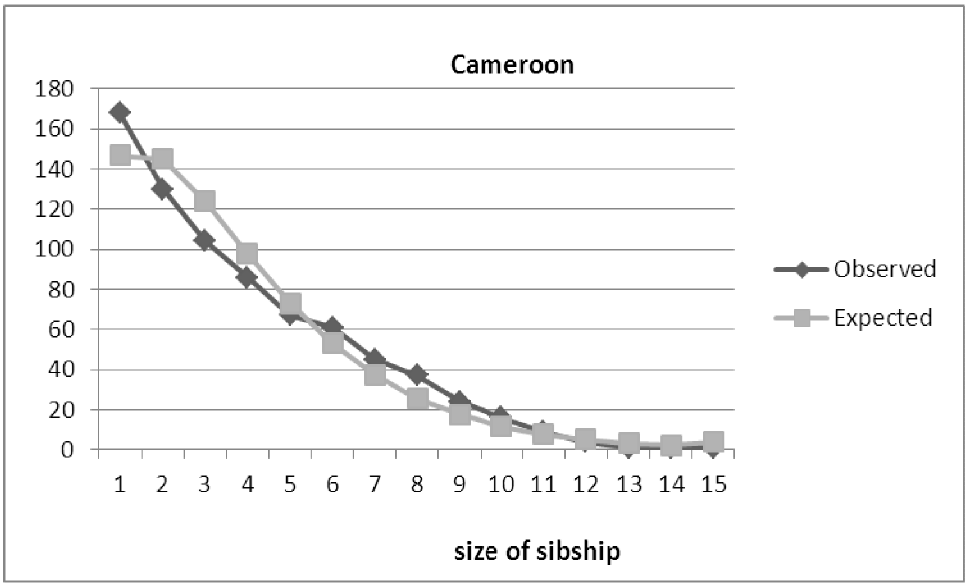

Burkina
180
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
size of sibship
Observed
Expected

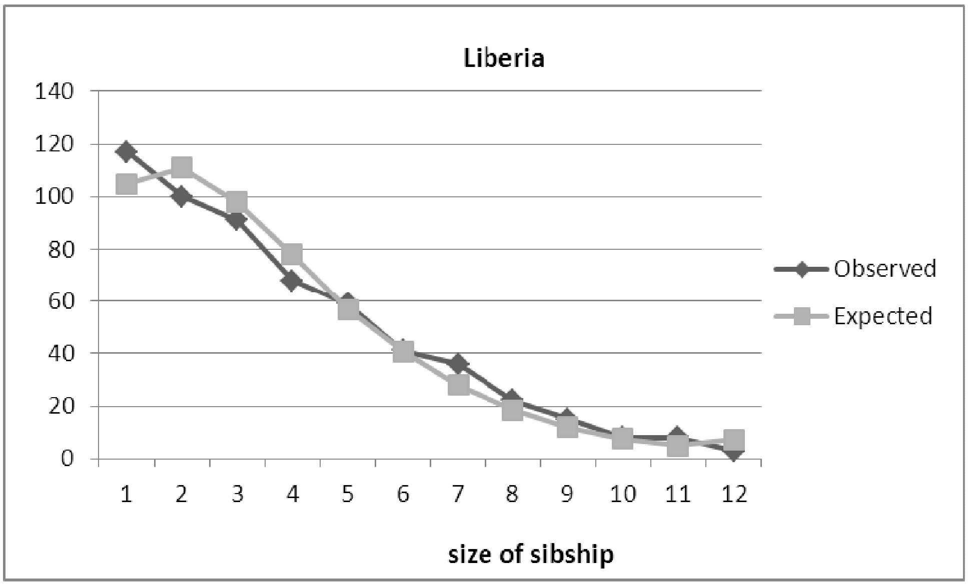

Guinea
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11
size of sibship
Observed
Expected

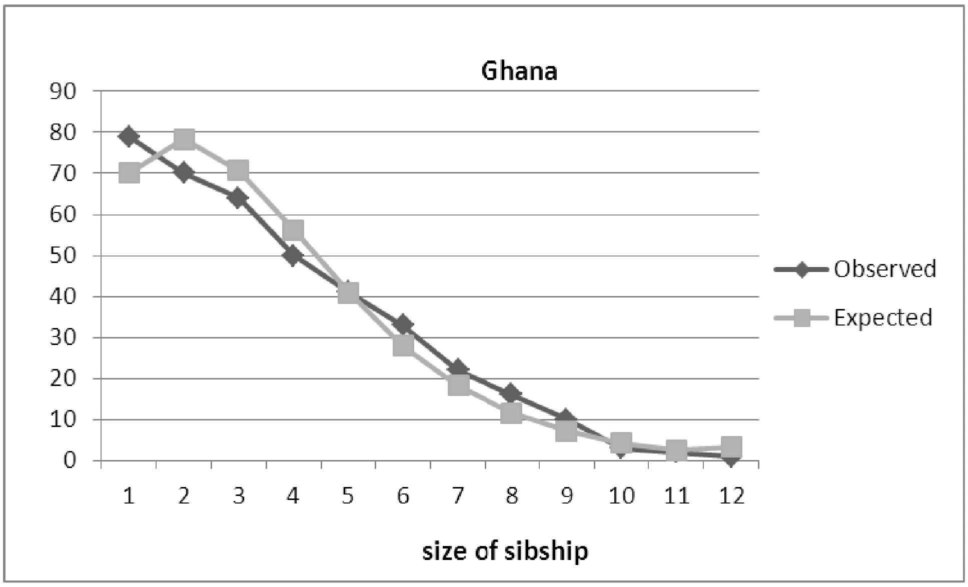

Congo Dem Republic
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
size of sibship
Observed
Expected

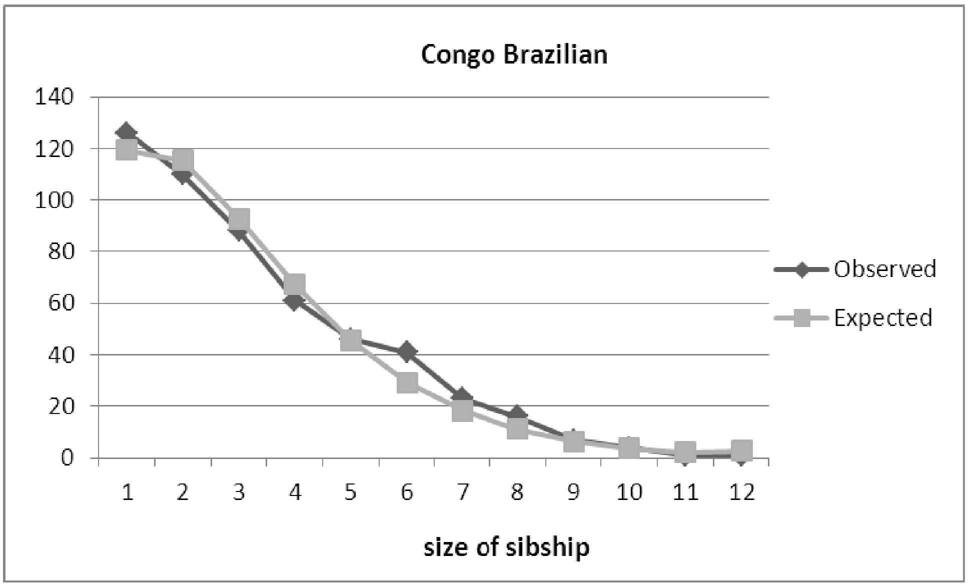

Senegal
200
180
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
size of sibship
Observed
Expected


Benin
300
250
200
150
100
50
0
1 2 3 4 5 6 7 8 9 10 11 12 13
size of sibship
Observed
Expected

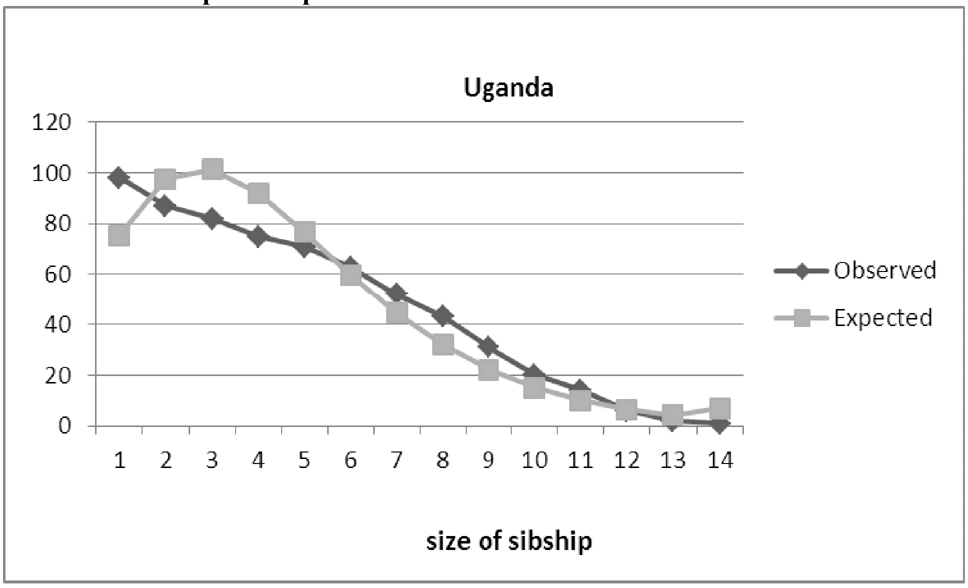

Cameroon
180
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
size of sibship
Observed
Expected

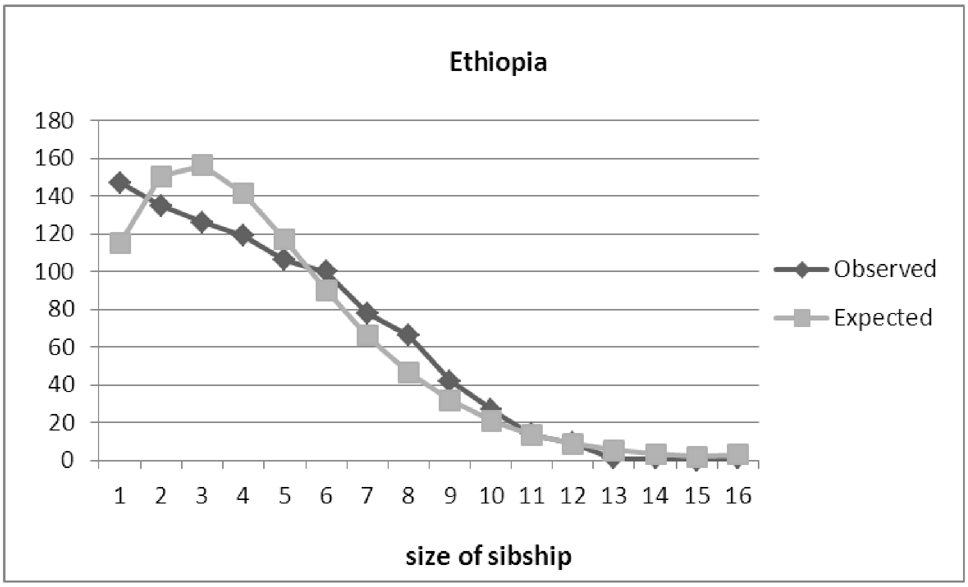

Liberia
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12
size of sibship
Observed
Expected

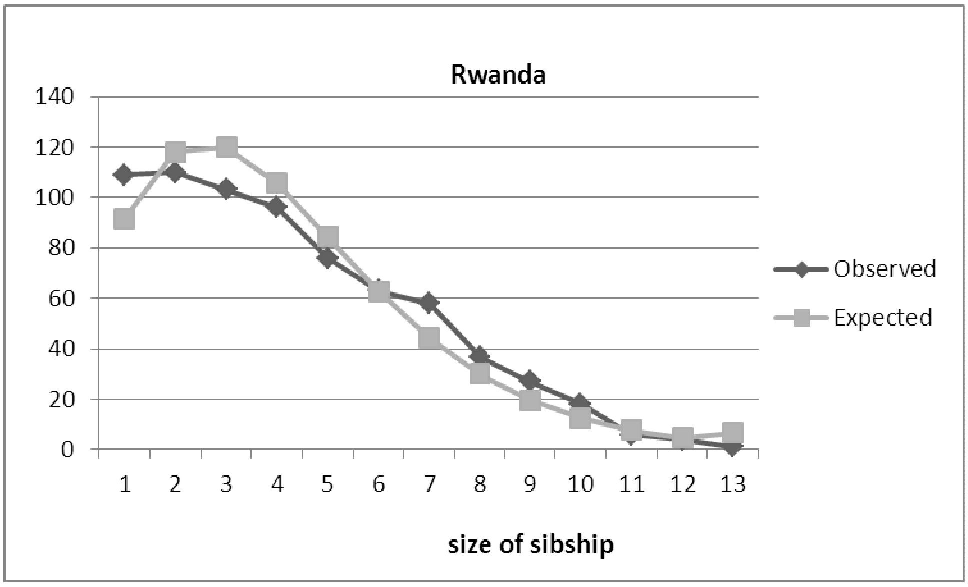

Ghana
90
80
70
60
50
40
30
20
10
0
1 2 3 4 5 6 7 8 9 10 11 12
size of sibship
Observed
Expected

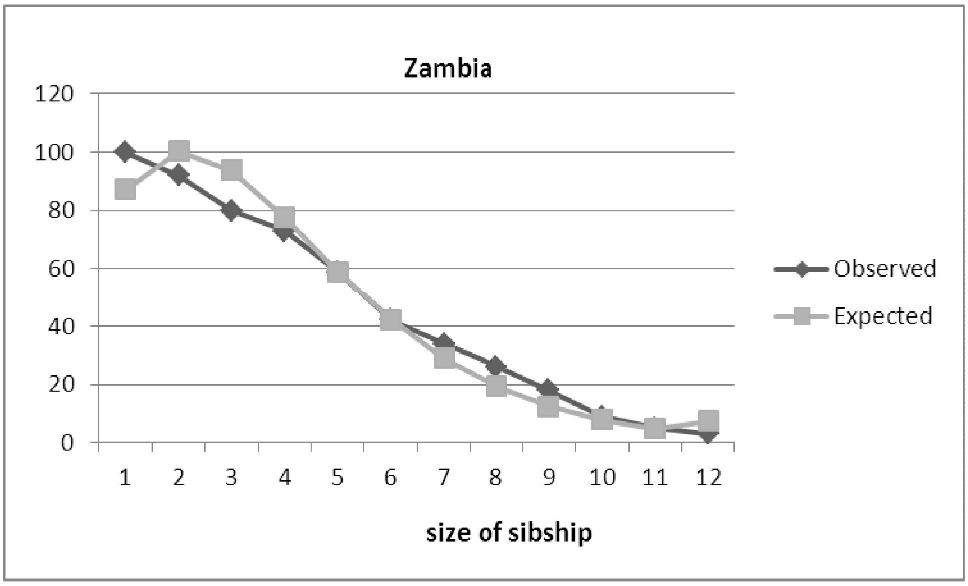


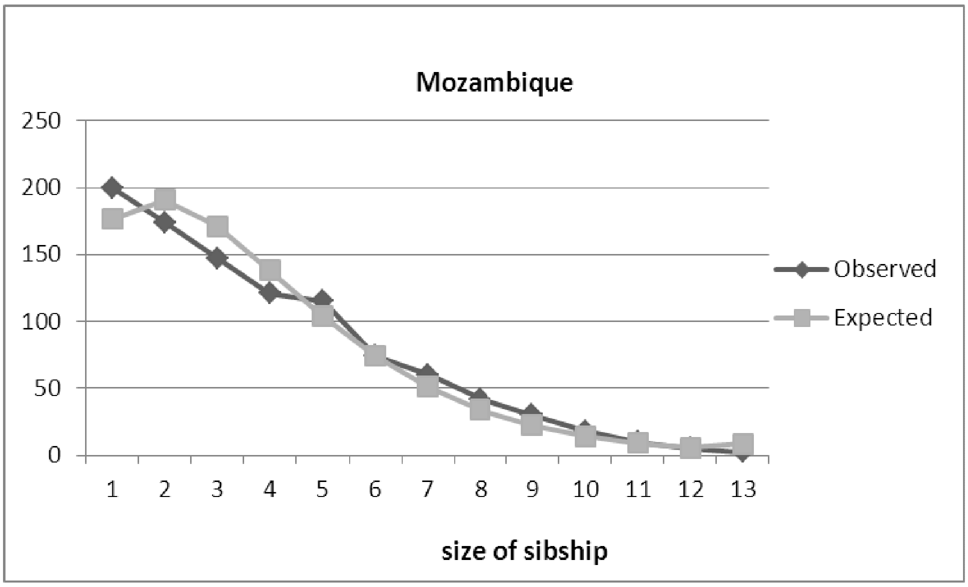
Graphical representation of Eastern and Southern Africa

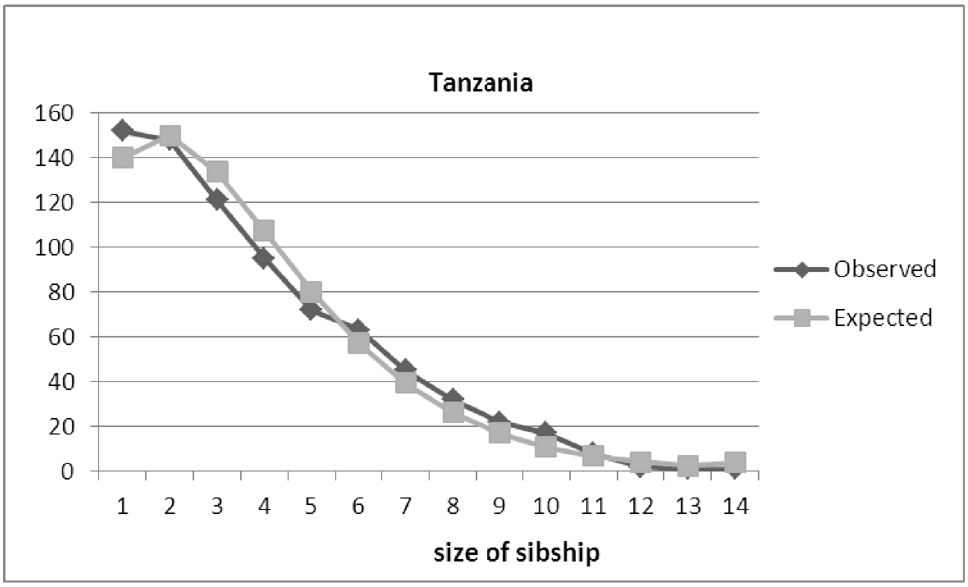


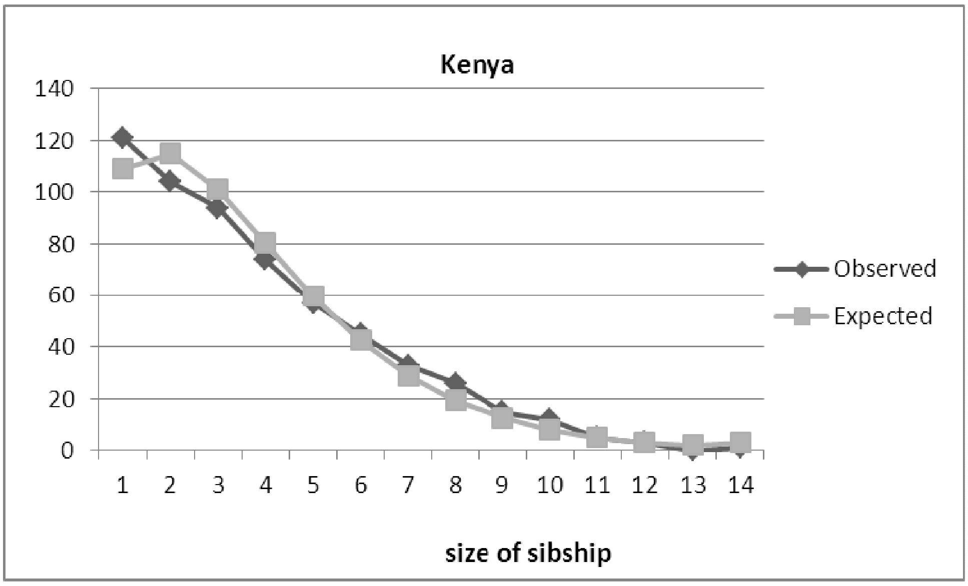


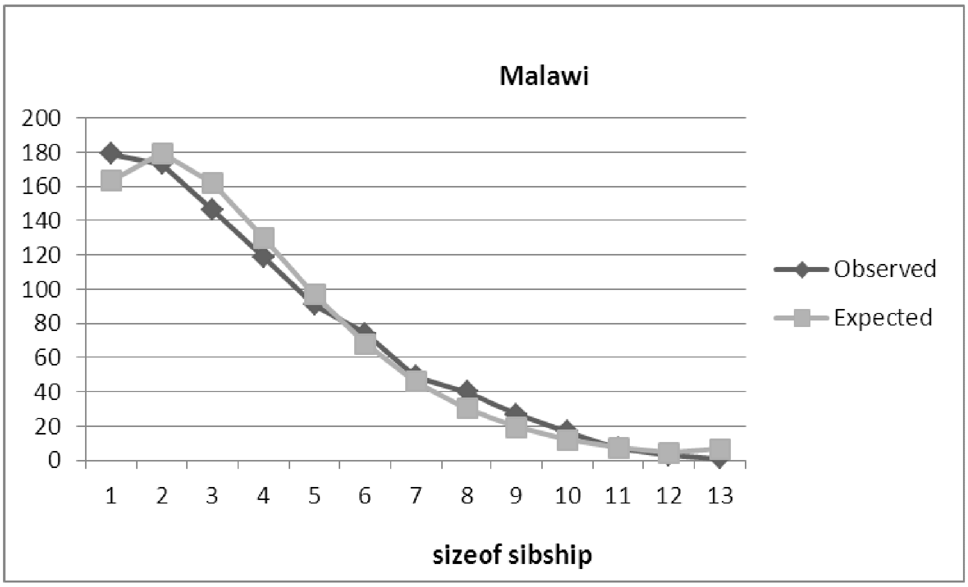

Rwanda
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13
size of sibship
Observed
Expected

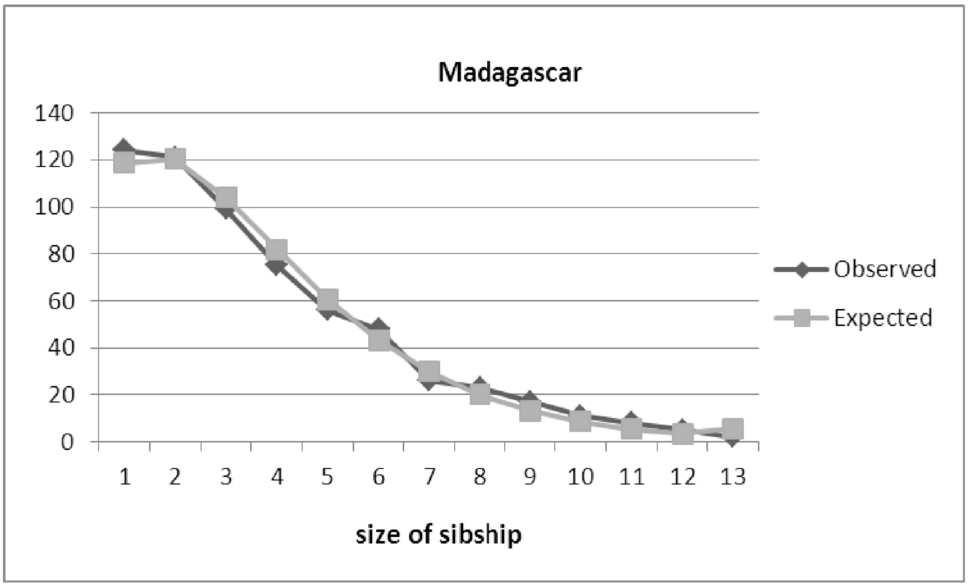

Zambia
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12
size of sibship
Observed
Expected

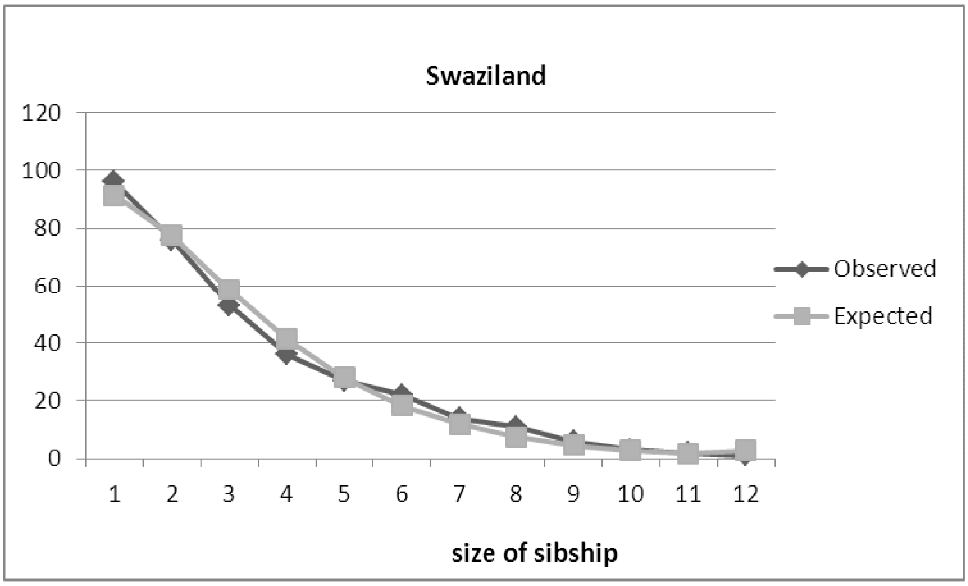

Mozambique
250
200
150
100
50
0
1 2 3 4 5 6 7 8 9 10 11 12 13
size of sibship
Observed
Expected

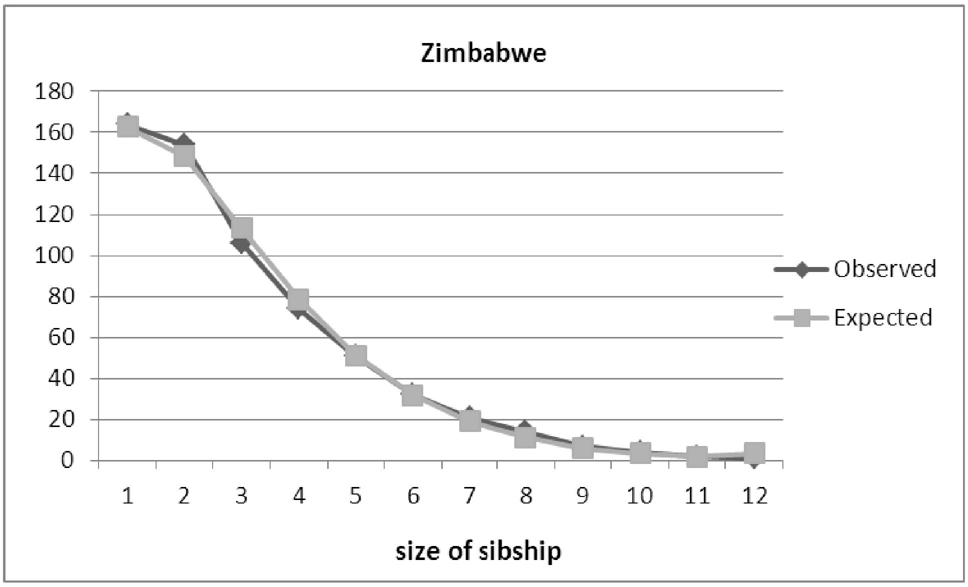

Tanzania
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
size of sibship
Observed
Expected

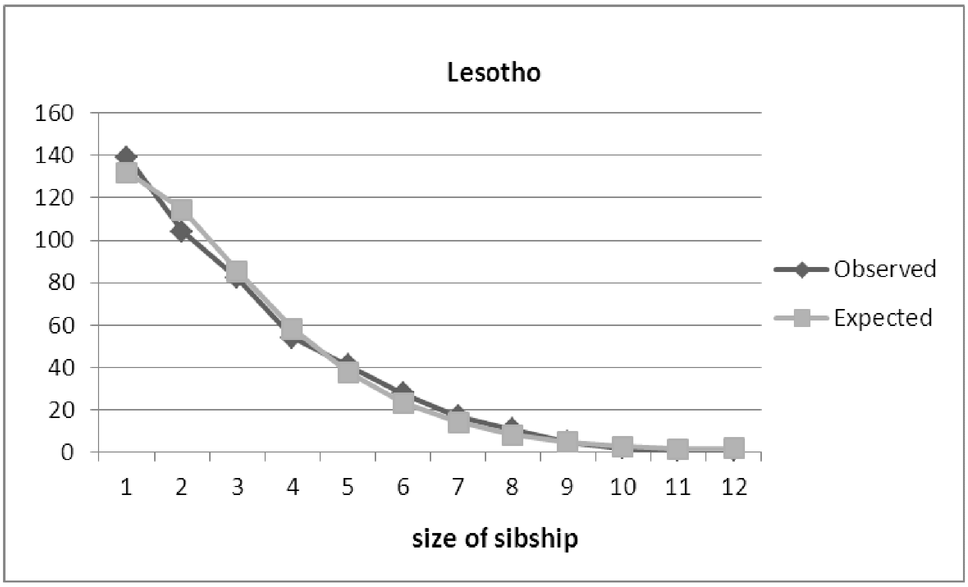

Kenya
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
size of sibship
Observed
Expected

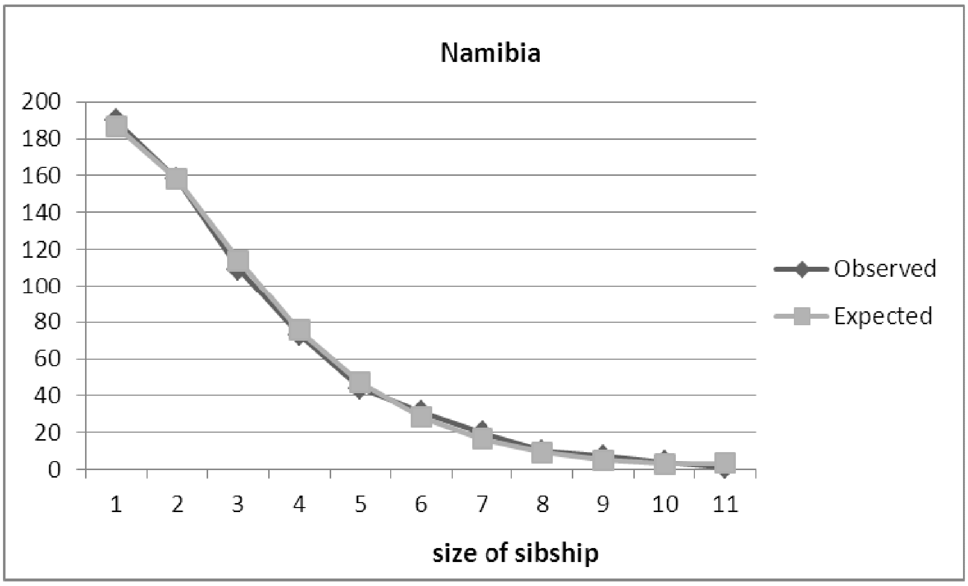

Malawi
200
180
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13
sizeof sibship
Observed
Expected


Madagascar
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13
size of sibship
Observed
Expected


Swaziland
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12
size of sibship
Observed
Expected


Zimbabwe
180
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12
size of sibship
Observed
Expected


Lesotho
160
140
120
100
80
60
40
20
0
1
2
3
4
5
6
7
8
9
10
11
12
size of sibship
Observed
Expected


Namibia
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
6
7
8
9
10
11
size of sibship
Observed
Expected