

An Analysis on Male and Female Junior High School Students' Van Hiele Levels of Geometric Thinking from Mathematical Ability

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Abstract
The purposes of this research were to examine the students' levels of thinking based on gender differences by investigating the students' levels of geometric thinking and making learning examples to improve the students' levels of thinking. Mathematical ability differences in this research referred to high, medium, and low mathematical abilities. This research was explorative research using a naturalistic qualitative approach. The use of the naturalistic qualitative approach was based on the consideration that the analysis for Van Hiele levels of thinking needed to be done by observing the students' thinking process characteristics. The research subjects consisted of 3 male students and 3 female students which were grouped based on mathematical abilities. In order to collect information about the students' levels of thinking, each research subject was interviewed with six activities which included: (1) Drawing quadrilaterals, (2) Identifying and defining quadrilaterals, (3) Sorting quadrilaterals, (4) Determining a mystery shape, (5) Equivalence of two definitions of a parallelogram, and (6) Application of quadrilaterals. The results of each activity were analyzed and the levels of thinking of each research subject were determined. It was found that there were five students at level 1 and only one student at level 0. All subjects had several weaknesses, among others using improper properties to distinguish, identify, and sort geometrical structures. All subjects still ignored class inclusion in quadrilaterals, although they were able to identify geometric shapes based on the properties of their components. From the analysis of the students' levels of thinking, learning examples were made based on Van Hiele five learning phases, i.e.: (1) Information Phase, (2) Guided Orientation Phase, (3) Explicitation Phase, (4) Free Orientation Phase, and (5) Integration Phase. The results showed that there was no difference between male and female students' levels of thinking since both male and female students were at level 1. It was different from the students' levels of thinking viewed from their mathematical abilities. All students in the high- and medium-scored groups were at level 1, while only one student was at level 1 and the others were at level 0 in the low-scored group. The learning examples were made in the form of Lesson Plan (*Rencana Pelaksanaan Pembelajaran*, RPP) and Student Worksheets (*Lembar Kerja Siswa*, LKS). It was hoped that the students could progress from level 1 to level 2 with these lesson plan and student worksheets.
Keywords: Van Hiele Levels of Geometric Thinking, Mathematical Ability

1. Introduction

Mathematics is a scientific discipline having a unique character compared to other disciplines because mathematical knowledge cannot be transferred completely from the teacher's mind to the students' minds during the lessons. According to Soedjadi (2000), a mathematics lesson is the mathematics elements or parts chosen on the basis of:

1. The meaning of education i.e. to develop the students' abilities and personality.
2. The real development demands from the surroundings that always change along with the advancement of science and technology.

With mathematics lesson, students are expected to not only capable of solving mathematical problems but also in applying mathematical problems in the real world. According to Soedjadi (2000), the purpose of teaching mathematics at every educational level basically refers to:

1. FORMAL objectives: i.e. the objectives that emphasize (1) the development of students' reasoning and (2) the establishment of students' attitudes.
2. MATERIAL objectives: i.e. the objectives that emphasize (1) calculation skills, (2) problem-solving, and (3) mathematical application.

It is expected that students not only understand mathematical concepts at schools but also have skills in the application in the real world. Thus, they can develop the ability to think logically, systematically, and critically in problem-solving and become diligent, creative, intelligent, and responsible persons.

Geometry is one of the subjects in mathematics with the quite worrying position. Based on the reality in the field, many students have low achievements in geometry. Soedjadi (2000) states that geometry topic is considered as a difficult topic in mathematics. Ponter (2001) and Molle (2000) also affirm that students do not master the geometric concept. One of the geometry topics in the eighth grade of junior high school is quadrilaterals. The quadrilaterals topic is important because this topic is one of the prerequisite topics in studying three-dimensional geometric shapes in the ninth grade. In order to master and understand three-dimensional geometric shapes, students should understand two-dimensional geometric shapes first, one of which is quadrilaterals. Van de Walle (2001) states the learning design of geometry for geometric thinking is through the van Hiele Levels of Geometric Thinking. The five levels of geometric thinking are level 0 (visualization), level 1 (analysis), level 2 (informal deduction), level 3 (deduction), and level 4 (rigor).

In relation to the van Hiele levels of geometric thinking above, learning of geometry should be adjusted to the students' levels of geometric thinking (Abdullah, 2011). The main purpose of learning of geometry is to increase the students' levels of geometric thinking (Gawlick, 2005). Learning of geometry will be effective if learning activities are carried out in accordance with the students' thinking skills structures (Kartono, 2010). In delivering geometric concepts to students, teachers should be creative in preparing the students' prior condition and in accommodating the students' learning style differences to achieve maximum learning outcomes (Narwanti, 2011: 26). Learning of geometry should be adjusted to the students' levels of geometric thinking. Learning adapted to students' thinking ability will enhance students' intellectual involvement so that they can improve their learning achievement in geometry. The levels of thinking used in this research were based on the Van Hiele levels of geometric thinking.

There are Five Levels of Geometric Thinking for students in understanding geometry. **Level 0: Visualization**. In this level, students reason about basic geometrical concepts, such as simple shapes, primarily by means of visual considerations of the concept as a whole without regard to the properties of its components. Students are already familiar with geometric shapes such as rectangles, squares, parallelograms, and so on based on the actual appearance of the shapes. Students are not able to characterize the shapes. For example, a trapezoid has a pair of parallel sides. Students only memorize geometric properties or concepts without any understanding. **Level 1: Analysis**. In this level, students reason about geometrical concepts by means of the informal analysis of component parts and attributes. Necessary properties of the concept are established. Students already know the properties of quadrilaterals (rectangles, squares, parallelograms, etc.). For example, the side pairs of a trapezoid are parallel and a kite has a pair of equal angles. **Level 2: Abstraction**. In this level, students logically order the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of properties in determining a concept. Students can determine the structures of quadrilaterals. In addition, students can be presented with an axiomatic-deductive method and can form an abstract and informal geometric structure. **Level 3: Deduction**. In this level, students reason formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions, and theorems. Students can be presented to the formal deductive method. Proofs of a theorem are no longer proven inductively but fully deductively by using geometric elements such as undefined terms, definitions, axioms, and theorems. However, students do not fully understand the use of axioms. Students cannot determine whether the axioms used are correct or not in the geometric structures. **Level 4: Rigor**. At this level, students can compare systems based on different axioms and can study various geometries in the absence of concrete models. Students understand the uses of certain axioms in a geometric structure. Once the axioms are changed, another geometric structure exists. Students can learn non-Euclidean geometries because they can compare different axiomatic systems. Students at the university level should reach this level.

2. Research Method

Research Subjects

The research subjects were chosen based on the gender differences and the mathematical abilities differences with at least 3 people involved. Hence, the research subjects were 6 students, consisting of 3 male students with high, medium, and low abilities and 3 female students with high, medium, and low abilities. The junior high school students of class VIIIA were selected because they already received quadrilaterals lesson in semester 2, so it was expected that students' levels of geometric thinking were easier to be examined.

Types of Research

The type of this research was explorative research using a naturalistic qualitative approach because the purpose of this study was to determine the students' levels of thinking by exploring the students' levels of geometric thinking. The use of the naturalistic qualitative approach was based on the consideration that the analysis for Van Hiele levels of thinking needed to be done by observing the thinking process characteristics of the research subjects while the research subjects conducted geometric activities. Observation on the research subjects was supported by interviews to enable the researchers in examining the research subjects' thinking process during these activities. Therefore, the appropriate approach was the naturalistic qualitative approach.

Research Instruments

In qualitative research, the researcher is the key instrument because qualitative research emphasizes the process rather than results. Although being the key instrument, during the data collection activities, the researchers used guidelines containing lists of geometric activities for the research subjects and interview guidelines. The interview guidelines used in this research were structured interview guidelines from Experimental Tasks in Appendix A (p. 35-53) from Final Report Assessing Children's Intellectual Growth in Geometry. This structured interview guidelines contained interview guidelines for triangle and quadrilateral concepts (Kho, 1996). However, the researchers only used interview guidelines for quadrilateral concepts. Interview guidelines for quadrilateral concepts consisted of six activities which involved 1) Drawing quadrilaterals, 2) Identifying and defining quadrilaterals, 3) Sorting quadrilaterals, 4) Guessing a mystery shape, 5) Equivalence of two definitions of a parallelogram, 6) Application of quadrilaterals.

Data Collection Techniques

In this research, data were taken from 6 students of class VIIIA based on gender differences and mathematical ability differences. First, students were grouped according to their gender. Then, their names were arranged based on the highest to the lowest mathematics scores in the first semester. From the topmost sorting, the students were grouped back into 3 groups, i.e. 30%, 40%, and 30% of the students' total number. The details of each group were as follows: The first 30% was a high-scored group, 40% was a medium-scored group, and the second 30% was a low-scored group. From the high-scored group, a student with the highest score was taken, from medium-scored groups, a student was chosen, and from the low-scored group, a student with the lowest score was taken. The selection results of the six research subjects were then consulted with the mathematics teacher to ensure the selection was in accordance with the students' mathematical abilities.

Data validity testing techniques were needed to get valid data. Data testing techniques in qualitative research can be through triangulation, i.e. data validity checking techniques which uses something out of the data for verification or as a comparison to the data (Moleong, 1992). Denzin (in Moleong, 1992) identifies four types of triangulation: (a) method triangulation, (b) investigator triangulation, (c) theory triangulation, and (d) data source triangulation.

In this research, the researchers used data source triangulation. According to Patton (in Moleong, 1992), data source triangulation means comparing and cross-checking the consistency of information derived at different times and by different means. In this study, data were collected by 1) Identifying, i.e. activities that utilized images of a shape, 2) Defining i.e. activities that required research subjects to express understanding of a shape with a sentence, 3) Classifying i.e. activities that utilized images of a shape or display of a cardboard-made shape, 4) Drawing of shape i.e. the activity of the research subjects in drawing quadrilaterals.

Data Analysis Technique

Data analysis in this study was based on three-stage data analysis from Miles and Huberman (in Kho, 1996), i.e.: (1) data reduction, (2) data display, and (3) conclusion drawing.

3. Findings and Discussion

The results showed that LA, a male student with high mathematical ability, could distinguish quadrilaterals based only on the side and angle attributes, but LA could not identify that the number of quadrilaterals could be infinite. Based on indicator 4 of level 0 and indicator 1 of level 1, it can be concluded that LA was between level 0 and level 1, meaning that LA showed a transition on the levels of thinking from level 0 to level 1. Based on the description above, it can be seen that LA was able to distinguish geometric shapes based on the properties of its components, but still ignored the class inclusion between quadrilateral structures. Based on indicator 1 and indicator 2 of level 1, it can be concluded that LA's level of thinking was at level 1. LA also could not identify the equivalence of two definitions of a parallelogram. Based on indicator 7 and indicator 8 of level 1, it can be concluded that LA was at level 1.

From the data, it was found that LA's level of thinking was mostly at level 1. From the results of the analysis of each activity carried out by LA, there were still some weaknesses LA had, among others, LA still could not imagine that the number of different quadrilaterals was infinite and LA still did not understand class inclusion, the equivalence of two definitions of a parallelogram, and mathematical proof. But LA's analysis in classifying quadrilateral structures based on the properties of each shape had begun to appear. So, it can be concluded that LA's level of thinking was at level 1.

PA was a male student with medium mathematical ability. In identifying squares, PA used the properties of all sides were of equal length and all angles were equal at 90°. PA could identify figure 9 and figure 12 as rectangles. Even though at first PA considered picture 6 as a rectangle, he then changed the answer with a parallelogram. PA identified figure 9 and figure 12 as rectangles because the opposite sides were of equal length with the same angle. In identifying a trapezoid, PA used the properties of one pair of opposite sides was parallel. PA could define the properties of squares, rectangles, parallelograms, rhombuses, trapezoids, and kites, but not yet perfect. PA used "opposite sides parallel and equal in length" in defining the properties of parallelograms.

LS, a male student with the low mathematical ability, could identify figure 7 as a square using the properties of all four sides were of equal length. LS could identify figure 9, figure 12, and figure 6 as rectangles using the properties of opposite sides were of equal length with the angle of 90°, figure 3, figure 10, and figure 13 as parallelograms using the properties of opposite sides were parallel. LS could identify figure 4 and Figure 8 as kites using the properties of two pairs of sides were of equal length. LS could define the properties of squares, rectangles, parallelograms, rhombuses, trapezoids, and kites, but not yet perfect. Some mistakes made by LS in defining the properties of quadrilaterals were in defining the number of rotational symmetries, alignment, and many ways a shape filled the frame.

PS was a female student with high mathematical ability. PS could distinguish quadrilaterals based only on the side attributes and rotational and fold symmetry, while could not identify that the number of rectangles could be infinite. Based on the indicator of level 0 and indicator 1 of level 1, it can be concluded that PS was between level 0 and level 1. PS could identify figure 2 and figure 7 as squares using the properties of all sides were of equal length. PS could identify figure 9 and figure 12 as rectangles using the properties of opposite sides were of equal length. PS could identify figure 3, figure 6, figure 10, and figure 13 as parallelograms. PS could identify figure 5 as a rhombus using the properties of opposite sides were of equal lengths and opposite angles were equal. PS could identify figure 1, figure 14, and figure 15 as trapezoids using the properties of two sloping sides would meet when extended. PS could identify figure 4 as a kite, but could not justify the reason. She identified the figure as the kite from the kite-like picture. PS could define the properties of squares, rectangles, parallelograms, rhombuses, and trapezoids, but used irrelevant properties in identifying kites. Besides, PS also used irrelevant properties and ignored class inclusion in identifying quadrilaterals.

LR was a female student with medium mathematical ability. LR could distinguish quadrilaterals based only on the side type and attributes, while LR did not know the number of quadrilaterals could be infinite. Based on indicator 1 and indicator 4 from level 0, it can be concluded that LR was at level 0. LR could identify figure 2 and figure 7 as squares using the properties of the four sides were of equal length. LR could identify figure 6, figure 9, and figure 12 as rectangles using the properties of a pair of long sides and a pair of short sides. LR could identify figure 3, figure 10, and figure 13 as parallelograms using the properties of the opposite sides were of equal length. LR could identify figure 5 as a rhombus using the properties of opposite sides were of equal length. LR could identify figure 1, figure 14, and figure 15 as trapezoids, but could not define the properties. LR could define the properties of squares, rectangles, parallelograms, rhombuses, trapezoids, and kites, but not yet perfect. LR made a number of mistakes in defining the properties of quadrilaterals, for example including irrelevant properties in mentioning the properties of a rhombus. In addition, LR also used inappropriate properties for trapezoids because she referred to isosceles trapezoid.

PR was a female student with low mathematical ability. PR could only draw 5 quadrilaterals. Based on indicator 4 from level 0, it can be concluded that PR's level of thinking was at level 0. PR could identify figure 6, figure 7, and figure 12 as squares. In sorting the squares, PR incorrectly identified figure 6 and figure 12 as squares because figure 6 was a parallelogram and figure 12 was a rectangle using the properties of all angles were right angles. PR could identify figure 9 as a rectangle, but cannot justify the reason. PR could identify figure 3, figure 10, and figure 13 as parallelograms, but cannot justify the reason. PR could identify figure 2 and figure 5 as rhombuses using the properties of all sides were of equal length. PR could identify figure 14 and figure 15 as trapezoids using the properties of unequal angles. PR could identify figure 4 and figure 8 as kites using the properties of the diagonals divided by two equal lengths. PR could define the properties of squares, rectangles, parallelograms, rhombuses, and kites, but could not define the properties of trapezoids. PR made a lot of mistakes in defining the properties of quadrilaterals. Analysis of PR on the equivalence of the two definitions of a parallelogram was PR agreed that a quadrilateral with parallel opposite sides had opposite sides with equal length. According to PR, the shape which met the above statement was a rectangle.

4. Conclusion

In accordance with the research results, it can be concluded that there was no difference between male and female students' levels of thinking because both male and female students were at Level 1. It was different from the students' level of thinking when viewed from the mathematical ability. All students in the high and medium-scored group were at level 1, whereas in the low-scored group only a student was at level 1 and the others were at level 0. Because the students at level 1 were dominant in this research, the proposed learning examples were made to progress the students' levels of thinking from the level of thinking 1 to the level of thinking 2.

The suggestions from this research are:

1. In the teaching of geometry, teachers should pay attention to the levels of geometric thinking students have. Students may have difficulty in understanding the concepts if the concepts are above their levels of thinking.
2. If teachers want to explain geometric concepts by classifying the students homogeneously, i.e. classifying students according to their levels of thinking, teachers should be able to distribute the

teaching time appropriately to achieve the learning objectives.

3. The model development on the continuous learning of geometry in junior high schools may use five phases of learning of geometry as van Hiele recommends.
4. Learning examples are still conceptual and need to be further developed through try-outs.

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