Research Journal of Finance and Accounting
ISSN 2222-1697 (Paper) ISSN 2222-2847 (Online)
Vol 2, No 9/10, 2011

Multidimensional Distress Analysis - A Search for New Methodology

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Abstract

Economists and management experts had been trying very hard to work out a model which will satisfy performance evaluation and distress analysis of an enterprise or a business unit. Almost all of them tried measuring performance and distress separately. May it be performance evaluation or distress analysis every scholar instead of reconciling the issues went on differentiating. This paper concentrates on distress analysis and tries to establish a new methodology by which both performance and distress position of an enterprise can be measured. This methodology is based on Fuzzy Set Logic and is also best fitted for ordinal data. In this paper we would like to take the privilege of re-writing certain terms like instead of writing distress we prefer to write subaltern and an enterprise or a business unit will be written as a unit. We are more focused in assessing the deprivation of a unit in different dimensions. This enables to analyze the financial position of a unit from different angles. The next question that comes is how much deprivation is compatible for survival? Or how many deprivations in dimensions are feasible? Our paper focuses on this issue by introducing a dual cut-off approach. We tried to look into the finest possible changes that we can make in our model so that it turns multidimensional instead of multivariate and suit to any form of enterprise. In this paper we had tried with equal weights (of dimensions) but it can be used with general weights.

Keywords: Bankruptcy, Deprivation, Dichotomous, Monotonicity, Multidimensional, Subalternity.

1. Introduction

Economists and management experts had been trying very hard to work out a model which will satisfy performance evaluation and distress analysis of an enterprise or a business unit. Almost all of them tried measuring performance and distress separately. May it be performance evaluation or distress analysis every scholar instead of reconciling the issues went on differentiating. An enterprise (or a business unit) when is in distress implies that it is not performing well, and when it is performing well it is far from any bankruptcy liquidation. Thus distress analysis and performance evaluation are the dual of each other. When anyone is concerned with distress analysis of an enterprise he is unknowingly analyzing the performance of that enterprise. Thus, the situation itself demands that there should be only one methodology that will measure the performance as well as the financial distress position the enterprise.

2. Review of literature

Let's get back to the history of financial distress analysis. Most of the scholars like Beaver (1966), Fitz Patrik (1974), Smith (1974) and Merwin (1974) tried to analyze corporate failure by some single variable, which is primarily known as *univariate analysis* of financial distress. Fitz (1974) examined the financial variables of companies that failed in 1920's and found that the best fitted financial variable for analyzing a corporate failure is Net profit- Net worth. Smith (1974) got with the opinion that the Working capital- Total assets are the best indicators of financial distress. Similarly Merwin (1974) also predicted that liquidity measurement indicator is the best indicator of financial distress. In all these researches financial distress is counted by a single variable. It was easy but not sufficient.

Then it was Altman (1968, 1983) came with a multivariate model based on multivariate discriminate analysis, where he deduced a distress function Z. He concluded that the critical value of Z will define the financial position of an enterprise. He divided the critical values in 3 sections, i.e.; too healthy (need not to bother), grey area (possibility of bankruptcy) and bankruptcy. When $Z \geq 3$ it is too healthy, $1.81 \leq Z < 3$ then it is in grey area and $Z < 1.81$ it is in immediate bankruptcy. Altman defined his distress function Z as;

$Z = 1.2\ X_1 + 1.4\ X_2 + 3.3\ X_3 + 0.6\ X_4 + X_5$,

Where $X_1 = \frac{\text{Working capital}}{\text{Total assets}}$, $X_2 = \frac{\text{Retained earnings}}{\text{Total assets}}$, $X_3 = \frac{\text{EBIT}}{\text{Total debts}}$,

$X_4 = \frac{\text{Market value of equity} + \text{Preference share capital}}{\text{Book value of total liabilities}}$, $X_5 = \frac{\text{Sales}}{\text{Total assets}}$

In 1983 he gave another equation for Z as:-

$Z = 0.717\ X_1 + 0.847\ X_2 + 3.107\ X_3 + 0.42\ X_4 + 0.998\ X_5$

where he altered only X_4. Instead of market of equity he considered book value of equity.

Other scholars like Blum (1974), Dombolena & Khonry (1980), Ohlson (1980), Zmijewski (1983), L.C. Gupta (1979), J. Aiyabei (2002), Mansur A. Mulla (2002), Selvam M. & Babu (2004), Ben McClure (2004), Prof. T.K. Ghosh (2004), Krishna Chaitanya (2005) and many others tried to analyze the financial distress of an enterprise from multivariate point of view. But they got stuck in the critical value of Z. That is only the critical value of Z determined the financial distress. So the models in spite of being multivariate were not multidimensional. Rather they were very much one-dimensional as they only concentrated on the value of Z. It was the value of Z that answered all the questions. Moreover the contribution of each variable towards the financial distress of an enterprise was constant for all business units (i.e.; 1.2 for X_1, 1.4 for X_2 etc.) so somehow the flexibility was missing in the earlier multivariate models.

3. Methodology

This paper concentrates on distress analysis and tries to establish a new methodology by which both performance and distress position of an enterprise can be measured. This methodology is based on **Fuzzy Set** approach and is also best fitted for ordinal data. We on our course of journey will mostly concentrate on the distress analysis part[1]. A business unit is in distress or acute bankruptcy which implies that it is deprived in certain dimensions. How would anyone define deprivation? In a nutshell deprivation is anything which is below a threshold limit. In this paper we would like to take the privilege of re-writing certain terms. Instead of writing distress we prefer to write subaltern[2] and an enterprise or a business unit will be written as a unit. We are more focused in assessing the deprivation of a unit in different dimensions. This enables to analyze the financial position of a unit from different angles. The next section of our paper deals with methodology and followed by illustrative example and conclusion. We first develop some definitions and concepts in terms of Fuzzy Set approach.

3.1. *Definitions*

Let, n be the no. of units and $d \geq 2$ be the no. of dimensions (factors) under consideration. Let, $y = [y_{ij}]$ denote the nXd matrix of achievements, where the typical entry $y_{ij} \geq 0$ is the achievement of units i = 1, 2, 3 …n and in dimensions j = 1, 2, 3….d. Each row vector y_i lists unit i's achievements, while each column vector y_{*j} gives the distribution of dimension j's achievements across the set of units. It is assumed that d is fixed and given and n is allowed to range across all positive integers. This allows comparing subalternity among populations of different sizes. Hence, the domain of matrices is given by, $Y = \{y € R_+^{nd} : n \geq 1\}$, this is due to the assumption that any unit's achievement can be nonnegative real no. This allows accommodating larger or smaller domain as per researcher's choice.

Let, $Z_j > 0$ denote the cut off below which any unit is considered to be deprived in dimension j. This leads Z to be a row vector of dimension specific cut offs. Also note that for any vector or matrix v, the expression $|v|$ denotes the sum of all its elements, and μ (v) represents the mean of v, which is $|v|$ divided by the total no. of elements in v.

A methodology 'M' (Alkire and Foster 2008) for measuring multidimensional subalternity is made up of an identification method and an aggregate method. The identification function (Bourguignon and Chakravarty 2003) $\Omega : R^d_+ \text{ X } R^d_{++} \rightarrow \{0,1\}$, which maps from unit i's achievement vector $y_i \in R^d_+$ and cut off vector $Z \in R^d_{++}$ to an indicator variable in such a way that $\Omega(y_i ; Z) = 1$ if unit i is deprived and $\Omega(y_i ; Z) = 0$ if unit i is not deprived.

Now, applying Ω to each unit's achievement vector in y, results the set $Z \in \{1,2….n\}$ of units who are deprived in y given Z. Next the aggregation step then takes Ω as given and associates with the matrix y and the cut off vector Z to an overall M(y; Z) of multidimensional subalternity. These results to a functional relationship M: Y X $R^{d}_{++} \rightarrow$ R which is the index or measure of multidimensional subalternity.

The methodology will be relevant if we replace the term achievement by deprivation. For any given y, let, $g^0 = [g^0_{ij}]$ denote the 0-1 matrix of deprivations associated with y. The element g^0_{ij} is defined as $g^0_{ij} = 1$ when $y_{ij} < Z_j$ and $g^0_{ij} = 0$ for $y_{ij} \geq Z_j$. From the matrix g^0 we can construct a column vector C of deprivation count, and $C_i = |g^0_i|$, where g_i^0 is unit i's deprivation vector. Thus C_i is no. of deprivation suffered by unit i. Note that when the variables in y are ordinal g^0 and C are still well defined i.e.; g^0 and C are both identical for all monotonic transformations of y_{ij} and Z_j.

For any given y, let, g^1 be the matrix of normalized gaps. And g^1 is defined as $g^1 = \frac{z_j - y_{ij}}{z_j}$ for $y_{ij} < Z_j$ or $g^1_{ij} = 0$ otherwise. Thus, g^1_{ij} is the measure of the extent to which the unit i is deprived in dimension j.

Similarly for $g^2_{ij} = \frac{(z_j - y_{ij})^2}{z_j}$ for $y_{ij} < Z_j$, or 0 otherwise. Here g^2_{ij} measures the vernulability of deprivation of i^{th} unit in j^{th} dimension.

3.2. *Identifying the deprived*

The basic question that comes who are deprived? In earlier definition section we had tried to give dimension specific cut offs. But the dimension specific cut offs alone do not suffice to identify which are deprived. So we must look for additional criteria that will focus across dimensions and arrive at a complete specification of identification methods. Thus for this reasons the cut off 'k' is introduced which considers deprivation across dimensions. The across dimension cut off k = {1, 2…d}. For some potential units Ω(y; Z), let, for one-dimensional aggregator function 'u' such that, $\Omega_u(y_i; Z) = 1$ for $u(y_i) < u(Z)$, or 0 otherwise.

The next question is what will be the value of k? To get an answer lets go by two methods i.e.; the union method and the intersection method.

The union approach is the most commonly used identification criteria. In this approach a unit i is said to be multidimensionality subaltern if there is at least one dimension in which the unit is deprived. The union based deprivation methodology may not be helpful for distinguishing and targeting the most subaltern units, since a unit is termed subaltern if it is deprived in any one dimension.

The other method commonly known as the intersection method which identifies unit i to be subaltern if it is deprived in all dimensions. This method successfully identifies a narrow slice of population which is deprived. Moreover it inevitably misses many units who are experiencing extensive but not universal deprivation.

Thus an alternative, is to use a cut off level for C_i that lies somewhere between two extremes of 1 and d. That is for k = 1, 2….d, let, Ω_k be the identification method defined by $\Omega_k(y_i ; Z) = 1$ for $C_i \geq k$, or 0 otherwise. That is to say, Ω_k identifies unit i as deprived when the no. of deprived dimensions in which i is deprived is at least k, otherwise it is not deprived. As because Ω_k depends both on within dimension cut offs Z_j and across dimension cut offs k, so Ω_k is called the dual cut off method of identification.

3.3. *Measuring Subalternity*

This is a process of measuring multidimensional subalternity M(y; Z) using dual cut off identification approach Ω_k.

To begin with is the percentage of units that are subaltern, i. e.; the head count ratio (H) = H (y ; Z) is defined as H = $\frac{q}{n}$, where q = q (y ; Z) is the no. of units in the set Z_k (no. of subaltern units using dual cut off approach) and n is the total no. of units. Note that H violates dimensional monotonicity. This means that if a unit becomes deprived in a dimension in which that unit had previously not been deprived, H remains unchanged. That is if a subaltern unit i becomes newly deprived in an additional dimension, then overall deprivation doesn't change.

So to combat this issue, an average deprivation share (A) across the deprived ones is introduced, which is defined by, $A = \frac{|c(k)|}{qd}$ where C (k) is the censored vector of deprivation counts and d is dimensions into consideration. The C (k) follows a rule i.e.; if $C_i \geq k$, then $C_i(k) = C_i$ or otherwise 0.

The first step is to measure the dimension adjusted head count ratio, which is given by $M_0 = HA = \frac{q}{n} \times \frac{|C(k)|}{qd} = \frac{|C(k)|}{nd}$. Again $M_0 = \mu(g^0(k))$

Dimension adjusted head count ratio is based on dichotomous data i.e.; whether deprived or not. So it doesn't give information on the depth of deprivation. To measure the sensitivity of the depth of deprivation lets go to the g^1 matrix of normalized gap. The censored version of g^1 is $g^1(k)$. Let the average deprivation gap (G) across all dimension in which the unit is deprived is given by, $G = \frac{|g1(k)|}{|g0(k)|}$

Thus the dimension adjusted deprivation gap $M_1 = HAG = \mu(g^1(k)) = \frac{|g1(k)|}{nd}$

Now M_1 satisfies monotonicity. But a natural question that comes, is it not also true that the increase in a deprivation has the same impact no matter whether the person is very slightly deprived or acutely deprived in that dimension. The latter's impact should be larger. So to combat this issue, the dimension adjusted M_2 can be calculated. M_2 is given by,

M_2 = HAS = $\frac{|g2(k)|}{nd}$, where average severity S = $\frac{|g2(k)|}{|g0(k)|}$

Thus in general the dimension adjusted measures M_α (y; Z) is given by, $M_\alpha = \mu\ (g^\alpha(k)) = \frac{|g\alpha(k)|}{nd}$ for $\alpha \geq 0$.

3.4. *Properties*

1. *Decomposability*: for any two data matrices x and y, M(x,y;Z) = $\frac{n(x)}{n(xy)}$M(x;Z) + $\frac{n(y)}{n(xy)}$M(y;Z).
2. *Replication invariance*: if x is obtained from y by a replication then M(x; Z) =M(y; Z).
3. *Symmetry*: if x is obtained from y by a permutation then M(x; Z) =M(y; Z).
4. *Subalternity focus*: if x is obtained from y by a simple increment among the non subalterns, then M(x; Z) =M(y; Z).
5. *Deprivation focus*: if x is obtained from y by a simple increment among the none deprived, then M(x; Z) =M(y; Z).
6. *Weak monotonicity*: if x is obtained from y by a simple increment, then M(x; Z) ≤ M(y; Z).
7. *Monotonicity*: M satisfies weak monotonicity and the following; if x is obtained from y by a deprived increment among the subalterns then M(x; Z) < M(y; Z).
8. *Dimensional monotonicity*: if x is obtained from y by a dimensional increment among the subalterns then M(x; Z) ≤ M(y; Z).
9. *Non-triviality*: M achieves at least two distinct values.
10. *Normalization*: M achieves a minimum value of 0 and a maximum value of 1.
11. *Weak transfer*: if x is obtained from y by an averaging of achievements among the subalterns, then M(x; Z) ≤ M(y; Z).
12. *Weak rearrangement*: if x is obtained from y by an association of decreasing rearrangement among the subalterns, then M(x; Z) ≤ M(y; Z).

3. Illustrations

In this section we had tried to apply our methodology. For illustration and our convenience we had taken four central public sector enterprises. From detailed analysis of their annual report we first calculated financial distress through Altman (1983) Z test next we went to our methodology of measuring multidimensional subalternity (for measuring deprivation).

TABLE-1 Necessary Details from Annual Reports of Different Companies

SL.NO.	NAME	NCA	RETAINED EARNING	EBIT	B.V.EQTY	B.V.T.L.	T.A.	SALES
1	ANDREW YULE	9677.6	5664.86	4504.13	6672.77	33063.29	27867.1	23211.7
2	BHARTI BHARI UDYOG LTD	49525.75	74.79	45.15	10698.06	54645.14	54645.14	1053.62
3	BALMER LAWRIE INVESTMENT	216736925	216236925	248463698	221972690	543513955	543513955	253029370
4	BBJ	4435.3	519.36	645.3	2026.5	5251.69	5251.61	15260.46

Source: Annual Reports of Selected Companies as on 31^{st} March 2011

TABLE-2 Calculation of Altman's Distress Co-efficient Z

SL.NO.	NAME	X1	X2	X3	X4	X5	Z	INTERPRETATION
1	ANDREW YULE	0.3472769	0.203281289	0.67500154	0.20181809	0.83294279	3.43444706	HEALTHY
2	BHARTI BHARI UDYOG LTD	0.9063157	0.001368649	0.00422039	0.19577331	0.01928113	0.76556774	BANKRUPT
3	BALMER LAWRIE INVESTMENT	0.3987698	0.397849812	1.11934355	0.40840293	0.46554347	4.73683871	HEALTHY
4	BBJ	0.84456	0.098895386	0.31843079	0.38587578	2.90586315	4.74079768	HEALTHY
	Z (CUT OFF)	0.24	0.35	0.45	0.4	4	6.02668	HEALTHY

Source: Computed from table-1

Multidimensional Subalternity Analysis:

ITERATION-1

SL.NO.	NAME	X1	X2	X3	X4	X5	
	Z(CUT OFF)	0.24	0.35	0.45	0.4	4	C(k)
1	ANDREW YULE	0	1	0	1	1	3
2	BHARTI BHARI UDYOG LTD	0	1	1	1	1	4
3	BALMER LAWRIE INVESTMENT	0	0	0	0	1	1
4	BBJ	0	1	1	1	1	4

ITERATION-2

	k =2						
SL.NO.	NAME	X1	X2	X3	X4	X5	M_0
1	ANDREW YULE	0	1	0	1	1	0.55
2	BHARTI BHARI UDYOG LTD	0	1	1	1	1	
3	BALMERLAWRIE INVESTMENT	0	0	0	0	0	
4	BBJ	0	1	1	1	1	
	CUNTRIBUTION OF EACH DIMENSION	0	0.15	0.1	0.15	0.15	0.55
	PERCENTAGE	0	27.27272727	18.1818182	27.2727273	27.2727273	100

ITERATION-3

SL.NO	NAME	X1	X2	X3	X4	X5	M1
1	ANDREW YULE	0	0.419196318	0	0.49545478	0.7917643	0.32587676
2	BHARTI BHARI UDYOG LTD	0	0.996089575	0.99062135	0.51056672	0.99517972	
3	BALMER LAWRIE INVESTMENT	0	0	0	0	0	
4	BBJ	0	0.717441753	0.29237602	0.03531054	0.27353421	

ITERATION-4

SL.NO.	NAME	X1	X2	X3	X4	X5	M2
1	ANDREW YULE	0	0.175725553	0	0.24547544	0.62689071	0.2474476
2	BHARTI BHARI UDYOG LTD	0	0.992194442	0.98133066	0.26067838	0.99038267	
3	BALMER LAWRIE INVESTMENT	0	0	0	0	0	
4	BBJ	0	0.514722669	0.08548374	0.00124683	0.07482096	

Source: All tables are computed from table-1

4. Conclusion

In earlier discussion it is clear that multivariate analysis of financial distress should be replaced by multidimensional subalternity analysis, since, it gives dimension specific result and allows flexibility for arriving a comprehensive interpretation. Altman's distress co-efficient Z shows that only **Bharti Bhari Udyog Ltd**. is on its way to bankruptcy. Our multidimensional subalternity analysis concludes that except **Balmer Lawrie Investment Co. Ltd.** all other companies are deprived. The dimensional adjusted M_0 is 0.55 and M_1 and M_2 are 0.33 and 0.25 respectively. The contribution of each dimension towards deprivation is X_1 = 0%, X2 = 27.27%, X_3 = 18.19%, X_4 = 27.27%, X_5 = 27.27%. The cut offs Z and k may be termed subjective but they still have some rationality. When X_1 = 0.24 and X_2 = 0.35, this implies that of Re. 1 of total asset Re. 0.24 is on account of working capital and Re. 0.35 is on account of retained earnings and the rest is on account of capital employed. X_3 being 0.45 indicates that Re.1 invested in equity yields Re.0.45 of EBIT. X_4 = 0.4 means of Re.1 of total liabilities 0.4 is the contribution towards equity and X_5 =4 means Re.1 of total asset increases sales by 4 times.

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Notes:

1. When we are concerned with the distress position of an enterprise we are also evaluating its performance.
2. Subalternity is staying subordinate in sex, caste, religion, office, business etc.

Abbreviations:

NCA = Net Current Asset

EBIT = Earnings before Interest and Tax

B.V.EQTY = Book Value of Equity (Since debt and equity of PSU are financed by govt. alone, so X_3 is calculated on B.V.EQTY)

B.V.T.L = Book Value of Total Liabilities

T.A. = Total Assets