

Achieving Optimal Investment Portfolio Using Dynamic Programming

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Abstract

Any time an investor makes an investment he must decide on the optimal investment strategy. Two very important strategies are Active portfolio management and Long term investing. The main objective of this study was to determine the optimal investment returns among six investments and the corresponding investments to be made. Data on six investments were collected – Government of Ghana's Treasury Bills, Barclays Bank Ghana Ltd, Ghana Commercial Bank, Data Bank, Guinness Ghana Limited and Fan Milk Limited. Sample period ranges from 2000 to 2008 and the price series were normalized such that each commodity's price changes have annualized volatility of 10%. Dynamic programming was used for data analysis. The results revealed that with ₵900 available for investment and given the corresponding returns from the various financial institutions we should not invest in Government of Ghana's Treasury Bills and Ghana Commercial Bank. However, we should invest ₵100 in Barclays Bank Ghana to get ₵15, ₵200 in Data Bank, Accra to get ₵40, ₵500 in Ghana Guinness Limited, Accra for a return of ₵75 and ₵100 in Fan Milk Limited, Accra for a return of ₵19. This gives the optimal returns of ₵149.

Keywords: Dynamic Programming, Optimal Investment

1. Introduction

People throughout ages have sought to look for avenues to increase their wealth. The best way to maintain and improve once wealth is to invest the little that one has. Individuals, entities, organizations and cooperate bodies make investments towards the future. One difficulty faced by all is where, how, when and for how long to invest, in the mist of all opportunities, to obtain the maximum satisfaction from the investment made. Usually investors might have a certain amount of money to invest. The investor may have various options of investment based on the returns.

Investment is a term frequently used in the fields of economics, business management and finance. It can mean savings alone, or savings made through delayed consumption. Investment can be divided into different types according to various theories and principles.

According to economic theories, investment is defined as the per-unit production of goods, which have not been consumed, but will however, be used for the purpose of future production. Examples of this type of investments are tangible goods like construction of a factory or bridge and intangible goods like 6 months of on-the-job training. In terms of national production and income, Gross Domestic Product (GDP) has an essential constituent, known as gross investment. This paper shares with a number of articles, such as Abel (1998), Epstein and Zin (1992), Telmer (1993), and Naik (1994), the objective of achieving optimal investment portfolio of Merton (1973) and Lucas (1978).

According to business management theories, investment refers to tangible assets like machinery and equipments and buildings and intangible assets like copyrights or patents and goodwill. The decision for investment is also known as capital budgeting decision, which is regarded as one of the key decision of management.

In finance, investment refers to the purchasing of securities or other financial assets from the capital market (Hassett, 2008). It also means buying money market or real properties with high market liquidity. Some examples are gold, silver, real properties, and precious items. Financial investments are in stocks, bonds, and other types of security investments. Indirect financial investments can also be done with the help of mediators or third parties, such as pension funds, mutual funds, commercial banks, and insurance companies. According to personal finance theories, an investment is the implementation of money for buying shares, mutual funds or assets with capital risk.

Usually a combination of any of the above investment possibilities may be considered by an investor or an individual. Any time an investor wishes to make an investment with a certain sum of money, say, he must decide on the optimal investment strategy to adopt. The optimal investment strategy could be long term or short term,

active portfolio management or long term investing. The choice of a particular investment should be based on the cumulative returns on all the investments.

2. Method

2.1 Data

Secondary data was used for the study. Interest rates and yield functions of various investments were collected from Banking, Financial and Non-Financial Institutions and interpolation used to obtain a single value of return. Data on six investments were collected – Government of Ghana's Treasury Bills, Barclays Bank Ghana Ltd, Ghana Commercial Bank, Data Bank, Guinness Ghana Limited and Fan Milk Limited.

Sample period ranges from 2000 to 2008 and the price series were normalized such that each commodity's price changes have annualized volatility of 10%.

Each commodity characteristic is its past returns at various time horizons. As such, in order to predict the 1-year return factors for the commodities, pooled panel regression on the data set is run to obtain the annual returns. Dynamic programming is then used to determine the optimal investment returns and the corresponding investments to be made.

The following notations were made:

Return from investment 1 - Government of Ghana's Treasury Bills

Return from investment 2 - Barclays Bank Ghana, Accra

Return from investment 3 - Ghana Commercial Bank

Return from investment 4 - Data Bank, Accra

Return from investment 5 - Ghana Guinness Limited, Accra

Return from investment 6 – Fan Milk Limited, Accra

2.2 Dynamic Programming

Dynamic Programming is a technique that can be used to solve many optimization problems. Dynamic programming obtains solutions by working backward from the end of a problem toward the beginning (Howard, 1960). It is a method for solving complex problems by breaking them down into simpler sub problems. It is applicable to problems exhibiting the properties of overlapping sub problems which are only slightly smaller and optimal substructure.

There are a number of characteristics that are common to all dynamic programming problems.

1. The problem can be divided into stage with a decision required at each stage.
2. Each stage has a number of states associated with it.
3. The decision at one stage transforms one state into a state in another stage.
4. Given the current state, the optimal decision for each of the remaining state does not depend on the previous states or decisions.
5. There exist a recursive relationship that identifies the optimal decision for stage j , given that stage $j + 1$ has already been solved.
6. The final stage must be solvable by itself. The last time properties are tied up in the recursive relationship given above.

Suppose an investor wants to invest and there are several opportunities available to him/her then there arise a problem of choice/allocation. It would be realized that each opportunity require deposits in financial terms and an expected return. The investor may allocate all the money to just one opportunity or split the money between the alternatives of investments all with the aim of obtaining the maximum returns from the investment made.

Let $f_i(x)$, $i = 1, 2, 3, \dots, n$ denote the return from investment i when x units of money are invested in investment i .

We define x_i ($i = 1, 2, 3, \dots, n$) as the number of units of money invested in investment, i .

The problem of determining how much to invest in each investment in order to maximize total returns can be approached through a multi-stage decision process by modeling a Mathematical program to find the optimal policy using Dynamic Programming.

$$\begin{aligned} & \text{Maximize} && \sum f_i(x_i) \\ & \text{Subject to} && \sum x_i \leq b \\ & && x_i \geq 0 \\ & && i = 1, 2, 3, \dots, n \end{aligned}$$

It must be noted that $f_i(x_i)$ are functions of a single variable, b is a known nonnegative integer and all $(x_i's)$ are positive integers.

2.3 Assumptions

The following assumptions were made;

1. The amount invested were restricted to be integral multiples of €100.
2. That there was a maximum of €900 available
3. Six investment options are available.

2.4 Algorithm for Dynamic Programming

We start the algorithm by first identifying the appropriate state, stage and decision (Hadley, 1962). We define stage such that when one stage is remained, the problem will be trivial to solve. That is if we arrange the investment in no specific order.

At Investment 6:

Investment 6 Returns = Investment 5 Returns + Investment 6 inputs

At Investment 1:

Investment 1 Returns = Investment 2 Returns + Investment 1 inputs

Hence Investment is the stage.

Define the state of the stage, which is at each stage (investment) the investor will have to decide how much money he will have to invest. To do this we need to know only the amount of money left at the beginning of the investment (stage). Hence State is the amount of money left to be invested.

3. Results

Letting $f_j(x_j)$ ($j = 1, 2, 3, 4, 5, 6$) denotes the return in (€) from investment j when x_j units of money are invested.

The return table from the investments has been written as shown below

TABLE 5: RETURNS FROM INVESTMENTS

RETURN FROM INVESTMENT	AMOUNT INVESTED									
	0	100	200	300	400	500	600	700	800	900
$f_1(x)$	0	10	20	25	33	35	40	45	55	60
$f_2(x)$	0	15	30	40	45	50	55	70	80	90
$f_3(x)$	0	12	22	32	35	40	50	60	65	70
$f_4(x)$	0	18	40	50	55	65	70	70	85	95
$f_5(x)$	0	20	28	38	50	75	75	80	80	85
$f_6(x)$	0	19	25	35	40	50	60	70	75	80

Return values of various units as the number of units of money invested in investment i .

Define

$x_i = 100, 200, 300, 400, 500, 600, 700, 800, 900$ as the number of units of money invested in investment i .

$M_j(i)$ = the best return beginning in stage j and state i .

$d_j(i)$ = Decisions taken at state that achieves $M_j(i)$

We note that $M_j(0) = 0$ and $d_j(0) = 0$

The model for solving the above problem is:

$$\text{Maximize: } Z = f_1(x) + f_2(x) + f_3(x) + f_4(x) + f_5(x) + f_6(x)$$

$$\text{Subject to } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \leq 900$$

We begin the solution by considering the last stage of the process, stage 6. We assume that the previous stages have been completed and we are to complete the allocation of the money to the investment 6. Since we do not know how much was allocated to the previous investment (investment 5), we do not know how many units are available for investment 6. Thus we consider all possibilities.

After the first five investments have been made there will be either $\text{¢}0$, $\text{¢}1$, $\text{¢}2$, $\text{¢}3$, $\text{¢}4$, $\text{¢}5$, $\text{¢}6$, $\text{¢}7$, $\text{¢}8$, $\text{¢}9$. It is clear from the definition of $f_6(x)$ that the best way to complete the process is to allocate all available units to investments 6.

From investment 6:

$$M_6(9) = \max[f_6(0), f_6(1), f_6(2), f_6(3), f_6(4), f_6(5), f_6(6), f_6(7), f_6(8), f_6(9)]$$

$$= \max[0, 19, 25, 35, 40, 50, 60, 70, 75, 80] \quad M_6(9) = 80 \text{ with } d_6(9) = 9$$

$$M_6(8) = \max[f_6(0), f_6(1), f_6(2), f_6(3), f_6(4), f_6(5), f_6(6), f_6(7), f_6(8)]$$

$$= \max[0, 19, 25, 35, 40, 50, 60, 70, 75] \quad M_6(8) = 75 \text{ with } d_6(8) = 8$$

$$M_6(7) = \max[f_6(0), f_6(1), f_6(2), f_6(3), f_6(4), f_6(5), f_6(6), f_6(7)]$$

$$= \max[0, 19, 25, 35, 40, 50, 60, 70] \quad M_6(7) = 70 \text{ with } d_6(7) = 7$$

$$M_6(6) = \max[f_6(0), f_6(1), f_6(2), f_6(3), f_6(4), f_6(5), f_6(6)]$$

$$= \max[0, 19, 25, 35, 40, 50, 60] \quad M_6(6) = 60 \text{ with } d_6(6) = 6$$

$$M_6(5) = \max[f_6(0), f_6(1), f_6(2), f_6(3), f_6(4), f_6(5)]$$

$$= \max[0, 19, 25, 35, 40, 50] \quad M_6(5) = 50 \text{ with } d_6(5) = 5$$

$$M_6(4) = \max[f_6(0), f_6(1), f_6(2), f_6(3), f_6(4)]$$

$$= \max[0, 19, 25, 35, 40] \quad M_6(4) = 40 \text{ with } d_6(4) = 4$$

$$M_6(3) = \max[f_6(0), f_6(1), f_6(2), f_6(3)]$$

$$= \max[0, 19, 25, 35] \quad M_6(3) = 35 \text{ with } d_6(3) = 3$$

$$M_6(2) = \max[f_6(0), f_6(1), f_6(2)]$$

$$= \max[0, 19, 25] \quad M_6(2) = 25 \text{ with } d_6(2) = 2$$

$$M_6(1) = \max[f_6(0), f_6(1)]$$

$$= \max[0, 19]$$

$$M_6(0) = \max[f_6(0)]$$

$$= \max[0]$$

$$M_6(1) = 19 \text{ with } d_6(1) = 1$$

$$M_6(0) = 0 \text{ with } d_6(0) = 0$$

From investment 5:

$$M_5(9) = \max[f_5(0) + M_6(9), \quad f_5(1) + M_6(8), \quad f_5(2) + M_6(7), \quad f_5(3) + M_6(6), \\ f_5(4) + M_6(5), \quad f_5(5) + M_6(4), \quad f_5(6) + M_6(3), \\ f_5(7) + M_6(2), \quad f_5(8) + M_6(1), \quad f_5(9) + M_6(0)]$$

$$M_5(9) = \max[0 + 80, 20 + 75, 28 + 70, 38 + 60, 50 + 50, 75 + 40, 75 + 35, 80 + 25, 80 + 19, 85 + 0] M_5(9) =$$

$$\max[80, 95, 98, 98, 100, 115, 110, 105, 99, 85] M_5(9) = 115 \text{ with } d_5(9) = 5$$

$$M_5(8) = \max[f_5(0) + M_6(8), \quad f_5(1) + M_6(7), \quad f_5(2) + M_6(6), \quad f_5(3) + M_6(5), \\ f_5(4) + M_6(4), \quad f_5(5) + M_6(3), \quad f_5(6) + M_6(2), \\ f_5(7) + M_6(1), \quad f_5(8) + M_6(0)]$$

$$M_5(8) = \max[0 + 75, \quad 20 + 70, \quad 28 + 60, \quad 38 + 50, \quad 50 + 40, \quad 75 + 35, \\ 75 + 25, \quad 80 + 19, \quad 80 + 0] M_5(8) =$$

$$= \max[75, 90, 88, 88, 90, 110, 100, 99, 80]$$

$$M_5(8) = 110 \text{ with } d_5(8) = 5$$

$$\begin{aligned}
 M_5(7) &= \max[f_5(0) + M_6(7), f_5(1) + M_6(6), f_5(2) + M_6(5), f_5(3) + M_6(4), f_5(4) + M_6(3), f_5(5) + M_6(2), f_5(6) + M_6(1), f_5(7) + M_6(0)] \\
 M_5(7) &= \max[0 + 70, 20 + 60, 28 + 50, 38 + 40, 50 + 35, 75 + 25, 75 + 19, 80 + 0] \\
 M_5(8) &= 100 \text{ with } d_5(7) = 5 \\
 M_5(6) &= \max[f_5(0) + M_6(6), f_5(1) + M_6(5), f_5(2) + M_6(4), f_5(3) + M_6(3), f_5(4) + M_6(2), f_5(5) + M_6(1), f_5(6) + M_6(0)] \\
 M_5(6) &= \max[0 + 60, 20 + 50, 28 + 40, 38 + 35, 50 + 25, 75 + 19, 75 + 0] \\
 M_5(6) &= \max[60, 70, 68, 73, 75, 94, 75] \\
 M_5(6) &= 94 \text{ with } d_5(6) = 5 \\
 M_5(5) &= \max[f_5(0) + M_6(5), f_5(1) + M_6(4), f_5(2) + M_6(3), f_5(3) + M_6(2), f_5(4) + M_6(1), f_5(5) + M_6(0)] \\
 M_5(5) &= \max[0 + 50, 20 + 40, 28 + 35, 38 + 25, 50 + 19, 75 + 0] \\
 M_5(5) &= \max[50, 60, 63, 63, 69, 75] \\
 M_5(5) &= 75 \text{ with } d_5(5) = 5 \\
 M_5(4) &= \max[f_5(0) + M_6(4), f_5(1) + M_6(3), f_5(2) + M_6(2), f_5(3) + M_6(1), f_5(4) + M_6(0)] \\
 M_5(4) &= \max[0 + 40, 20 + 35, 28 + 25, 38 + 19, 50 + 0] \\
 M_5(4) &= \max[40, 55, 53, 57, 50] \\
 M_5(4) &= 57 \text{ with } d_5(4) = 4 \\
 M_5(3) &= \max[f_5(0) + M_6(3), f_5(1) + M_6(2), f_5(2) + M_6(1), f_5(3) + M_6(0)] \\
 M_5(3) &= \max[0 + 35, 20 + 25, 28 + 19, 38 + 0] \\
 M_5(3) &= \max[35, 45, 47, 38] \\
 M_5(3) &= 47 \text{ with } d_5(3) = 2 \\
 M_5(2) &= \max[f_5(0) + M_6(2), f_5(1) + M_6(1), f_5(2) + M_6(0)] \\
 M_5(2) &= \max[0 + 25, 20 + 19, 28 + 0] \\
 M_5(2) &= \max[25, 39, 28] \\
 M_5(2) &= 39 \text{ with } d_5(2) = 1 \\
 M_5(1) &= \max[f_5(0) + M_6(1), f_5(1) + M_6(0)] \\
 M_5(1) &= \max[0 + 19, 20 + 0] \\
 M_5(1) &= \max[19, 20] \\
 M_5(1) &= 20 \text{ with } d_5(1) = 1 \\
 M_5(0) &= \max[f_5(0) + M_6(0)] \\
 M_5(0) &= \max[0 + 0] \\
 M_5(0) &= \max[0] \\
 M_5(0) &= 0 \text{ with } d_5(0) = 0
 \end{aligned}$$

Investment 4

$$\begin{aligned}
 M_4(9) &= \max[f_4(0) + M_5(9), f_4(1) + M_5(8), f_4(2) + M_5(7), f_4(3) + M_5(6), f_4(4) + M_5(5), f_4(5) + M_5(4), f_4(6) + M_5(3), f_4(7) + M_5(2), f_4(8) + M_5(1), f_4(9) + M_5(0)] \\
 M_4(9) &= \max[0 + 115, 18 + 110, 40 + 100, 50 + 94, 55 + 75, 65 + 57, 70 + 47, 70 + 39, 85 + 20, 95 + 0] \\
 M_4(9) &= \max[115, 128, 140, 144, 130, 122, 117, 109, 105, 95] \\
 M_4(9) &= 144 \text{ with } d_4(9) = 3 \\
 M_4(8) &= \max[f_4(0) + M_5(8), f_4(1) + M_5(7), f_4(2) + M_5(6), f_4(3) + M_5(5), f_4(4) + M_5(4), f_4(5) + M_5(3), f_4(6) + M_5(2), f_4(7) + M_5(1), f_4(8) + M_5(0)] \\
 M_4(8) &= \max[0 + 110, 18 + 100, 40 + 94, 50 + 75, 55 + 57, 65 + 47, 70 + 39, 70 + 20, 85 + 0] \\
 M_4(8) &= \max[110, 118, 134, 125, 112, 112, 112, 90, 85] \\
 M_4(8) &= 134 \text{ with } d_4(8) = 2 \\
 M_4(7) &= \max[f_4(0) + M_5(7), f_4(1) + M_5(6), f_4(2) + M_5(5), f_4(3) + M_5(4), f_4(4) + M_5(3), f_4(5) + M_5(2), f_4(6) + M_5(1), f_4(7) + M_5(0)] \\
 M_4(7) &= \max[0 + 100, 18 + 94, 40 + 75, 50 + 57, 55 + 47, 65 + 39, 70 + 20, 70 + 0] \\
 M_4(7) &= \max[100, 112, 115, 107, 102, 104, 90, 70] \\
 M_4(7) &= 115 \text{ with } d_4(7) = 2
 \end{aligned}$$

$$\begin{aligned}
 M_4(6) &= \max[f_4(0) + M_5(6), f_4(1) + M_5(5), f_4(2) + M_5(4), f_4(3) + M_5(3), \\
 &\quad f_4(4) + M_5(2), f_4(5) + M_5(1), f_4(6) + M_5(0)] \\
 M_4(6) &= \max[0 + 94, 18 + 75, 40 + 57, 50 + 47, 55 + 39, 65 + 20, 70 + 0] M_4(6) = \\
 &= \max[94, 92, 97, 97, 94, 84, 70] M_4(6) = 97 \text{ with } d_4(6) = 2 \\
 M_4(5) &= \max[f_4(0) + M_5(5), f_4(1) + M_5(4), f_4(2) + M_5(3), f_4(3) + M_5(2), \\
 &\quad f_4(4) + M_5(1), f_4(5) + M_5(0)] \\
 M_4(5) &= \max[0 + 75, 18 + 57, 40 + 47, 50 + 39, 55 + 20, 65 + 0] M_4(5) = \\
 &= \max[75, 75, 87, 89, 75, 65] M_4(5) = 89 \text{ with } d_4(5) = 3 \\
 M_4(4) &= \max[f_4(0) + M_5(4), f_4(1) + M_5(3), f_4(2) + M_5(2), f_4(3) + M_5(1), f_4(4) + \\
 &\quad M_5(0)] M_4(4) = \max[0 + 57, 18 + 47, 40 + 39, 50 + 20, 55 + 0] M_4(4) = \\
 &= \max[57, 65, 79, 70, 55] M_4(4) = 79 \text{ with } d_4(4) = 2 \\
 M_4(3) &= \max[f_4(0) + M_5(3), f_4(1) + M_5(2), f_4(2) + M_5(1), f_4(3) + M_5(0)] M_4(3) = \\
 &= \max[0 + 47, 18 + 39, 40 + 20, 50 + 0] M_4(3) = \max[47, 57, 60, 50] M_4(3) = \\
 &= 60 \text{ with } d_4(3) = 2 \\
 M_4(2) &= \max[f_4(0) + M_5(2), f_4(1) + M_5(1), f_4(2) + M_5(0)] M_4(2) = \max[0 + 39, 18 + \\
 &\quad 20, 40 + 0] \\
 M_4(2) &= \max[39, 38, 40] M_4(2) = 40 \text{ with } d_4(2) = 2 \\
 M_4(1) &= \max[f_4(0) + M_5(1), f_4(1) + M_5(0)] \\
 M_4(1) &= \max[0 + 20, 18 + 0] M_4(1) = \max[20, 18] M_4(1) = 20 \text{ with } d_4(1) = 0 \\
 M_4(0) &= \max[f_4(0) + M_5(0) f_4(0) + M_5(0)] \\
 M_4(0) &= \max[0, 0] M_4(0) = 0 \text{ with } d_4(0) = 0
 \end{aligned}$$

From investment 3:

$$\begin{aligned}
 M_3(9) &= \max[f_3(0) + M_4(9), f_3(1) + M_4(8), f_3(2) + M_4(7), f_3(3) + M_4(6), \\
 &\quad f_3(4) + M_4(5), f_3(5) + M_4(4), f_3(6) + M_4(3), \\
 &\quad f_3(7) + M_4(2), f_3(8) + M_4(1), f_3(9) + M_4(0)] \\
 M_3(9) &= \max[0 + 144, 12 + 134, 22 + 115, 32 + 97, 35 + 89, 40 + 79, 50 + 60, \\
 &\quad 60 + 40, 65 + 20, 70 + 0] M_3(9) = \\
 &= \max[144, 146, 137, 129, 124, 119, 107, 100, 85, 70] M_3(9) = 146 \\
 &\text{with } d_3(9) = 1 \\
 M_3(8) &= \max[f_3(0) + M_4(8), f_3(1) + M_4(7), f_3(2) + M_4(6), f_3(3) + M_4(5), f_3(4) + \\
 &\quad M_4(4), f_3(5) + M_4(3), f_3(6) + M_4(2), f_3(7) + M_4(1), f_3(8) + M_4(0)] M_3(8) = \\
 &= \max[134, 127, 119, 121, 114, 100, 90, 80, 65] M_3(8) = 134 \text{ with } \\
 &d_3(8) = 0 \\
 M_3(7) &= \max[f_3(0) + M_4(7), f_3(1) + M_4(6), f_3(2) + M_4(5), f_3(3) + M_4(4), \\
 &\quad f_3(4) + M_4(3), f_3(5) + M_4(2), f_3(6) + M_4(1), \\
 &\quad f_3(7) + M_4(0)] \\
 M_3(7) &= \max[0 + 115, 12 + 97, 22 + 89, 32 + 79, 35 + 57, 40 + 40, \\
 &\quad 50 + 20, 60 + 0] M_3(7) \\
 &= \max[115, 109, 111, 111, 92, 80, 70, 60] \\
 M_3(7) &= 115 \text{ with } d_3(7) = 0 \\
 M_3(6) &= \max[f_3(0) + M_4(6), f_3(1) + M_4(5), f_3(2) + M_4(4), f_3(3) + M_4(3), \\
 &\quad f_3(4) + M_4(2), f_3(5) + M_4(1), f_3(6) + M_4(0)] \\
 M_3(6) &= \max[0 + 97, 12 + 89, 22 + 79, 32 + 57, 35 + 40, 40 + 20, \\
 &\quad 50 + 0] M_3(6) = \max[97, 101, 101, 89, 75, 60, 50] \\
 M_3(6) &= 101 \text{ with } d_3(6) = 1 \\
 M_3(5) &= \max[f_3(0) + M_4(5), f_3(1) + M_4(4), f_3(2) + M_4(3), f_3(3) + M_4(2), \\
 &\quad f_3(4) + M_4(1), f_3(5) + M_4(0)]
 \end{aligned}$$

$$\begin{aligned}
 M_3(5) &= \max[0 + 89, 12 + 79, 22 + 57, 32 + 40, 35 + 20, 40 + 0] M_3(5) = \\
 &\max[89, 91, 79, 72, 55, 40] M_3(5) = 91 \text{ with } d_3(5) = 1 \\
 M_3(4) &= \max[f_3(0) + M_4(4), f_3(1) + M_4(3), f_3(2) + M_4(2), f_3(3) + M_4(1), \\
 &f_3(4) + M_4(0)] \\
 M_3(4) &= \max[0 + 79, 12 + 57, 22 + 40, 32 + 20, 35 + 0] M_3(4) = \\
 &\max[79, 69, 65, 62, 35] M_3(4) = 79 \text{ with } d_3(4) = 0 \\
 M_3(3) &= \max[f_3(0) + M_4(3), f_3(1) + M_4(2), f_3(2) + M_4(1), f_3(3) + M_4(0)] \\
 M_3(3) &= \max[0 + 60, 12 + 40, 22 + 20, 32 + 0] M_3(3) = \\
 &\max[60, 52, 42, 32] M_3(3) = 60 \text{ with } d_3(3) = 0 \\
 M_3(2) &= \max[f_3(0) + M_4(2), f_3(1) + M_4(1), f_3(2) + M_4(0)] M_3(2) = \max[0 + 40, \\
 &12 + 20, 22 + 0] \\
 M_3(2) &= \max[40, 32, 22] M_3(2) = 40 \text{ with } d_3(2) = 0 \\
 M_3(1) &= \max[f_3(0) + M_4(1), f_3(1) + M_4(0)] M_3(1) = \max[0 + 20, 12 + 0] \\
 M_3(1) &= \max[20, 12] M_3(1) = 20 \text{ with } d_3(1) = 0 \\
 M_3(0) &= \max[f_3(0) + M_4(0)] \\
 M_3(0) &= \max[0 + 0] M_3(0) = \max[0] M_3(0) = 0 \quad \text{with } d_3(1) = 0
 \end{aligned}$$

From investment 2:

$$\begin{aligned}
 M_2(9) &= \max[f_2(0) + M_3(9), f_2(1) + M_3(8), f_2(2) + M_3(7), f_2(3) + M_3(6), \\
 &f_2(4) + M_3(5), f_2(5) + M_3(4), f_2(6) + M_3(3), \\
 &f_2(7) + M_3(2), f_2(8) + M_3(1), f_2(9) + M_3(0)] \\
 M_2(9) &= \max[0 + 146, 15 + 134, 30 + 115, 40 + 101, 45 + 91, 50 + 79, 55 + 60, \\
 &70 + 40, 80 + 20, 90 + 0] M_2(9) = \\
 &\max[146, 149, 145, 141, 136, 129, 115, 111, 120, 90] M_2(9) = 149 \\
 &\text{with } d_2(9) = 1 \\
 M_2(8) &= \max[f_2(0) + M_3(8), f_2(1) + M_3(7), f_2(2) + M_3(6), f_2(3) + M_3(5), \\
 &f_2(4) + M_3(4), f_2(5) + M_3(3), f_2(6) + M_3(2), \\
 &f_2(7) + M_3(1), f_2(8) + M_3(0)] \\
 M_2(8) &= \max[0 + 134, 15 + 115, 30 + 101, 40 + 91, 45 + 79, 50 + 60, 55 + 40, \\
 &70 + 20, 80 + 0] M_2(8) = \\
 &\max[134, 130, 131, 131, 124, 107, 95, 90, 80] M_2(8) = 134 \text{ with } \\
 &d_2(8) = 0 \\
 M_2(7) &= \max[f_2(0) + M_3(7), f_2(1) + M_3(6), f_2(2) + M_3(5), f_2(3) + M_3(4), \\
 &f_2(4) + M_3(3), f_2(5) + M_3(2), f_2(6) + M_3(1), \\
 &f_2(7) + M_3(0)] \\
 M_2(7) &= \max[0 + 115, 15 + 101, 30 + 91, 40 + 79, 45 + 60, 50 + 40, \\
 &55 + 20, 70 + 0] M_2(7) = \\
 &= \max[115, 116, 121, 119, 105, 90, 75, 70] \\
 M_2(7) &= 121 \text{ with } d_2(7) = 2 \\
 M_2(6) &= \max[f_2(0) + M_3(6), f_2(1) + M_3(5), f_2(2) + M_3(4), f_2(3) + M_3(3), \\
 &f_2(4) + M_3(2), f_2(5) + M_3(1), f_2(6) + M_3(0)] \\
 M_2(6) &= \max[0 + 101, 15 + 91, 30 + 79, 40 + 60, 45 + 40, 50 + 20, \\
 &55 + 0] M_2(6) = \max[101, 106, 109, 97, 100, 95, 70] \\
 M_2(6) &= 109 \text{ with } d_2(6) = 2 \\
 M_2(5) &= \max[f_2(0) + M_3(5), f_2(1) + M_3(4), f_2(2) + M_3(3), f_2(3) + M_3(2), \\
 &f_2(4) + M_3(1), f_2(5) + M_3(0)] \\
 M_2(5) &= \max[0 + 91, 15 + 79, 30 + 60, 40 + 40, 45 + 20, 50 + 0] \\
 M_2(5) &= \max[91, 94, 90, 80, 65, 50] M_2(5) = 94 \text{ with } d_2(5) = 1
 \end{aligned}$$

$$M_2(4) = \max[f_2(0) + M_3(4), f_2(1) + M_3(3), f_2(2) + M_3(2), f_2(3) + M_3(1), f_2(4) + M_3(0)]$$

$$M_2(4) = \max[0 + 79, 15 + 60, 30 + 40, 40 + 20, 45 + 0] M_2(4) = \max[79, 75, 70, 60, 45] M_2(4) = 79 \text{ with } d_2(4) = 0$$

$$M_2(3) = \max[f_2(0) + M_3(3), f_2(1) + M_3(2), f_2(2) + M_3(1), f_2(3) + M_3(0)] M_2(3) = \max[0 + 60, 15 + 40, 30 + 20, 40 + 0] M_2(3) = \max[60, 55, 50, 40]$$

$$M_2(3) = 60 \text{ with } d_2(3) = 0$$

$$M_2(2) = \max[f_2(0) + M_3(2), f_2(1) + M_3(1), f_2(2) + M_3(0)]$$

$$M_2(2) = \max[0 + 40, 15 + 20, 30 + 0]$$

$$M_2(2) = \max[40, 35, 30] M_2(2) = 40 \text{ with } d_2(2) = 0$$

$$M_2(1) = \max[f_2(0) + M_3(1), f_2(1) + M_3(0)]$$

$$M_2(1) = \max[0 + 20, 15 + 0] M_2(1) = \max[20, 15] M_2(1) = 20 \text{ with } d_2(1) = 0$$

$$M_2(0) = \max[f_2(0) + M_3(0)] M_2(0) = \max[0 + 0]$$

$$M_2(0) = \max[0] M_2(0) = 0 \text{ with } d_2(0) = 0$$

From investment 1:

$$M_1(9) = \max[f_1(0) + M_2(9), f_1(1) + M_2(8), f_1(2) + M_2(7), f_1(3) + M_2(6), f_1(4) + M_2(5), f_1(5) + M_2(4), f_1(6) + M_2(3), f_1(7) + M_2(2), f_1(8) + M_2(1), f_1(9) + M_2(0)]$$

$$M_1(9) = \max[0 + 149, 10 + 134, 20 + 121, 25 + 109, 33 + 94, 35 + 79, 40 + 60, 45 + 40, 55 + 20, 60 + 0]$$

$$M_1(9) = \max[149, 144, 141, 134, 127, 114, 100, 85, 75, 60] M_1(9) = 149 \text{ with } d_1(9) = 0$$

ALLOCATION

The optimal return from the investment is 149 which we obtained by starting the allocation from stage 1, then to stage 2 up to stage 6 as follows:

- i. With 9 units, available allocate to stage 1, $f_1(9) = 0$, leaving $9 - 0 = 9$.
- ii. With 9 units, available allocate to stage 2, $f_2(9) = 1$, leaving $9 - 1 = 8$.
- iii. With 8 units, available allocate to stage 3, $f_3(8) = 0$, leaving $8 - 0 = 8$.
- iv. With 8 units, available allocate to stage 4, $f_4(8) = 2$, leaving $8 - 2 = 6$.
- v. With 6 units, available allocate to stage 5, $f_5(6) = 5$, leaving $6 - 5 = 1$.
- vi. With 1 units, available allocate to stage 6, $f_6(1) = 1$, leaving $1 - 1 = 0$.

TABLE 6: OPTIMAL ALLOCATIONS AND RETURNS FROM INVESTMENTS

RETURN FROM INVESTMENT	AMOUNT INVESTED									
	0	100	200	300	400	500	600	700	800	900
$f_1(x)$	0*	10	20	25	33	35	40	45	55	60
$f_2(x)$	0	15*	30	40	45	50	55	70	80	90
$f_3(x)$	0*	12	22	32	35	40	50	60	65	70
$f_4(x)$	0	18	40*	50	55	65	70	70	85	95
$f_5(x)$	0	20	28	38	50	75	75*	80	80	85
$f_6(x)$	0	19*	25	35	40	50	60	70	75	80

4. Conclusion

The results from this study are clear. We investigated to determine the optimal investment returns among six investments and the corresponding investments to be made. Results from the dynamic programming indicates that with ₵900 available for investment and given the corresponding returns from the various financial institutions we should not invest in Government of Ghana's Treasury Bills and Ghana Commercial Bank. However, we should invest ₵100 in Barclays Bank to get ₵15, ₵200 in Data Bank, Accra to get ₵40, ₵500 in Ghana Guinness Limited, Accra for a return of ₵75 and ₵100 in Fan Milk Limited, Accra for a return of ₵19. This gives the optimal returns of ₵149.

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