

Measuring Business Cycle and Inflation Forecast: The Case of Pakistan

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Abstract
The output gap and inflation forecast are important factors to analyze current state of the economy and stance of monetary policy. In this study we have measured business cycle through estimating output gap using different methods namely the Linear Time Trend (LTT) method, Quadratic Time Trend (QTT) method, Hodrick-Prescott (HP filter), Band Pass Baxter-King Filter (BP), Double Exponential Smoothing Method (DES), Structural Vector Autoregressive (SVAR) method and Production Function (PF) method. For the analysis we have used annual data over the period 1960 to 2014 for Pakistan. Moreover, the inflation is forecasted with univariate and multivariate models. The results suggest that Quadratic Time Trend (QTT) method and Structural Vector Autoregression (SVAR) captures the history of Pakistan economy well. Whereas, output gap estimated through SVAR generate better inflation forecast compared to other methods.
Keywords: output gap, inflation forecast, univariate, multivariate, business cycle
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1. INTRODUCTION

The economic management of an economy is based on continuous monitoring and analysis of different economic indicators, particularly, output gap and inflation. Monetary policy is an important tool of the public policy to stabilize these economic indicators.

In monetary policy formulation of most of the contemporary central banks, inflation forecast is the main ingredient. The prime objective of monetary policy, in most of the countries, is price stability along with real stabilization. To incorporate typical time lags, monetary policy needs to be concerned with future expected inflation. The Current inflation level which is the result of past monetary policies may provide only insufficient information as claimed by Lucas (1976). However, inflation forecasts that link future inflation to current developments can cover this gap.

The strong pressure of excess demand may provide some signal of future inflation to a central bank, which is operating in an inflation-targeting monetary policy regime. Currently, the output gap is a commonly used measure of inflationary pressure in the economy. Moreover, the output gap has a well-known role in the theoretical and applied literature in explaining price and wage inflation (De Brouwer, 1998). The output gap is defined as the difference between the actual output of the economy and its potential output.

$$Y_t^g = Y_t - Y_t^* \qquad (1.1)$$

It is a temporary component of output which is usually stationary but not necessarily periodical. The potential output is the production capacity of the economy that is mostly related with permanent component of output (which is normally non-stationary) (Nelson & Plosser 1982). The potential output of economy at any time is consistent with stable inflation (Bjørnland *et al*. 2005). The output gap is positive when actual output is greater than potential output. The output gap is negative when actual output is less than potential output. A positive output gap means that there is an excess demand and rising inflation in the economy as the economy is operating above its potential level. A negative output gap means that there is an excess supply or unused capacity and falling inflation in the economy as the economy is operating below its potential level.

In many cases, the analysis of the output gap is regarded as the starting point for studying business cycles (Ladiray *et al.* 2003). Since, the output gap is not directly observable, and must therefore need to be estimated by using different methods. The different calculation methods, however, produce different values of the output gap. If the measurement of business cycle is only based on the output gap which is measured by one method, there is a risk of misjudging the economic situation. Therefore, measurements of the output gap must be based on professional judgment and additional indicators (Bjornland *et al*. 2005).

Output gap can be estimated by structural as well as statistical methods. The former includes Structural Vector Autoregression Model and Production Function Approach, while the later includes detrending method, Band pass filter, Exponential Smoothing, Hodrick-Prescott Filter. However, none of the method is free from errors. Different measures, at any particular time, may indicate different phases of economic activity[1]. At the same time, at different times, different measures can be useful for inflation forecasting. Therefore, one should not rely on the results of only one method.

The trends and fluctuations in business cycle have compelled both the researchers and policy makers to contribute in the literature that may be supporting for the stabilization of the economy. The previous literature has the ability to meet just the basic requirements of business cycle for example Arby (2004) separate, the real GDP of Pakistan in to three component i.e. long run trend, business cycles and short run shocks by using HP filter. Bukhari & khan (2008), attempt to calculate potential output and output gap by using six different statistical and structural methods. The study found that although the output gaps calculated by using different method are not close to each other's, but there is some degree of association between them.

The previous studies are unable to compare each method with the economic history. They also remained unable to conduct further analysis such as forecasting of inflation, by using the output gaps, that has been the important objective of central banks in terms of price stability. Within this context, this study makes the following contributions. The study counsels the output gaps by applying the famous statistical and structural techniques and then results of each method are compared with the historical events that have taken place. The study forecasts inflation by using different measures of output gaps and then results are compared on the basis of forecast accuracy.

The study consists of five sections. The second section deals with the data and econometric techniques used for the estimation of the models. The third section discusses the estimated results based on different statistical and structural approaches. Finally, the last section concludes the study.

2. DATA AND ECONOMETRIC METHODOLOGY

This section presents the variables, sources of data and econometrics methodologies used in this study to find empirical results. In this study we have used annual time series data for Pakistan over the period 1960 to 2014. Data on Real GDP, Labor Force, Real Gross Fixed Capital Formation, and Unemployment have been taken from various issues of Economic Survey of Pakistan, published by Ministry of Finance, Government of Pakistan and Annual Reports of State Bank of Pakistan. Data on Consumer Price Index have been taken from International Financial Statistic (IFS) of International Monetary Fund (IMF).

Real GDP is measured as the market value of all final goods and services newly produced domestically. Annual inflation from annual CPI figures is calculated by growth rate of consecutive years. Labor force is a core variable of labor market which consist of all employed and unemployed workers. Unemployment rate (U_r) is the fraction of labor force that is unemployed. Annual data series of capital stock is measured by using Perpetual Inventory Method which takes capital stock as the accumulation of the stream of past investments:

$$K_t = I_t + \left(1-\delta\right)K_{t-1} \qquad (2.1)$$

We use (Nehru & Dhareshwar 1993) method to obtain capital stock series:

$$K_t = \left(1-\delta\right)^t K_0 + \sum_{i=0}^{t-1} I_{t-i}(1-\delta)^i \qquad (2.2)$$

Where δ *and* K_0 stands for the rate of geometric decay and initial capital stock in period 0, respectively. K_0 can be calculated in number of ways as mentioned in (Nehru and Dhareshwar 1993) but we utilize the method introduced by Harberger (1978) modified by (Nehru and Dhareshwar 1993), to obtain K_0. The first period investment is estimated through a linear regression of the log of investment against time. The fitted value of initial investment is used to calculate initial capital stock using the following equation:

$$K_{t-1} = \frac{I_t}{(g+\delta)} \qquad (2.3)$$

Where g and δ are the growth rates of output and depreciation rate of capital, respectively and δ is taken as 4%[2].

[1] See for discussion, de Brouwer (1998), Scacciavillani and Swagel (1999), Nelson and Plosser (1982), and Watson (1986).
[2] We are following the work of Nehru & Dhareshwar (1993)

2.1 ECONOMETRIC METHODOLOGY

The underlying section discusses the econometric methodologies used in the study. This section is divided in to two sub-sections. In first and second sub-section, we discuss the methodology of different statistical and structural methods to calculate the output gap, respectively. The third sub-section discusses the inflation forecast methodology using Autoregressive Moving Average (ARMA) and Vector Autoregressive (VAR) models.

2.1.1 STATISTICAL METHODS

These are the methods that are based on some statistical procedure rather than referring explicitly on an economic theory (Cogley 1997). We discuss here five statistical methods to calculate output gap in case of Pakistan. These methods include Linear Time Trend (LTT), Quadratic Time Trend (QTT), Hodrick-Prescott (HP filter), Band Pass Baxter and King Filter (BP) and Double Exponential Smoothing method (DES).

2.1.1.1 Linear Time Trend Method (LTT)

LTT method is simple and straightforward. The potential output is calculated from a linear equation of output or log of output on a time trend. The fitted line is taken as potential output while the difference between the potential output and actual output is considered to be the estimate of the output gap. The fitting of linear time trend on output is calculated as follows:

$$Y_t^* = \alpha_0 + \alpha_1 T \qquad (2.4)$$

$t = 1, 2 \cdots\cdots$

Where Y^* and T stand for potential output in natural log form and time trend, respectively. While α_0 *and* α_1 are estimated coefficients of fitting time trend.

We can use potential output estimate from equation (2.4) to calculate output gap as follows

$$Y_t^g = (Y_t - Y_t^*) * 100 \qquad (2.5)$$

$t = 1, 2 \cdots\cdots$

Where, Y^g and Y stand for the output gap and actual output, respectively.

2.1.1.2 Quadratic Time Trend Method (QTT)

The Quadratic time trend method is more flexible as compared to linear time trend method. At the end points of data set, quadratic trend method performs very well. The output gap by quadratic time trend method is estimated as percentage deviation of real GDP from its quadratic trend. The fitting of Quadratic time trend on GDP is calculated as follows.

$$Y_t^* = \alpha_0 + \alpha_1 T + \alpha_2 T^2 \qquad (2.6)$$

$t = 1, 2 \cdots\cdots$

Where Y^* T *and* T^2 stand for potential output, time trend and square of time trend, respectively. While α_0, α_1, and α_2, are estimated coefficients of fitting trend. We can use potential output estimate from equation (2.6) to calculate output gap as follows

$$Y_t^g = \left(\left(Y - Y_t^*\right) \middle/ Y_t^* \right) * 100 \qquad (2.7)$$

$t = 1, 2 \cdots\cdots$

Where Y^g *and* Y stand for the output gap and actual output, respectively.

2.1.1.3 Hodrick-Prescott Filter Method (HP Filter)

HP method is flexible to attain the fluctuation in potential output growth by setting different values of smoothing parameter. The HP method (1997) is commonly used to estimate potential output from actual output by fitting a smooth curve along a point. The conceptual framework given by Hodrick-Prescott (1997) is that Let Y_t denote an observable time series. The HP filter decomposes Y into a non-stationary trend "Y^*", and a stationary residual component, Y^g that is:-

$$Y_t = Y_t^g + Y_t^* \qquad (2.8)$$

For $t = 1, 2 \cdots\cdots T$.

Where, Y^g *and* Y^* are unobservable components and Y^* (potential output) is obtained by HP filter that allocate the weight to Y^* against the signal Y. HP solves the problem by minimizing:-

$$\underset{\{Y_t^*\}_{t=-1}^{T}}{Min}\left\{\sum_{t=1}^{T} Y_t^{g^2} + \lambda \sum_{t=1}^{T}\left[\left(Y_t^* - Y_{t-1}^*\right) - \left(Y_{t-1}^* - Y_{t-2}^*\right)\right]^2\right\} \qquad (2.9)$$

Where $Y_t^g = Y_t - Y_t^*$ Let λ be is a positive (weight) number, which penalizes the variability in the Y^* series. The larger value of λ will make the solution series smoother. If there is no disturbance then the signal is fully informative and it is better to set λ=0. If λ approaches infinity, then the function in equation (2.9) is minimized by penalizing changes in potential growth, which is done by making potential output growth constant or Y^* approaches the ordinary least squares estimate of Y against a linear time trend. In Eviews software, the default values of λ are 100 for annual data. Hodrick and Prescott (1980) did use $\lambda = 100$ for annual data. We set $\lambda = 400$ for annual data used by Correia, *et al.* (1992) and Cooley & Ohanian (1991).

2.1.1.4 The Band Pass Baxter and King Filter (BP)

As compared to HP filter, in Band pass we can make use of historical experience with regard to the duration of business cycle (by considering the frequency of cyclical fluctuations) when estimating the output gap. Therefore, we can say that our business cycle has the length that has historically been observed for business cycles. We use the BP filter developed by Baxter & King (1995). The most important contribution of Baxter & King (1995) is the derivation of a band pass filter to estimate directly the cyclical component C_t. Burns & Mitchell (1946) define the business cycle as fluctuations in some macroeconomic series lasting no less than 6 and no more than 32 quarters. The cyclical component C_t is extracted by applying to Y_t *as:*

$$C_t = \sum_{j=-k}^{k} \alpha_j Y_{t-j} \qquad (2.10)$$

Where, α_j are corresponding weights of the frequency response function. These weights are derived from the inverse Fourier transformation. When using the Baxter & King filter, k observations are lost at the beginning and at the end of the sample period, according to the required degree of approximation to the ideal filter. The common choice $k = 3$ for annual data. In order to reduce the loss of data at the beginning and at the end of the sample, truncated versions of the filter can be used. Alternatively, it is possible to forecast and backcast the series before applying the filter so as to use the complete moving average[1].

2.1.1.5 Double Exponential Smoothing Method (DES)

The Exponential smoothing technique can be useful in time series data, either to smooth data or to make forecasts which assign exponentially decreasing weights over time. The data series to whom exponential smoothing is applied, represented by { Y_t }, and the output of the exponential smoothing algorithm is commonly written as { S_t }. The simplest form of exponential smoothing at time t=0 is given by the formulas as follows:

$$S_1 = Y_0$$

$$S_t = \alpha Y_{t-1} + (1-\alpha) S_{t-1} \qquad (2.11)$$

Where α and t stand for the smoothing factor and time period, while $0 < \alpha < 1$ *and* $t > 1$. The Values of α close to one have less of a smoothing effect and give greater weight to recent changes in the data, while values of α closer to zero have a greater smoothing effect and less responsive to recent changes. There is no formally correct procedure for choosing α. Sometimes, the statistician's judgment chooses an appropriate factor and default value of $\alpha = 0.3$ as a smoothing factor. The smoothed statistic S_t is a simple weighted average of the previous observation Y_{t-1} and the previous smoothed statistic S_{t-1}.

If the trend as well as the mean is varying slowly over time, a higher-order smoothing model is needed to track the varying trend. Simple exponential smoothing does not follow the important component of time series "Trend" that is handled in double exponential smoothing method. The Brown's linear exponential smoothing

[1] In our analysis, the Band Pass Baxter and King filter is extended by means of a simple, mechanical projection.

(LES) model introduced by Brown (1963) is the simplest time-varying trend model, which uses two different smoothed series that are centered at different points in time. The standard form of this model is usually expressed as follows: Let S' denote the singly-smoothed series obtained by applying simple exponential smoothing to series Y as in equation (2.11):

$$S'(t) = \alpha Y(t) + (1-\alpha) S'(t-1) \quad (2.12)$$

Then, let S" denote the double-smoothed series obtained by applying simple exponential smoothing (using the same α) to series S':

$$S''(t) = \alpha S'(t) + (1-\alpha) S''(t-1) \quad (2.13)$$

Finally, the forecast Ý (t+1) is given by:

$$\dot{Y}(t+1) = a(t) + b(t) \quad (2.14)$$

Where:

$$a(t) = 2S'(t) - S''(t) \cdots \textit{the estimated level at period } t$$

$$b(t) = (\alpha/(1-\alpha))(S'(t) - S''(t)) \cdots \textit{the estimated trend at period } t$$

2.1.2 Structural Methods

Statistical methods are easy to use but have some shortcomings as Quah (1992) argues that it is impossible to separate the relative importance of demand and supply shocks in an unvariate framework. Conversely, Structural methods rely on a specific economic theory (Chagny & Döpke 2001). We discuss here two structural methods to calculate output gap. These methods include Production Function (PF) method and Structural Vector Autoregressive (SVAR) method.

2.1.2.1 Production Function Method (PF)

Production function method is mostly associated with the basic structure of economy while calculating potential output. We use simplest form of Cobb-Douglas as two factor production function used by Giorno, *et al.* (1995) followed by Froyland & Nymoen (2000).

The aggregated production function stated in Cobb-Douglas production function for the economy, at time t, is assumed as:

$$Y_t = A_t + \alpha L_t + (1-\alpha) K_t \quad (2.15)$$

$$t = 1, 2 \cdots\cdots T.$$

Where Y, A, L, K stands for real output, total factor productivity, total labor force and capital stock, respectively. All the variables are measured in natural logarithms. The coefficient α and (1- α) are the shares of labor and capital stock, respectively under the assumption of constant return to scale. A is calculated as a residual from equation (2.15).

Potential output is then calculated by the combination of total factor productivity with the actual capital stock and potential employment.

$$Y_t^* = A_t^* + 0.56 L_t^* + 0.44 K_t^* \quad (2.16)$$

We set value of labor and capital share as 0.56 and 0.44 respectively, used by Khan (2006). Where, the potential capital stock is assumed equal to actual capital stock. Potential employment is labor force[1]. The resulting residual from equation (2.15) is smoothed using HP filter to get potential level of total factor productivity. The output gap is then calculated by following equation (1.1).

2.1.2.2 Structural Vector Autoregressive Method (SVAR)

In order to apply Structural Vector Autoregressive (SVAR) method, there is need to check the stationarity of variables and cointegration among them. The cointegration is conducted between real output and unemployment. We use a Structural VAR developed by Blanchard and Quah (1989). Suppose the variables discussed above are differenced stationary, not cointegarted and can be represented by the Vector autoregressive (VAR). That's why; we use Structural VAR in order to calculate potential output, originally developed by (Blanchard & Quah 1989). According to them, real GNP is affected both by demand and supply side disturbances, where demand side disturbances have no long-run effect on real GNP. While, on supply side, productivity shocks are assumed to have long run effect on output. We follow the Blanchard & Quah (1989) SVAR method by using variables i.e. real GDP and unemployment rate.

The demand and supply shocks are not observed, but the problem can be handled by the estimation of VAR. Given that the variables are stationary, there exists a VAR representation given that:

[1] It is difficult to calculate measure of potential employment as the level of labor resources that might be employed without resulting in additional inflation.

$$\begin{bmatrix}\Delta Y_t \\ Z_t\end{bmatrix} = \begin{bmatrix}A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L)\end{bmatrix}\begin{bmatrix}\Delta Y_{t-1} \\ Z_{t-1}\end{bmatrix} + \begin{bmatrix}e_{1t} \\ e_{2t}\end{bmatrix} \quad (2.17)$$

OR

$$X_t = A(L)X_{t-1} + e_t \quad (2.17a)$$

Whereas X_t, the column vector of $(\Delta Y_t, Z_t)$, e_t the column vector of (e_{1t}, e_{2t}) and $A(L)$= the 2×2 matrix with elements equal to the polynomials $A_{ij}(L)$ and the coefficient of $A_{ij}(L)$ are denoted by $a_{ij}(k)$[1].

The VAR residuals are composite of the pure innovation ε_{1t} and ε_{2t}. For example, e_{1t} is the one-step-ahead forecast error of Y_t; i.e. $e_{1t} = \Delta Y_t - E_{t-1}\Delta Y_t$. From the bivaraite moving average (BMA), the one-step-ahead forecast error is $b_{11}(0)\varepsilon_{1t} + b_{12}(0)\varepsilon_{2t}$, so we can write it as follows:

$$e_{1t} = b_{11}(0)\varepsilon_{1t} + b_{12} \quad (2.18)$$

Similarly, since e_{2t} is the one-step-ahead forecast error of Z_t

$$e_{2t} = b_{21}(0)\varepsilon_{1t} + b_{22}(0)\varepsilon_{2t} \quad (2.19)$$

or, combing (2.18) and (2.19), we get

$$\begin{bmatrix}e_{1t} \\ e_{2t}\end{bmatrix} = \begin{bmatrix}b_{11}(0) & b_{12}(0) \\ b_{21}(0) & b_{22}(0)\end{bmatrix} + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}$$

If $b_{11}(0)$, $b_{12}(0)$, $b_{21}(0)$ and $b_{22}(0)$ were known, it would be possible to recover ε_{1t} and ε_{2t} from the regression residual e_{1t} and e_{2t}. We need four restrictions to identify these structural shocks. Blanchard and Quah identify these restrictions as follows:

Given (2.18) and noting that $E\varepsilon_{1t}\varepsilon_{2t} = 0$, the normalization $\text{var}(\varepsilon_1) = \text{var}(\varepsilon_2) = 1$ means that the variance of e_{1t} is

$$\text{var}(e_1) = b_{11}(0)^2 + b_{12}(0)^2 \quad (2.20)$$

Similarly, using (2.19) the variance of e_{2t} is

$$\text{var}(e_2) = b_{21}(0)^2 + b_{22}(0)^2 \quad (2.21)$$

The covariance of VAR residual is

$$COV(e_{it}, e_{2t}) = b_{11}(0)b_{21}(0) + b_{12}(0)b_{22}(0) \quad (2.22)$$

For all possible realization of the $\{\varepsilon_{1t}\}$ sequence, ε_{1t} shocks will have only temporary effects on the ΔY_t sequence if

$$b_{21}(0)\left[1 - \sum_{a22}(k)\right] + b_{21}(0)\sum_{a12}(k) = 0 \quad (2.23)$$

If we apply these four restrictions then we can identify structural shocks i.e., ε_{1t} and ε_{2t} . Now the potential output component ΔY^p is determined by cumulating supply-side shocks for example, set all $\{\varepsilon_{1t}\}$ shocks equal to zero and use the actual $\{\varepsilon_{2t}\}$ series to obtain permanent changes in $\{Y_t\}$ as

$$\Delta Y_t^p = \sum_{k=0}^{\infty} b_{12}(k)\varepsilon_{2t-k} \quad (2.24)$$

After calculating potential output we can identify the output gap by following equation (1.1).

2.1.3 Inflation Forecasting

We use two empirical approaches to forecast inflation. As a benchmark, we estimate a univariate Autoregressive and Moving Average (ARMA) model. Next, we use a bivariate Vector Autoregressive (VAR) model that includes inflation and output gaps estimated from different statistical and structural filters. Short theoretical

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[1] For example $A_{11}(L) = a_{11}(0) + a_{11}(1)L + a_{11}(2)L^2 + \cdots$

descriptions of both models are given below.

2.1.3.1 UNIVARIATE FORECASTING MODEL

In the simplest form, inflation can be modeled as an ARMA process. ARMA models in our study provide a benchmark for inflation's forecasting.

The equation for an ARMA model is given as:

$$\pi_t = \alpha + \sum_{i=1}^{p} \phi_i \pi_{t-i} + \sum_{j=0}^{q} \theta_j \varepsilon_{t-j} \qquad (2.25)$$

Where π denote inflation rate and ε denotes white noise error term. Modeling and forecasting of various ARMA time series models based on Pakistan's annual Inflation data would be carried out by following methodology based on Feridun *et al.* (2006).

1. Specification and estimation of various possible types of ARMA models through equation (2.25).
2. Obtaining ex-post forecast after empirically estimating the various types of ARMA models.
3. Comparison of forecasting performance of various types of ARMA models by using certain statistical measures such as Mean Square Error (MSE) or Root Mean Square Error (RMSE). The Root Mean Square Error (RMSE) is defined as:

$$RMSE(n) = \sqrt{(T)^{-1} \sum_{i=1}^{T} \left(\pi_i^a - \pi_i^f(n)\right)^2} \qquad (2.26)$$

Where $\pi^f(n)$ shows the models prediction for the rate of inflation *n* steps into the future and π^a shows the corresponding actual inflation. T is the number of forecasts computed.

2.1.3.2 VECTOR AUTOREGRESSIVE (VAR) MODELS

VAR provides a systematic way to capture rich dynamics in multiple time series, and the statistical tools from VAR are easy to use and interpret. In data description and forecasting, VAR is proven to be powerful and reliable tool (Sims 1980). In order to conduct the out of sample inflation forecasting, the simple VAR model is used, in which inflation depends on its past values and past values of output gap. The output gap also depends on its past values and past values of inflation.

The mathematical form of a VAR is

$$Y_t = A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + BX_t + \varepsilon_t \qquad (2.27)$$

Where Y_t, X_t and $A_1 \cdots A_p$, B stands for k vector of endogenous variables, vector of exogenous variables, and matrices of coefficients to be estimated, respectively. However ε_t , a vector of innovations that may be contemporaneously correlated with each other but are uncorrelated with their own lagged values and all of the right-hand side variables. The procedure of out of sample *h* steps ahead forecast is computed by estimating the VAR through a given year, making the forecast *h* steps ahead, reestimating the VAR through the next year, making the next forecast and so on through the forecast period (Stock & Watson 2001).

Finally, Forecasting performance of the VAR Model would be compared by computing statistics such as RMSE as given in equation (2.26).

3. ESTIMATION RESULTS AND INTERPRETATION

This section contains the results of the estimation along with their interpretation. It is divided in to two sub sections. In first section, we study the results and discussion of output gap. In second section, we describe the results and discussion of inflation forecasting.

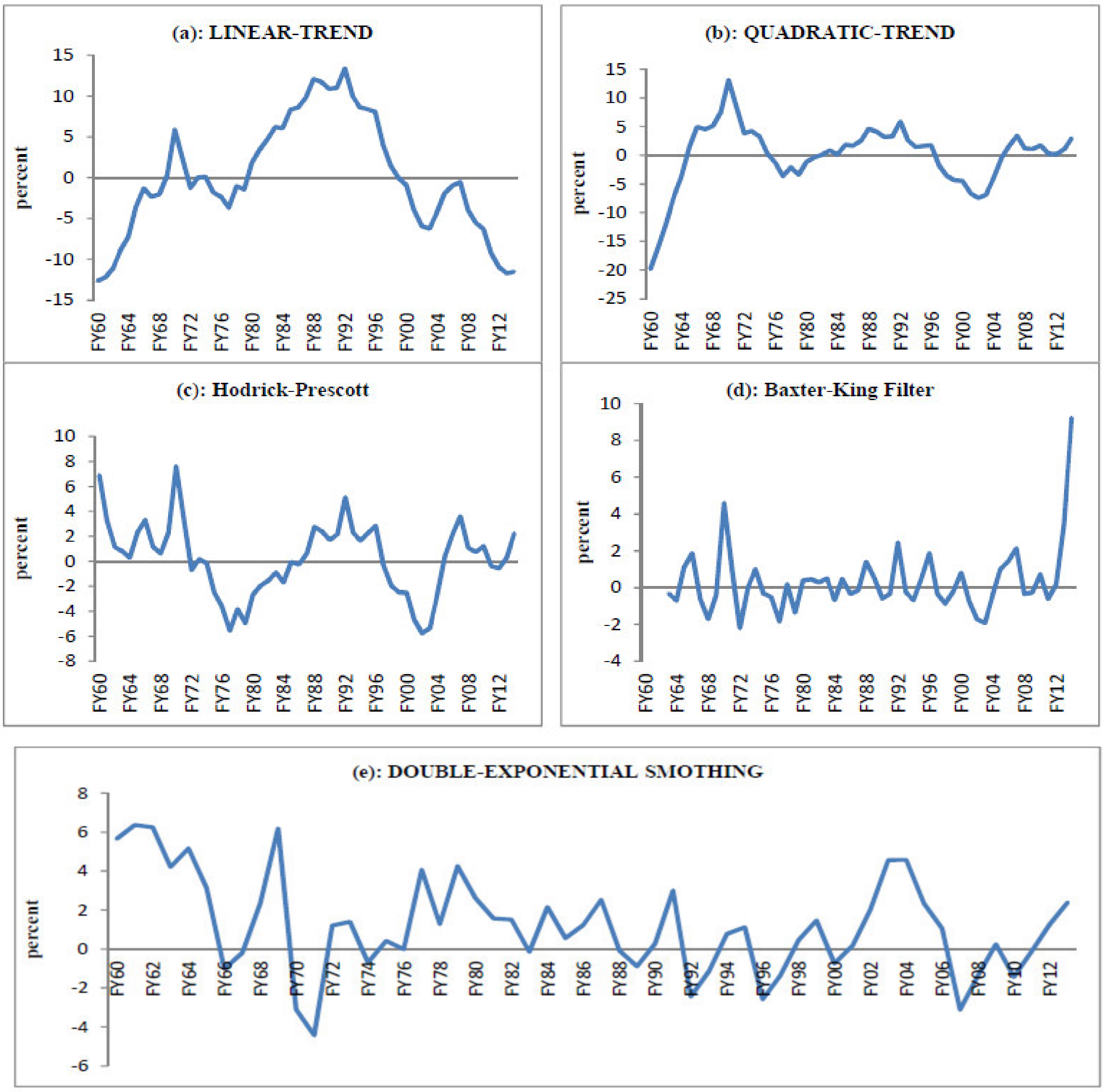
3.1 Results and Discussion of Output Gap Estimates

In this section we discuss results and interpretation of seven output gap measure. This section is divided in to two subsections. In first subsection, we study discussion relating to statistical methods. In second subsection, we study discussion relating to structural methods.

3.1.1 Statistical Methods

The results of five statistical methods are shown in Figure 3.1.1[1].

Fig 3.1.1: Output Gap Calculated from Statistical Methods



Discussion

The output gap from linear and quadratic detrending, as shown in fig 3.1.1 are almost similar in fluctuations, indicating that there is an expansion in the economic activity, on average, during 1960 to 1970. However, in the beginning of 1960, there is strong evidence of trough in economic activity as output gap is -12.63% and -19.81% calculated from linear and quadratic trend, respectively but HP shows that economy is in boom in 1960 as the output gap is 6.91%. After that overall economic activity follow contraction at ongoing period since 1965. All mentioned methods show that at 1970, economy reaches its peak point as the out gap calculated from linear, quadratic, HP, BP and exponential smoothing method are 5.87%, 13.13%, 7.59%, 4.59% and 6.18%, respectively, which strongly support the boom period. After 1970 all methods show almost consistent results i.e. economy is in recessionary phase till 1977. At the same period, economy reaches its trough as the output gap in linear, quadratic, HP and BP filter is -3.70%, -3.63%, -5.57% and -1.83%, respectively. In sum, we can say that the complete business cycle starting from 1960 up to 1977 is apparent that is also strongly supported by the historical or practical view point (Mahmood *et al.* 2008).

Pakistan was considered a model for economic development during the 1960s in the world, and there was much praise for its progress in the field of processing and developing as its GDP approached Rs. 447876.3

[1] The results of output gap from statistical methods are given in Appendix A.1

million. In the sixties, government priorities shifted from the industrial sector towards the agricultural sector resulting in rapid growths for each along with significant increase in per capita income. After the 1965 war, there was a decrease in the inflow of foreign loans, slow down in the industrial growth, and increase in defense expenditures. The GDP growth was 6.8% on average. The huge trade deficit was financed mainly through foreign loans. Lower yields in agricultural crops during 1966 and 1967 resulted in import of food grains increasing the burden on import financing to almost 8 % per annum.

After the separation of eastern Pakistan in 1971, deterioration of economic growth continued to prevail and the country was faced with the challenges of rehabilitation of a post-war economy, high rates of inflation and shrinking agriculture and industrial sectors. In the seventies, economic management shifted towards nationalization. In the early 1972, the government took some drastic steps and brought fundamental structural reforms like land reforms, labor reforms, nationalization of key industries, banks, insurance companies and exports like trade of two major export products i.e. rice and cotton. Although these reforms were introduced to improve the efficiency of the manufacturing sector, the share of both the manufacturing and agriculture to GDP declined. Due to a decline in the two major sectors, the entire economy saw a decline in GDP growth to 3.6 percent per annum.

After 1977, economy began to recover by taking almost 15 years as identified by linear, quadratic, HP and BP filter. All these methods show that economy is in expansion from 1978 to 1992 and it approaches its peak point in 1992 as gap in this period from linear, quadratic; HP, BP and exponential smoothing method are 13.35%, 5.87%, 5.12%, 2.45% and 3%, respectively. However, during 1992 to 2002, economy again faced recessionary period and the year “2002” clearly evaluates the trough point indicating by the output gap, where linear, quadratic, HP and BP are -5.93%, -7.48%, -5.76% and -1.72%, respectively. The second business cycle from 1977-2002 is also consistent with the history of Pakistan. In 1980s, institutions were liberalized and denationalized and the role of public sector was reduced. Pakistan’s economic performance remained at 7.1% on average and the country was considered as one of the most developing countries of South Asia. The impressive growth rates were seen due to diminishing imports along with increasing remittances. Large foreign assistance helped in improving the balance of payments position of the country that might be the result of Afghan war.

In 1990s, dwindle in economic performance of Pakistan appeared as GDP growth rate was 4.4% on average throughout this period. Both the fiscal and current account deficits caused uncontrollable levels of public debt. Fiscal deficit as a ratio of GDP remained above 6%, while the current account deficit in 1990s was 5.9 percent of GDP as compared to 2.7 percent of GDP in 1980s. It remained negative mostly because of persistent trade deficits caused by economic sanctions and large declines in exports. Pakistan, as a consequence of the nuclear tests conducted in May 1998 by both India and Pakistan, had to face financial hardships brought on by the imposition of international economic sanctions. As a result of the sanctions, capital inflows ceased entirely which caused the economic conditions to become severely difficult. Macroeconomic indicators, like low tax to GDP ratio, a double-digit inflation, low levels of investment, poor social sector indicators and poor governance of institutions also worsened the economy further.

After 2002, all methods show that economy is recovering till 2007. All methods, except for the linear detrending, identify that economy is in expansionary phase from 2002 to 2007 that is consistent with historical analysis. However, in 2008 to present economy is in recessionary face. On average, yearly growth through the period of 2000-2007 remained 7.0% with the highest at 9% in 2004-05 and lowest 1.9% in 2000-01. In 2000-07 real GDP of Pakistan increased from $60 billion to $170 billion, with per capita income rising from under $500 to over $1000. The volume of international trade increased from $20 billion to $60 billion.

In the 2000s, more liberal economic strategies have been adopted with the aim to enhance the share of Pakistan’s’ exports in the world economy. The privatization process started with a focus on banking, telecommunication, oil and gas and energy sectors. More importantly, in the aftermath of 9/11, the foreign grants of about $1 billion to $1.5 billion per annum during 2003-08 come in after easing of sanctions. Foreign exchange reserves increased due to remittances amounting to $18.5 billion during 2003-08. The current account surplus was recorded at an annual average of 1.9% of the GDP. The rising remittances since September 11, 2001 onward kept the current account balance in surplus.

The floods of 2010 and 2011, decline in security situation, uncertainty and lack of focus are one of the reasons behind Pakistan poor economic performance in last five years.

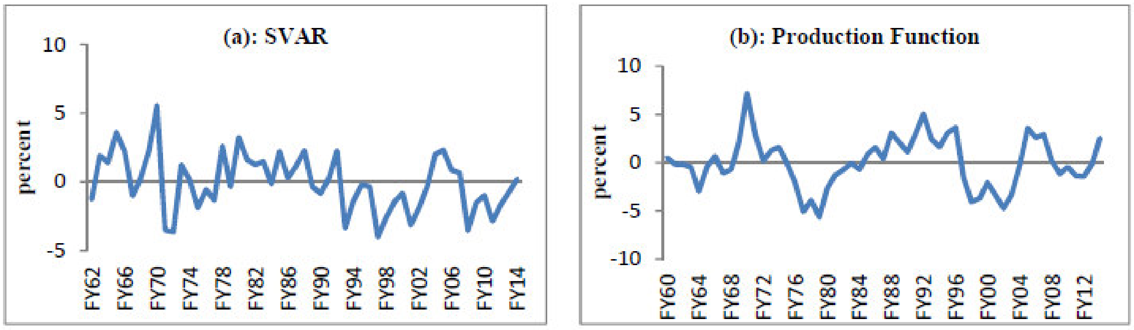
After 2007 international prices of food and fuel prices start increasing, and energy crisis of Pakistan’s initiate began to visible itself, the momentum of growth was break up. The average GDP growth rate (annually) 3 percent over the period being discussed is less as compared to the period from FY 2003 to FY 2007 in which it was 7%.

In sum up, we can say that the economy of Pakistan since 1960 has pass through three complete business cycles. Regarding comparison of results from different measures of output gap, it is found that results of quadratic time trend captures the history well. The second method which produced factual results is HP filter. However, the results of quadratic time trend are even better, especially in the beginning of sample period.

3.1.2 Structural Methods

The results of structural methods are shown in Figure 3.1.2[1].

Fig 3.1.2: Output Gap Calculated from Structural Methods



Discussion

The results of production function method are roughly similar to quadratic time trend method except in starting period. However, the structural VAR method produces different result as compare to these that indicates demand pressure in economy from 1960 to 1970. However, in first half of 60s SVAR method indicates increasing growth trend in economy. In contrast, production function method shows decreasing trend in output. In second half, both methods show economy is expanding and reached its peak point in 1970 as the gap calculated from SVAR and production function are 5.56% and 7.18%. After 1970, production function method indicates that economy fell in to recession till 1979 (trough point) as the gap in 1979 is -5.65%. However, SVAR method shows recessionary phase from 1970 to 1977 and two minimum points in 1972 and 1975 where the gap is -3.64% and -1.90%, respectively.

Following 1979, economy began to recover by taking almost 13 years as identified by both methods. Both methods show that economy is in expansion from 1979 to1992 and it approaches its peak point in 1992 as gap in this period from SVAR and PF are 2.24% and 5.05%, respectively. However, during 1992 to 2002, economy again faced recessionary period and the year "2002"clearly evaluates the trough point indicating by the output gap calculated from PF i.e. -4.76%. However, SVAR method indicates that trough point is not clear but economy has been facing recession since 1992. Both methods identify that economy is in expansionary phase from 2002 to 2007 as the gap in 2007 from PF is 2.92%. After 2007 to present economy is in recessionary phase.

In sum up, we can say that structural method also explain that the economy of Pakistan, since 1960, has gone through three complete business cycles, that is strongly supported by the historical view point as clearly discussed above. Moreover, the two structural methods produce almost similar pattern of economic activity and this pattern is consistent with the economic history of Pakistan. Furthermore, barring beginning of the sample period, results of structural methods are in conformity with those from quadratic time trend method.

3.2 RESULTS AND DISCUSSION OF INFLATION FORECASTING

This section discusses the comparison of results based on various ARMA and VAR models and, on the basis of this comparison, evaluates which model forecast inflation better than the other and which measure of output gap is more helpful. The results are given in table 3.2.

Table 3.2 Forecast Comparison

Out of Sample 1-Step Ahead Forecast from 2010 to 2014			
S.NO	**Methods**	**MSE**	**RMSE**
	ARMA		
1	ARMA(2,5)	4.04	2.01
	Statistical		
1	Linear Time Trend (LTT)	14.00	3.74
2	Quadratic Time Trend (QTT)	11.19	3.34
3	Hodrick-Prescott (HP filter)	11.99	3.46
4	Band Pass Baxter and King Filter Method (BP)	11.96	3.46
5	Double Exponential Smoothing (DES)	12.76	3.57
	Structural		
1	Structural Vector Autoregressive (SVAR)	10.28	3.21
2	Production Function (PF)	12.72	3.57

[1] The results of output gap from structural methods are given in Appendix A.1

Discussion

The results indicate that the forecast ability of the output gap model based on the structural methods gives better results than the statistical method as the mean square error (MSE) and root mean square error (RMSE), are minimum in structural methods compared to that with statistical methods. In addition to that, MSE and RMSE calculated from SVAR is minimum as compare to all other methods which means SVAR perform better forecast. It is consistent with the empirical studies [For Example Chagny & Döpke (2001), Bjørnland *et al*. (2005) and Cesaroni (2010)]. The second best inflation forecast is generated using output gap Method based on quadratic time trend and third best measure is through Band Pass Baxter and King Filter Method approach. The reason is that structural approach assume that particular economic theory exist that explains the economic behavior very well (Menashe & Yakhin 2004). However, decomposing output based on purely statistical criteria can be random from an economic perspective. For example, the supply side of the economy is assumed to be non-stochastic if a linear deterministic time trend is fitted in output and demand side changes are the prime factor in economic fluctuations.

Moreover, the univariate ARMA model perform better forecast than bivariate VAR model as the value of MSE and RMSE is minimum in case of ARMA (2,5) model that is consistent with the empirical studies. In addition to that, small VARs of two or three variables are often unstable and thus poor predictors of the future (Stock & Watson 1996). And univariate autoregressions are generally better than standard non linear models (Stock & Watson 1999). Feridun *et al.* (2006) also conclude that the VAR models don't perform better than the ARIMA (2, 1, 2) models.

4. SUMMARY AND CONCLUSION

The measurement of output gap to study business cycle and its use in inflation forecasting has been an important debate in current economic literature. Most of the modern central banks in the world have adopted an inflation forecast as an intermediate target. There exists abundant literature on empirical side that scrutinizes the given issue but there is little empirical evidence in case of Pakistan. In order to overcome this deficiency on empirical grounds, different statistical and structural methods are used to estimate output gap. In addition to that, the inflation is forecasted using univariate, Autoregressive and Moving Average (ARMA) and bivariate, Vector Autoregressive (VAR) models.

In order to calculate output gap, five statistical and two structural methods are used, where statistical methods consist of Linear Time Trend, Quadratic Time Trend, Hodrick-Prescott (HP filter), Band Pass Baxter and King Filter (BP) and Exponential Smoothing method. Whereas, the structural methods include Structural Vector Autoregressive (SVAR) method and Production Function (PF) approach. The results show that the economy of Pakistan since 1960 has gone through three complete business cycles, which is strongly supported by the historical facts and empirical evidence (see Mahmood *et al.* 2008).

According to the comparison of statistical measures of output gap, it is found that results of quadratic detrending captures the history well. The second method which produced realistic results is HP filter. However, the results of quadratic detrending are even better, especially in the beginning of sample period. According to the comparison of structural measures of output gap, it is found that the two structural methods produced almost similar pattern of economic activity and this pattern are consistent with the economic history of Pakistan. Moreover, the results of structural methods at the beginning of the sample period are in accordance with those from quadratic detrending method.

The comparisons of output gap to forecast inflation indicate that structural methods perform better than the statistical methods. Moreover, the SVAR is the best model to forecast inflation as compare to all others methods. The output gap Method is the second best model which forecast inflation better than the other methods. The results also indicate that univariate ARMA model forecast inflation better than bivariate VAR models do using any of the output gap measure.

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APPENDIX

Table A.1, Results of output gap by statistical and structural methods

YEARS	LTTG	QTTG	HPG	BPG	DESG	SVARG	PFG
FY60	-12.63	-19.81	6.91	-		-	0.50
FY61	-12.19	-15.94	3.27	-	5.66	-	-0.17
FY62	-11.09	-11.92	1.16	-	6.37	-1.29	-0.19
FY63	-8.86	-7.31	0.81	-0.32	6.24	1.91	-0.47
FY64	-7.28	-3.73	0.30	-0.71	4.21	1.39	-2.98
FY65	-3.61	1.49	2.31	1.11	5.16	3.60	-0.42
FY66	-1.32	4.97	3.29	1.87	3.13	2.27	0.66
FY67	-2.33	4.55	1.17	-0.61	-1.01	-1.02	-1.11
FY68	-2.03	5.09	0.63	-1.71	-0.20	0.27	-0.71
FY69	0.29	7.41	2.30	-0.42	2.36	2.30	2.13
FY70	5.87	13.13	7.59	4.59	6.18	5.56	7.18
FY71	2.34	8.52	3.39	0.95	-3.11	-3.53	2.96
FY72	-1.29	3.84	-0.69	-2.20	-4.44	-3.64	0.19
FY73	-0.01	4.22	0.18	-0.02	1.20	1.23	1.24
FY74	0.10	3.31	-0.15	1.00	1.39	0.11	1.58
FY75	-1.79	0.33	-2.51	-0.31	-0.70	-1.90	-0.13
FY76	-2.35	-1.29	-3.64	-0.52	0.42	-0.56	-2.07
FY77	-3.70	-3.63	-5.57	-1.83	-0.02	-1.34	-5.06
FY78	-1.07	-2.08	-3.80	0.19	4.07	2.61	-3.90
FY79	-1.44	-3.40	-4.96	-1.36	1.27	-0.37	-5.65
FY80	1.83	-1.12	-2.65	0.39	4.25	3.23	-2.65
FY81	3.45	-0.36	-1.93	0.46	2.60	1.62	-1.29
FY82	4.72	0.11	-1.53	0.29	1.57	1.25	-0.75
FY83	6.21	0.89	-0.86	0.51	1.51	1.48	-0.06
FY84	6.08	0.13	-1.69	-0.69	-0.15	-0.15	-0.73
FY85	8.32	1.85	-0.07	0.48	2.15	2.22	0.84
FY86	8.61	1.69	-0.26	-0.34	0.56	0.28	1.56
FY87	9.79	2.54	0.60	-0.15	1.23	1.14	0.41
FY88	12.07	4.63	2.74	1.40	2.52	2.28	3.07
FY89	11.72	4.09	2.39	0.53	-0.07	-0.36	2.08
FY90	10.89	3.15	1.72	-0.58	-0.89	-0.85	1.11
FY91	11.00	3.29	2.18	-0.35	0.27	0.24	2.99
FY92	13.35	5.87	5.12	2.45	3.00	2.24	5.05
FY93	10.03	2.61	2.32	-0.24	-2.44	-3.36	2.45
FY94	8.62	1.48	1.65	-0.68	-1.15	-1.40	1.63
FY95	8.40	1.63	2.27	0.50	0.77	-0.22	3.07
FY96	8.06	1.76	2.84	1.88	1.12	-0.36	3.63
FY97	4.00	-1.75	-0.34	-0.34	-2.59	-4.03	-1.58
FY98	1.45	-3.62	-1.94	-0.86	-1.33	-2.60	-4.09
FY99	-0.03	-4.36	-2.49	-0.23	0.46	-1.48	-3.71
FY00	-0.93	-4.47	-2.52	0.82	1.47	-0.82	-2.05
FY01	-4.03	-6.59	-4.71	-0.75	-0.75	-3.15	-3.43
FY02	-5.93	-7.48	-5.76	-1.72	0.17	-1.89	-4.76
FY03	-6.27	-6.86	-5.36	-1.91	2.02	-0.36	-3.36
FY04	-4.23	-3.91	-2.68	-0.46	4.56	2.00	-0.24
FY05	-1.91	-0.53	0.38	0.99	4.57	2.32	3.59
FY06	-0.99	1.62	2.17	1.44	2.33	0.85	2.57
FY07	-0.53	3.39	3.60	2.14	1.06	0.64	2.92
FY08	-4.02	1.19	1.10	-0.33	-3.12	-3.57	0.14
FY09	-5.52	1.09	0.74	-0.26	-1.35	-1.51	-1.16
FY10	-6.32	1.75	1.21	0.73	0.25	-1.00	-0.45
FY11	-9.31	0.27	-0.40	-0.61	-1.43	-2.87	-1.34
FY12	-10.96	0.18	-0.56	0.14	-0.11	-1.68	-1.45
FY13	-11.72	1.05	0.29	3.45	1.26	-0.75	-0.13
FY14	-11.52	2.97	2.23	9.21	2.39	0.20	2.53

Source: self calculation and estimation

Where LTTG, QTTG, HPG, DESG, BPG, SVARG, PFG are output gap from Linear Time Trend method, output gap from Quadratic Time Trend method, output gap from Hodrick-Prescott (HP filter), output gap from Baxter-King Method (BP Filter), output gap from double Exponential Smoothing method, output gap from Structural Vector Autoregressive method and output gap from Production Function method, respectively.