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Threshold autoregressive (TAR) &Momentum Threshold autoregressive (MTAR) models Specification

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Abstract
This paper proposes a testing integration and threshold integration procedure of interest rate and inflation rate. It locates whether there have been a cointegrating relationship between them and recognizes the procedures of addressing structural break. The most important issue is the testing of the hypothesis that whether effect of inflation on interest rates depends on the movement of inflation declining or increasing. In this study we analyze the inflation and interest rate of Canada for their long term relationships by applying co integration technique of EG-Model. Further we test it for the structural break and threshold autoregressive (TAR) and Momentum Autoregressive (MTAR) integration and stationarity. We generate real interest rate and test it for the same. We come to an end that TAR best capture the adjustment process. We discover that our selected series are integrated at level one. There is cointegration relationship between interest rate and inflation with the cointegrating vector (1,-1).We find no asymmetry in our series and therefore conclude that there is same effect of inflation increase or decrease on interest rate.
Keywords: Threshold autoregressive, Momentum autoregressive, Co integration, Stationarity

1. Introduction

Time series run into practice may not always exhibit characteristics of a linear process. Many processes occurring in the real world exhibit some form of non-linear behavior. This has led to much interest in nonlinear time series models including bilinear, exponential autoregressive, Threshold autoregressive (TAR) and many others. In recent years non-linear time series has captured the attention of many researchers due to its uniqueness. Among the family of non linear models, the threshold autoregressive (TAR), momentum threshold autoregressive (MTAR) and bilinear model are perhaps the most popular ones in the literature. However, the TAR model has not been widely used in practice due to the difficulty in identifying the threshold variable and in estimating the associated threshold value. The threshold autoregressive model is one of the nonlinear time series models available in the literature. It was first proposed by Tong (1978) and discussed in detail by Tong and Lim (1980) and Tong (1983). The major features of this class of models are limit cycles, amplitude dependent frequencies, and jump phenomena. Much of the original motivation of the model is concerned with limit cycles of a cyclical time series, and indeed the model is capable of producing asymmetric limit cycles. The threshold autoregressive model, however, has not received much attention in application. This is due to (a) the lack of a suitable modeling procedure and (b) the inability to identify the threshold variable and estimate the threshold values. A TAR model is regarded as a piecewise –linear approximation to a general non-linear model. This model can be seen as a piecewise linear AR model, with somewhat abrupt change, from one equation or regime to another dependent on whether or not a threshold value © is exceeded by $z_{t-d.}$ In the TAR models the regime is determined by the value of $zt_{-d.}$ where d=1,2,3….here d=delay parameter show that the timing of the adjustment process is such that it takes more than one period for the regime switch to occur.TAR models capture the deepness asymmetry in the data. A similar model which captures the steepness in the data is momentum threshold autoregressive model (MTAR).If we say "zt" is stationary series then testing for deepness hypothesis by TAR model is simply equivalent to the testing of no skewness in the data zt. Whereas testing the steepness hypothesis is equivalent to the testing the no skewness in Δzt.

The main focal point of this paper is application of the TAR model and MTAR model with two regimes. We illustrate the proposed methodology by analysis of artificial data set. The problem of estimating the threshold parameter, i.e., the change point, of a threshold autoregressive model is studied here.
The primary goal of this study is to suggest a simple yet widely applicable model-building procedure for threshold autoregressive models. Based on some predictive residuals, a simple statistic is proposed to test for threshold nonlinearity and specify the threshold variable.

2. Methodology

We address the issue of time series stationarity e.g unit root testing, co integration (long run relationship) and presence of structural break. Structural change is an important problem in time series and affects all the inferential procedures. A structural break in the deterministic trend will lead to misleading conclusion that there is a unit root, when in fact there might be not.

In order to find the long term relationship (co integration) we apply the co integration technique of EG-model (1979). Further, we test the stationary and integration in TAR and MTAR. Generally for analysis, Firstly, we need to plot the series and identify the presence of deterministic trends, secondly, we need to detrend the series and obtain the filtered series, which are fluctuations around zero-horizontal line. Thirdly, we need to define the Heaviside function and Estimate the specific model defined, then testing the related hypothesis. Finally, we need to test for deepness and steepness asymmetric root for TAR and MTAR specification.

We collect the time series data of Canada from www.rba.gov.au . We generate the yearly inflation series from CPI data by applying the formula.

Inflation= (CPI Current-CPI last/CPI Last)*4 (1)

Real Interest rate=Inflation-Interest rate (2)

2.1-Test for integration

We follow the following steps for it.

Consider a AR (1) model;

$Yt=\phi Yt-1+et$

Case1-if $|\phi|<1$ the series is stationary.

Case2-if $|\phi|>1$ the series is non-stationary and explodes.

Case3-if $|\phi|=1$ series contain a unit root and non-stationary.

A test for the order of integration is a test for the number of unit roots; it follows the step as under:

Step (a) - we Test "Yt" series if it is stationary then Yt is I(0). If No then Yt is I (n); n>0.

Step (b) - we Take first difference of Yt series as $\Delta yt=yt-yt-1$ and test Δyt to see if it is stationary.

If yes then yt is I (1); if no then yt is I (n); n>0 and so on.

2.2 TAR& MTAR Models Specification

We detrend the series first and obtain the filtered series residuals. Define the Heaviside Indicator and decompose the series in positive and negative. Estimate the models for testing the asymmetric unit root under the Null .We used Enger and Granger (1998) critical values against F-statistics tabulated in order to draw the conclusion.

2.2.1- Testing for evidence of threshold stationarity

Actually when we conduct testing under TAR or MTAR model, the threshold parameter is unknown .We follow the following steps to estimate the threshold parameters;

(a): define the first-differenced series: $\Delta y_t=y_t-y_{t-1}$

(b): making the series Δy_t sorted in ascending order

(c): taking off 15% largest values and 15% smallest values.

(d): using remaining 70% values to estimate threshold parameters.

(e): estimate model $\Delta y_t=I_t P_1 y_{t-1}+(1-I_t)P_2 y_{t-1}+e_t$

For each possible threshold parameter, we find the lowest AIC, it is best estimate of threshold parameter.

To test y_t is stationary or non-stationary: we do the following procedure:

H_0: $P_1=P_2=0$

H_1: $P_1<0$ and $P_2<0$

Construct $F=[(SSE_R-SSE_{UR})/J]/[SSE_{UR}/(N-K)]$ (4)

If this value is larger than critical value then null hypothesis is rejected, series is stationary. Then we can test steepness asymmetric roots as:

H_{0a}: $P_1=P_2$

H_{1a}: $P_1 \neq P_2$

Estimate model under H_{0a} and H_{1a}.

Construct $F=[(SSE_R-SSE_{UR})/J]/[SSE_{UR}/(N-K)]$

If F-statistics is less than critical value, then accept H_{0a}, that there is no steepness asymmetry.
In order to find the result of increasing and decreasing interest rate on inflation we follow MTAR procedure**.** We divide the series into two regimes by defining the indicator functions for positive and negative values and then test for these two regimes. If there is asymmetric adjustment it indicates that there is different effect of decling and increasing interest and vice versa.

2.2.2 Structural break

There have been several studies deriving tests for structural change in cointegration relationship. We consider a simple diagnostic test for structural change as used in previous studies.

Wright (1993) extends the CUMSUM test to non-stationary trended variables and to integrated variables. Hoa and Inder (1996) extends the OLS-based CUMSUM test discussed by Ploberger and Kramer (1992), they suggest its use as a diagnostic test for structural change. They consider FM-OLS residuals and long run variance estimate. Then derive the asymptotic distribution of the FM-OLS based CUMSUM test statistic, tabulate the critical values, and show that the test has nontribal local power irrespective of the particular type of structural change. If there is break then we apply unit root testing in the presence of structural break by defining dummy variables, three model under the unit root null and under trend stationary alternative hypothesis was tested by tabulating the critical values as Perron(1989) did in its paper.

However we can also use dummy variable to address this issue:

$DT_t=t$ if $t \le T$ or $DT_t=0$ if otherwise. "T" shows the time series and "t" shows the point where structural break occur. Then put the dummy variable into the model and estimate the model, get the final results.

Sometimes, if the structural break is really affecting the final results in the cointergrating relationship, we can delete these data and use remaining data to estimate the model.

3. Empirical Findings

We take the series of inflation and interest rate data for Canada from (1980-2009).It is usually consider about Time series data it is volatile and may have autocorrelation problem. In this situation OLS estimation becomes biased and spurious. To addresses all such problems we applied different test to our series.

Table 1,explains the results of unit root testing in interest rate, inflation rate and real interest rate series while the results for TAR and MTAR stationaity of these series are in Table 2.

For the interest rate, we test whether it is I (1). From our output statistic we came to know that interest rate is non-stationary at I (0) but stationary at first difference. Later on, we test for inflation and real interest rate; they all are stationary at first difference. It shows that our series have unit root at level.

TABLE-1

Unit root results

variable	difference	Test value	Critical value	Decision about null of unit root
Interest rate	no	-2.4588	-3.45	Null of unit root accepted
Interest rate	yes	-6.8017	-3.45	Rejected
Inflation rate	no	-2.1876	-3.45	accepted
Inflation rate	yes	-10.87643	-3.45	rejected
Real interest rate	no	-2.12	-3.45	accepted
Real interest rate	yes	-8.8602	-3.45	rejected

Critical value is taken from Fuller (1976) for the model (constant and trend) at 5% level of significance. Table1 result shows all our series are stationary at I(1).

Cointegration is an econometric property of time series variables. If the interest rate and inflation are themselves non-stationary, but a linear combination of them is stationary then the series are said to be co integrated. We get our residuals from the relation and find the value of -9.87102 is less than critical value – 2.8775 at 5% significant level. We reject the null hypothesis of non statinarity for residuals. AS u_t is I (0).Therefore, inflation rate and interest rate are cointegrated .when we check it for cointegration vector we find , they are cointegrated with cointegrating vector (1,-1).It depicts there is long term relationship between both variables. They cause each other. When we investigate for structural break we find there is consistent relationship during our period of analysis. There is no break.

If financial variables have asymmetric roots, then these variables should model to accommodate these roots, otherwise, model would be miss-specified and will results in misleading inferences. We test asymmetry root for interest rate, real interest rate and inflation. We test the deepness and steepness asymmetric root of interest rate and inflation.

In the testing for the presence of asymmetric roots, we find that both of interest rate and inflation do not have asymmetric roots. We use nominal interest rate and inflation to calculate real interest rate. For real interest rate, we use TAR and MTAR model to test the presence of asymmetric root. The final result we get that there is no threshold asymmetry or steepness asymmetry in real interest rate. These results show there is same effect of increasing and decreasing interest on inflation and on the other side increasing and decreasing inflation on interest rate because there is no-asymmetry. When we find the MTAR results for inflation we also find there is no asymmetric adjustment in the inflation series too. Results are reported in Table 2

TABLE-2

TAR and MTAR results

		Interest rate	**Inflation rate**	**Real Interest Rate**	**F-stat**	**Critical-values**	**Decision**
	under H_0 and H_1						
TAR	ESSr	11.6725	3655.049	571450.3			
	ESSur	11.07116	2833.475	364201.6			
	F-stat	4.7526	5.2726	49.79			
	Granger – critical value	4.56	4.56	4.56			
	Decision	Null of non stationary is rejected.	Null of non stationary is rejected.	Null of non stationary is rejected			
	under H0a and H1a						
	ESSr	11.13526	2876.175	364247.1			
	ESSur	11.07116	2833.475	364201.6			
	F-stat	1.0132	2.63	0.00218			
	F-critical	3.92	4.64	3.92			
MTAR	**Decision**	Null accepted. Series does not have asymmetric roots	Null accepted. Series does not have asymmetric roots .	Null accepted. Series does not have asymmetric roots			
	ESSr			571451.3	50.017	4.56	Null of non stationary rejected
	ESSur			364358			
Real Interest Rate	under H0a and H1a						
	ESSr			364610	0.1217	3.92	NO asymmetry accepted
	ESSur			364358			

3. Conclusion

The aim of this study is not to explore the economic issues related to inflation rate series or interest rate series of Canada. Our objective is this study is to explain the TAR and MTAR model application on the data and issues related to them for the further research.

The crucial point is that the dynamic adjustment process is usually assumed to be linear. But in the presence of asymmetric adjustment it become non linear so in such situation test for unit root has low power, because it does not capture appropriately the dynamic adjustment process. This study tries to explain the procedure for this dynamic adjustment when we observe the unit root for our real interest rate and inflation series it shows high power of ADF test which was indication that there is symmetric adjustment. The AIC and SBC both select the TAR model over MTAR in our study because its value is lower here. Hence we conclude that TAR best capture the adjustment process. The adjustment coefficient values are significant and according to hypothesis. We find that our selected series are integrated at level one. There is cointegration relationship between interest rate and inflation with the cointegrating vector (1,-1).We find no asymmetry in our series and therefore conclude that there is same effect of inflation increase or decrease on interest rate.

References

Enders, W., and C. W. J. Granger, 1998, “Unit-Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates,” *Journal of Business and Economic Statistics* 16:3, 304-311.

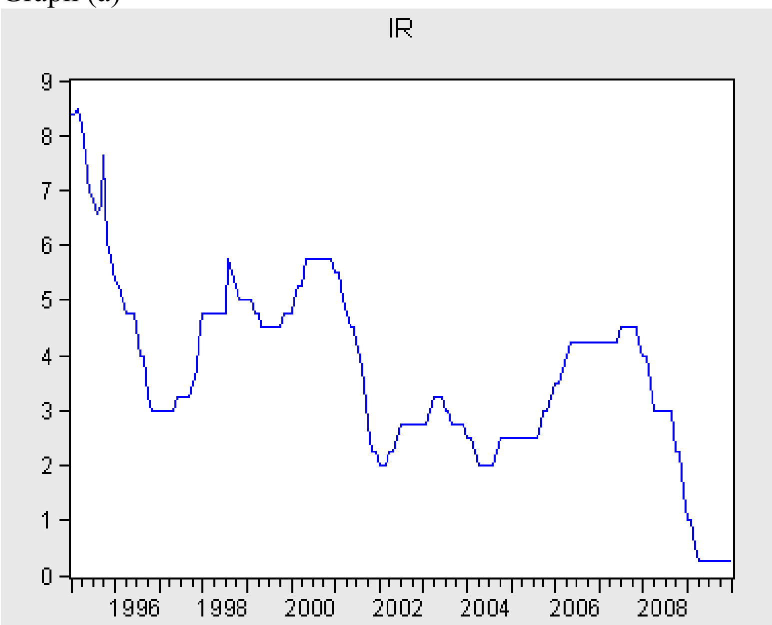
Enders, W., K. S. Im, J. Lee, and M. C. Strazicich, 2010, “IV Threshold Cointegration Tests and the Taylor Rule,” *Economic Modelling* 27, 1463-1472.

Li, J and J. Lee, 2010, “ADL Tests for Threshold Cointegration,” *Journal of Time Series Analysis* 31, 241-254.

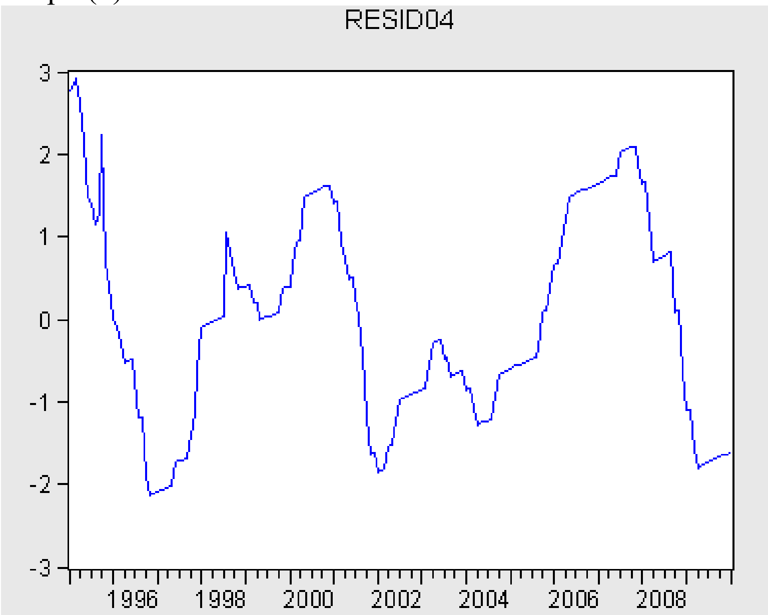
Tong, H., 1983, *Threshold Models in Non-Linear Time Series Analysis*, New York: Springer-Verlag.

Appendix:

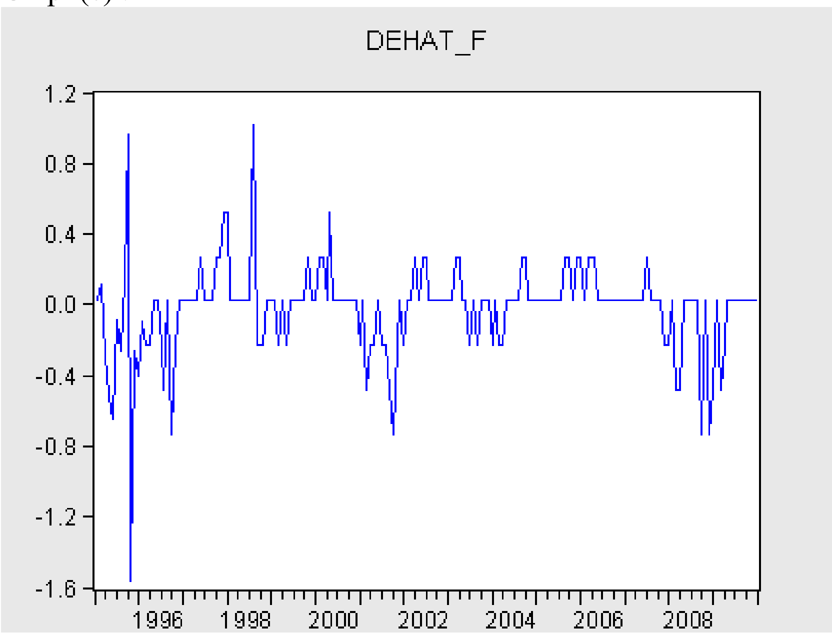
Graph (a)



Graph (b)



Graph(c) :



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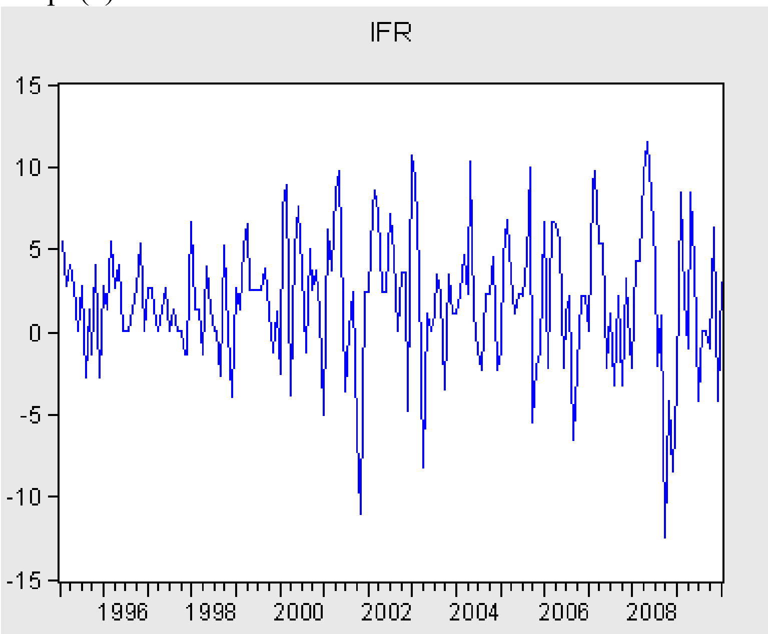
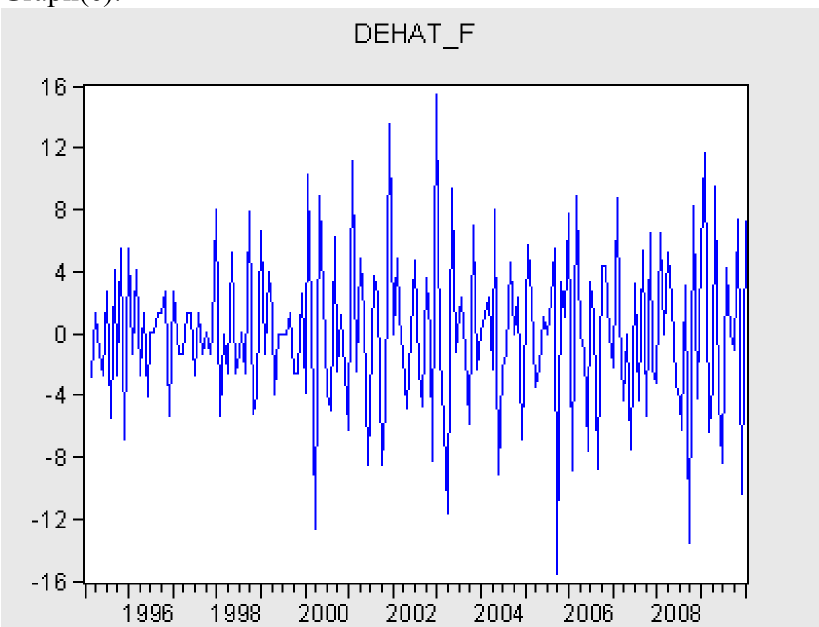
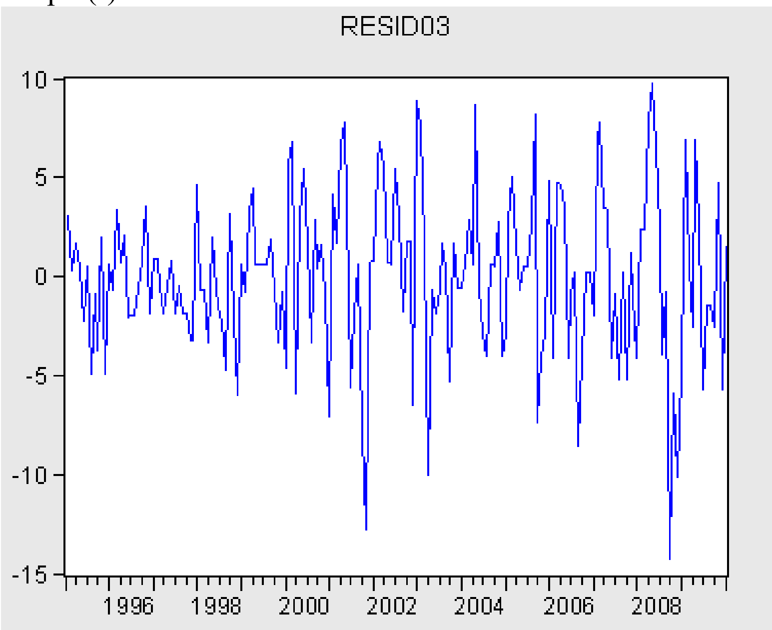
Graph(d):



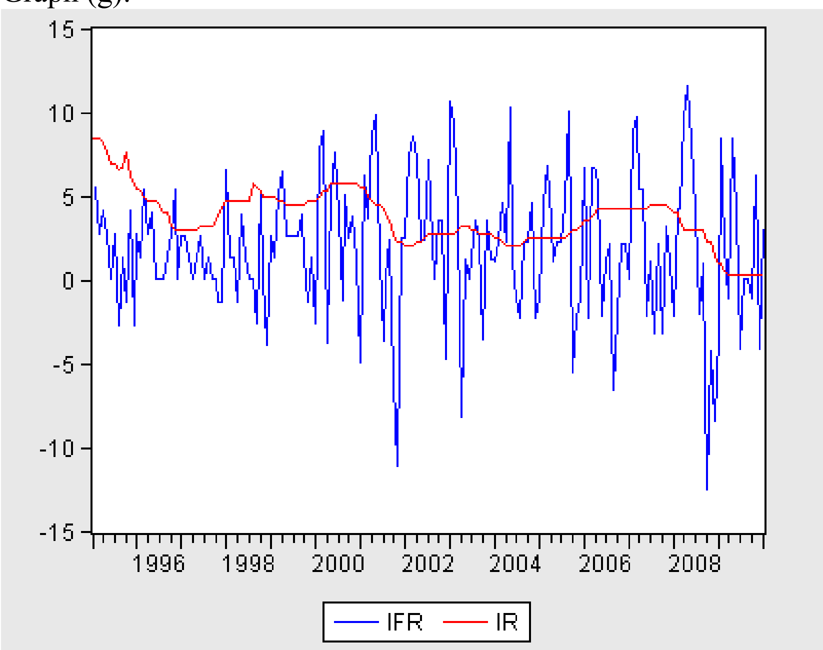
Graph(e):



Graph (f):



Graph (g):



Output (a):

Dependent Variable: IR
Method: Least Squares

Sample (adjusted): 1995M05 2010M01
Included observations: 177 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.120158	0.057256	2.098588	0.0373
IFRI	0.000363	0.007121	0.050952	0.9594
IFRII1	0.027957	0.010646	2.626054	0.0094
IR(-1)	1.088802	0.074433	14.62793	0.0000
IR(-2)	0.066046	0.110900	0.595539	0.5523
IR(-3)	-0.049301	0.111167	-0.443489	0.6580
IR(-4)	-0.138715	0.073147	-1.896390	0.0596

R-squared	0.974138	Mean dependent var	3.623785
Adjusted R-squared	0.973225	S.D. dependent var	1.534746
S.E. of regression	0.251131	Akaike info criterion	0.113057
Sum squared resid	10.72131	Schwarz criterion	0.238667
Log likelihood	-3.005526	Hannan-Quinn criter.	0.164000
F-statistic	1067.225	Durbin-Watson stat	1.990541
Prob(F-statistic)	0.000000		

Output (b)

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	1.929705	(2, 177)	0.1482
Chi-square	3.859411	2	0.1452

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2)	-0.046193	0.045463
C(3)	0.132727	0.068170

Restrictions are linear in coefficients.