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Double Exponential Jump Diffusion Model: An Empirical Assessment for the Turkish Stock Market

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Abstract

In continuous time option pricing and portfolio optimization problems generally Geometric Brownian Motion risky asset dynamic is used. However, normality of risky asset returns is not always supported by empirical studies. In empirical studies it is seen that asymmetric leptokurtic feature and the volatility smile are observed in emerging market asset returns. For this purpose, in this paper the applicability of Kou's Double Exponential Jump Diffusion model to İstanbul Stock Exchange main index ISE100 is investigated. The results are compared with Geometric Brownian Motion price dynamic. It is seen that Kou's model perform better to capture the leptokurtic property of returns.

Key Words: Double Exponential Jump Diffusion Model, Geometric Brownian Motion, Empirical Characteristic Exponent

1. Introduction

In asset pricing literature, papers using common time series models assume that the past behavior of prices gives information about future prices. However, according to Bachelier (1900) the asset price's current movement is a function not only of earlier fluctuations, but also of the present market position. His study proposed the theory of random walks. Random walk theory states that the asset price changes are independent, identically distributed random variables (Fama, 1965).

Bachelier's (1900) study is a crucial study about the asset price dynamic. His simplest and most important model goes as follows: Suppose S(t) is the price of an asset or a commodity at the end of time period t. Then it is assumed that the differences of the form S(t+T)-S(t) are independent, Gaussian, random variables with zero mean and variance proportional to the differencing interval resulted in the construction of Brownian Motion (BM) (Mandelbrot, 1963).

After the construction of BM, in the early 1970s, assumption of Geometric Brownian Motion (GBM) model for the risky asset dynamics was started to be used in option pricing and portfolio optimization problems following the study of Black and Scholes (1973). In GBM pricing dynamic, over a finite time interval the returns on a common stock are log normally distributed. Since it is easy to drive explicit solutions with GBM it is preferred in practice. However, some of the studies especially the study of Mandelbrot (1963) and Fama (1965) stated that the GBM dynamic is inconsistent with market data.

In 1963, Mandelbrot stated that the asset price distributions have leptokurtic and asymmetric features and the stable Paretian family of distributions is suggested to characterize the stochastic properties of prices.

In 1965 Fama, discussed the random walk theory for asset prices by testing the independence of returns and investigating the distribution of asset prices. The first main result of his study was that there were departures from normality in the distributions of the first differences of the logarithms of stock prices. In addition, the daily changes in log price of stocks of large mature companies followed stable Paretian distributions with characteristic exponents close to 2.

When the studies about distributional properties of asset returns are taken into account, number of extensions of GBM have been proposed. The most popular extension is the Merton's (1976) Jump diffusion model. In jump diffusion model Poisson jumps are added to the GBM model to describe discontinuous changes of asset returns upon arrival of new information. In this case, asset returns are modeled with a mixture of both continuous and jump processes.

Another extension to the GBM is Double Exponential Jump Diffusion (DEJD) model proposed by Kou (1999) and Kou (2002). It gained acceptance in many studies since it takes into account the leptokurtic feature and volatility smile of asset returns and besides the model is simple enough to produce analytical solutions for pricing problems. In this model, the continuous part of asset prices is driven by GBM and jump part is modeled by logarithm of the jump sizes having a double exponential distribution. Since, the double exponential distribution has both a high peak and heavy tails, it can be used to model both the overreaction and under reaction to outside news (Kou, 2002).

The theory of finance depends on the efficient market hypothesis which states that the asset prices follow random walk. The vast majority of theoretical and empirical studies in this area focus on developed countries. However, in recent years, with the globalization of the financial markets, investors began to deal with new emerging markets which can provide higher returns. As a result, emerging markets became crucial for the international financial system. When relevant literature is reviewed it can be observed that emerging markets tend to have different features when compared to the developed markets.

Researches on emerging financial markets suggest that there are four main characteristics of emerging markets: high average return, high volatility and low correlations with developed markets and more predictable returns (Bekaert et al. (1998)). In addition, the normality of the emerging market returns is argued in studies such as; Harvey (1995), Harvey and Bekaert (1997) and Bekaert et al. (1998)) The emerging market asset returns has leptokurtic features, that means higher peak and heavier tails than those of the normal distribution with significant skewness. The results of Mandelbrot (1963) and Fama (1965) capture developed countries though the results are also valid for emerging market.

The predictability of emerging markets asset returns contradicts with the random walk Theory of Efficient Market Hypothesis. There are many studies testing the efficiency and random walk theory of the Turkish stock exchange. The results of some studies revealed the violation of the random walk theory such as: Balaban et al.(1996) Kasman and Kırkkulak (2007) Özcan ve Yılancı (2009) Çevik (2012) Kapusuzoglu(2013). In other studies concluded with the weak form efficiency of the İstanbul stock exchange are : Buguk and Brorsen (2003), Eken and Adalı (2008), Ergül (2009), Korkmaz and Akman (2010), Zeren and Kara-Arı (2013). There are considerable number of studies that cannot reject the efficient market hypothesis for Turkish stock market. Then, as an emerging market we cannot conclude that the asset prices do not have random walk dynamic.

The distributional properties of asset prices have important implications for several financial models. Solutions to the financial problems such as asset pricing, portfolio optimization and option pricing are dependent to the asset price dynamics.

Most of the papers written by using ISE 100 index return data proposed that the returns had leptokurtic features with significant skewness such as: Tufan and Hamarat (2004), Erdem et al. (2005), Demireli (2010), Er and Fidan (2013), Altun and Tatlıdil (2015).

In this paper, by taking into account the results of studies about the non-normality of emerging market returns and the studies that cannot reject the efficient market hypothesis for Turkish stock market the applicability of Kou's DEJD model to İstanbul Stock Exchange main index ISE 100 is investigated. The results are compared with GBM price dynamics. To evaluate the performance of DEJD process first of all unknown parameters are estimated with empirical characteristic exponent method. Then, performances of the price dynamics are compared by using simulations of the diffusion models.

1.1. Literature Review

In 1990 with the study of Bachelier, BM process is stated for the stock price movements. Since, it is easy to derive explicit solution, GBM stock price model is used in many option pricing and portfolio optimization problems. However, the normality assumption of logarithmic returns in GBM is criticized in many studies. As a result, number of extensions of GBM has been proposed.

One of the most popular extensions to the GBM is Merton's (1976) Jump diffusion model. Another extension to the GBM is DEJD model proposed by Kou (1999) and Kou (2002). DEJD model is proposed for the option pricing model as an alternative model for Black Sholes option pricing model. Kou (2002) stated that the model is simple enough to produce analytical solutions for a variety of option-pricing problems and can reflect the leptokurtic feature and volatility smile of asset returns.

After the studes of Kou (1999) and Kou (2002), the DEJD is used in asset price dynamics to solve the different option pricing problems.

Kou and Whang (2004) extended the DEJD option pricing study and derived pricing formula for finite-horizon American options (by extending the Barone-Adesi and Whaley method) and for popular path-dependent options (such as lookback, barrier, and perpetual American options).

Sepp (2004) derived formulas for double and single barriers and touch options with time-dependent rebates by assuming the underlying asset price follows DEJD process.

Guohe (2007) derived the closed-form solution of the European call option for the DEJD model with two different market structure risks that there exist CIR stochastic volatility of stock return and Vasicek or CIR stochastic interest rate in the market.

AitSahlia and Runnemo (2007), presented a simple numerical approach to compute accurately the values and optimal exercise boundaries for American options when the underlying process is a DEJD model.

Zhang et al. (2012) solved the European option pricing problems with the fuzzy interest rate, fuzzy drift, fuzzy volatility and fuzzy jump intensity. They presented the fuzzy pricing formula of European options based on the DEJD model.

Fuh et al (2013) provided methodologies to price discretely monitored exotic options when the underlying evolves according to a DEJD process.

DEJD leads to tractable pricing formulas for different types of options therefore, it is widely used in option pricing problems nevertheless the empirical assessment of the model is also needed. There are some studies to assess the performance of DEJD for different risky assets by using parameter estimation techniques.

Tuzov (2006) empirically investigated the performance of Generalized Method of Moments in estimating the parameters of DEJD model.

Ramezani and Zeng (2007), by using daily returns for the S&P-500 and the NASDAQ indexes and individual stocks, predicted the parameters of DEJD model with maximum likelihood estimation. They assessed the performance of DEJD relative to log-normally distributed jump-diffusion (LJD) and the GBM and found that DEJD performs better than these alternatives for both indexes and individual stocks.

Cont and Tankov (2009) investigated the behavior of Constant Proportion Portfolio Insurance

(CPPI) strategies in models where the price of the underlying portfolio may experience downward jumps. As a case study, they used the DEJD model and estimated the parameters of DEJD by using the empirical characteristic exponent.

Emenogu (2012) used the Merton (1976) and Kou (2002) extensions to model the dynamics of Hedge fund indices. In the study, the parameters of each models was estimated by Maximum Likelihood Estimation method, Empirical Characteristic Function method, The Cumulant Matching method and Generalized Method of Moments. Calibration results of the study showed that these two models fit the data well, however, the empirical comparison showed DEJD results are more consistent with the empirical data.

In emerging market modelling the underlying risky asset is a very important concept for option pricing and portfolio optimization problems. In relevant literature, there are a few studies that took into account the Lévy dynamics in asset pricing for Turkish stock market.

Önalan (2011) reviewed the connections between Lévy processes with jumps and self-decomposable laws asset returns are representing by a Normal Inverse Gaussian process.

Göncü et al (2015) investigated the goodness-of-fit of three Lévy processes, namely Variance-Gamma, Normal Inverse Gaussian and Generalized Hyperbolic distributions.

Sezgin-Alp (2015) evaluated the performance of GBM and Pure Jump Diffusion models for ISE-100 index. In addition, the performances of those models are assessed in continuous time mean-variance portfolio selection problem.

One of the aims of this study is to fulfill this gap and give insight to the future studies on how to use Lévy assest pricing dynamics in option pricing and portfolio optimization problems.

2. Methodology

2.1. The Asset Pricing Model

In this paper for asset pricing two different price dynamics are used. First one is the GBM process. When risky asset dynamic is assumed to be GBM process with constant model parameters, it is modeled with the following stochastic differential equation.

$$dS(t)=S(t)(\mu dt+\sigma dW_t) \tag{1}$$

In this equation, *S(t)* denotes the price of stock at time *t*, W_t is the BM, μ is the drift parameter and σ is the constant volatility. Solving the stochastic different equation by using Ito's Lemma gives the dynamics of the asset price as follows:

$$S(t)=S(0)\exp\left\{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t\right\} \tag{2}$$

Then, the logarithmic return will be;

$$\ln(S(t)/S(0))=\exp\left\{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t\right\} \tag{3}$$

As a result, the characteristic function of GBM can be derived as given in Equation 4.

$$E\left[e^{iuXt}\right] = \exp\left\{t\left(i\gamma u-\frac{\sigma^2u^2}{2}\right)\right\} \tag{4}$$

Second asset pricing process used in this paper is the DEJD. When risky asset price dynamic is assumed to be DEJD model then the asset price at time *t* with constant parameters is modeled with following stochastic differential equation given in Equation 5.

$$dS(t)=S(t)\left(\mu dt+\sigma dW_t+d\left(\sum_{i=1}^{N_t}V_i-1\right)\right) \tag{5}$$

V_i is a sequence of independent identically distributed (i.i.d.) nonnegative random variables. Such that $Y=\log(V)$ has a double exponential distribution with the density;

$$f_Y(y)=p\eta_1e^{-\eta_1 y}1_{\{y\geq 0\}}+q\eta_2e^{\eta_2 y}1_{\{y<0\}} \quad \eta_1>1,\eta_2>0 \tag{6}$$

Here, $pq \geq 0$, $p+q=1$, represent the probabilities of upward and downward jumps. The requirement $\eta_1 > 1$ is needed to ensure that $E(V)<\infty$ and $E(S(t))<\infty$ (Kou, 2002). Y can be represented as:

$$Y=\begin{cases}\xi^{+} \text{ with probability } p\\ -\xi^{-} \text{ with probability } q\end{cases} \tag{7}$$

Where, ξ^{+} and ξ^{-} are exponential random variable with mean $1/\eta_1$ and $1/\eta_2$. That means Y has an IID mixture distribution two exponential distribution with probabilities *p* and *q*, respectively (Ramezani and Zeng, 2006).

Solving the stochastic different equation by using Ito's Lemma gives the dynamics of the asset price as follows:

$$S(t)=S(0)\exp\left\{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t\right\}\prod_{i=1}^{N(t)}V_i \tag{8}$$

Here, all the source of randomness is assumed to be independent. Since, the solution of the DEJD model given as in Equation 8 Then, the logarithmic return will be;

$$\ln(S(t)/S(0))=\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t+\sum_{i=1}^{N(t)}\ln V_i \tag{9}$$

The logarithmic return is such an exponential Levy process as given in Equation 10 (Tankov, 2007).

$$X_t=\gamma_t+\sigma W_t+\sum_{i=1}^{N(t)}Z_t \tag{10}$$

Here, $\gamma=\left(\mu-\frac{1}{2}\sigma^2\right)$ and $(Z_t)_{t\geq 1}=(\ln V_t)_{t\geq 1}$ is a jump process with (possibly) infinitely many jumps with probability density function of *F* describes the distribution of jump size.

The characteristic function of a Lévy process is given by the Lévy-Khintchine formula given in Equation 11.

$$E\left[e^{iuXt}\right] = \exp\left\{t\left(i\gamma u-\frac{\sigma^2u^2}{2}+\int_R\left(e^{iux}-1-iux1|x|\leq 1\right)\nu(dx)\right)\right\} \tag{11}$$

Where, ν is a positive Lévy measure on *R* describing the jumps of the process. If *X* is compound Poisson, then ν(dx) = λF(dx) (Tankov, 2007).

Since $(Z_t)_{t\geq 1}=(\ln V_t)_{t\geq 1}$ has a double exponential distribution then the Levy measure for DEJD will be as given in Equation 12.

$$\nu(x)=\lambda\left(p\eta_1e^{-\eta_1 y}1_{\{y\geq 0\}}+q\eta_2e^{\eta_2 y}1_{\{y<0\}}\right) \tag{12}$$

As a result, the characteristic function of DEJD can be derived as given in Equation 13 and Equation14.

$$E\left[e^{iuXt}\right] = \exp\left\{t\left(i\gamma u-\frac{\sigma^2u^2}{2}+\int_0^\infty(e^{iux}-1)p\eta_1e^{-\eta_1x}dx+\int_{-\infty}^0(e^{iux}-1)q\eta_2e^{\eta_2x}dx\right)\right\} \tag{13}$$

$$E\left[e^{iuXt}\right] = \exp\left\{t\left(i\gamma u-\frac{\sigma^2u^2}{2}+\lambda\left(\frac{p\eta_1}{\eta_1-iu}+\frac{q\eta_2}{\eta_2+iu}-1\right)\right)\right\} \tag{14}$$

2.2. Parameter Estimation

When calibrating the models, in parameter estimation stage, Maximum Likelihood Estimation (MLE) method, Empirical Characteristic Function (ECF) estimation method, The Cumulant Matching Method (CMM) and Generalized Method of Moments (GMM) can be used. Generally ML method is preferred because of its desirable properties such as consistency, asymptotical normality and asymptotical efficiency.

Stochastic diffusion asset pricing dynamics are continuous models. The maximum likelihood (ML) estimator for diffusion processes are based on discrete observations. Then, for maximum likelihood estimation of diffusion process transition density is needed to derive likelihood function. The main problem in maximum likelihood function is that the transition density of the process should be specified completely (Ramezani and Zeng, 2004). When the GBM market dynamic is assumed, the log-returns are normally distributed. Then, the maximum likelihood parameter estimation method can be directly used. In this paper, when GBM is considered for asset pricing, the maximum likelihood parameter estimation method is used to estimate the unknown parameters.

In 2004, Ramenzani and Zeng specified the transition density of DEJD model. However, it is not as tractable as the transition density of GBM. In addition, Honoré (1998) stated that in jump diffusion asset pricing models the log-return is equivalent to a discrete mixture of normally distributed variables, where N goes to infinity and for such cases the likelihood function can be unbounded which causes inconsistency. For this purpose, in this paper for DEJD asset pricing model calibration, Empirical Characteristic Function (ECF) estimation method, initiated in 1970's, reviewed by Yu in 2004 and applied for DEJD model by Cont and Tankov (2009), is used for Turkish stock market.

The basic idea of ECF estimation method is to match the characteristic function derived from the model and the empirical characteristic function obtained from the market data. It has desirable properties in such circumstances where the MLE approach encounters difficulties such as the likelihood function is not tractable but the characteristic function has a tractable expression (Yu, 2004).

In ECF estimation method the unknown parameters θ are found by minimizing the distance measure between the characteristic exponent and the empirical characteristic exponent given in Equation 15 (Cont and Tankov, 2009).

$$\int_{-K}^{K}\left|\psi_\theta(u)-\hat{\psi}(u)\right|^2 w(u)du \tag{15}$$

Here, $\hat{\psi}(u)$ is the empirical characteristic exponent given in Equation 16, $\psi_\theta(u)$ is the characteristic exponent which is given in Equation 17, $\theta=(\mu,\sigma,\lambda,p,\eta_1,\eta_2)$ is the unknown parameters vector and $w(u)$ is continuous weight function.

$$\hat{\psi}(u) = \frac{1}{t}\log\left(\frac{1}{N}\sum_{k=1}^{N} e^{iuX_k}\right) \tag{16}$$

$$\psi_\theta(u) = \left(i\gamma u - \frac{\sigma^2 u^2}{2} + \lambda\left(\frac{p\eta_1}{\eta_1 - iu} + \frac{q\eta_2}{\eta_2 + iu} - 1\right)\right) \tag{17}$$

Weight function basically matches the ECF and CF continuously over an interval (Yu, 2004). Depending on the choice of weighting function used, their estimator may be more or less efficient (Singleton, 2001). The weights should reflect the relative precision of as an estimate of $\psi_\theta(u)$ (Cont and Tankov, 2009). In this paper, as in the paper of Cont and Tankov (2009) by assuming the logarithmic return distribution is relatively close to Normal distribution the weight w is approximated as:

$$w(u) \approx \frac{e^{-\sigma_{est}{}^2 u^2}}{1 - e^{-\sigma_{est}{}^2 u^2}} \tag{18}$$

Here, σ_{est} is the Standard deviation of the empirical data. Cont and Tankov (2009) stated that the estimated parameter values are not very sensitive to K for K >50. After trying different choices K is taken as 60 as in Cont and Tankov (2009).

3. Data and Empirical Results

In this paper, to evaluate the applicability of Kou's DEJD model in Turkish stock market, daily closing prices of the ISE-100 index are used. The study concentrates on the period of January 2004-October 2015. This period covers the 2008 global economic and financial crisis. The global crisis began in the United States with the bursting of mortgage market that deteriorated the prices of mortgage back securities in the summer of 2007 and the effect of the dramatic financial crisis in US market rapidly spread to the rest of the world in 2008. With the bankruptcy of Lehman Brothers in September 2008 sharpe declines are observed in global financial markets.

In Figure 1 the effect of global crisis for Turkish stock market can be observed. It is seen that the decline effect of crisis starts at the beginning of 2008 till the end of year. Financial crisis may create a big volatility in prices of financial assets. Therefore, to see the performance of DEJD model exactly the data is divided into three periods, namely, the pre-crisis period from January 2004, to December 2007; the crisis period, which began on January 2008, and ended on December 2008; and the post-crisis period, which began on January 2009 till the end of October 2015.

Figure 1 ISE 100 closing prices between January 2004-October 2015

In parameter estimation stage, the distributions of log-returns is needed so they are calculated. Table 1 shows the descriptive statistics and Kolmogorov Simirnov (KS) normality test results of logarithmic returns. The Turkish stock market faced negative average returns during crisis period, while pre and post crisis periods the average returns are positive. The standard deviation of ISE 100 index increased during crisis period which means volatility that measures the risk is higher in this period. As seen from the Table 1 the distribution of log-returns show positive or negative skewness values which means flatter tails than the normal distribution. The kurtosis statistics are deviated from 3. The skewness and kurtosis parameters give an insight about the distribution of log-returns but to be able to speak more precisely about normality KS test results are needed. KS test results show that the normality of logarithmic returns are rejected at %5 significance level in all periods.

Table1 Descriptive statistics and Normality test results of log-returns for each period

	Pre-Crisis	**Crisis**	**Post-Crisis**
N	1003	250	1716
Mean	0.001	-0.0028	0.0006
Std.Dev.	0.01735	0.0273	0.0154
Skewness	-0.285	0.249	-0.426
Kurtosis	1.139	2.304	3.203
KS Test Statistic	0.034	0.06	0.056
KS Test P-value	0.007	0.029	0

In application part, firstly the unknown parameters are estimated for GBM and DEJD model by using the log-return series for each sub-periods. The estimated model parameters are given in Table 2. The expected return and volatility parameters are close to each other for both models and consistent with the descriptive statistics given in Table 1. The other four parameters intensity parameter, probability of upward jumps and upward and downward jump distributional parameters are special parameters for DEJD model.

Table 2 Estimated parameters for GBM and DEJD for each periods

	Pre-Crisis		**Crisis**		**Post-Crisis**	
Parameters	**GBM**	**DEJD**	**GBM**	**DEJD**	**GBM**	**DEJD**
μ	0.0010	0.0011	-0.0029	-0.0023	0.0006	0.0006
σ	0.0173	0.0155	0.0272	0.0238	0.0154	0.0111
λ	-	0.5046	-	0.2292	-	0.5448
p	-	0.7738	-	0.7626	-	0.5948
η_1	-	268.5313	-	759.4995	-	120.3681
η_2	-	68.9586	-	29.9183	-	80.6244

After the parameter estimation step, with the estimated parameters by GBM and DEJD models the log-returns are simulated for ISE 100 index for each periods and results are compared with the actual returns.

Figure 2, Figure 3 and Figure 4 show the actual returns and simulated returns for each periods respectively. As an emerging market in actual return data there are upward and downward jumps during time periods. However, the GBM simulations have deviations around average returns as expected because of its nature. DEJD simulations are more consistent with the empirical data. In DEJD trajectories there are upward and downward jumps but as the time of jumps are random and as it is just a simulation study, perfect capturing is not satisfied.

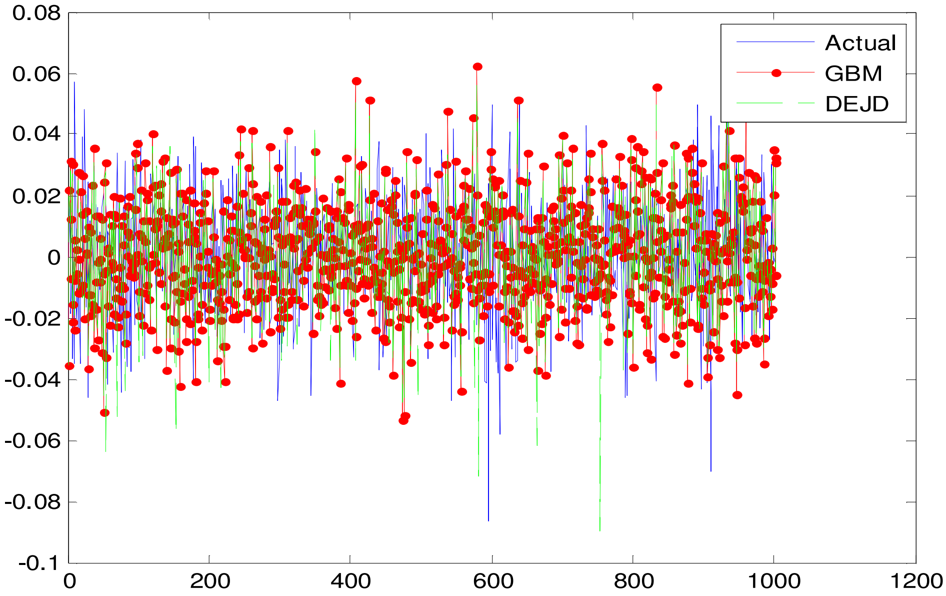

Figure 2 Actual and simulated log-returns in pre-crisis period

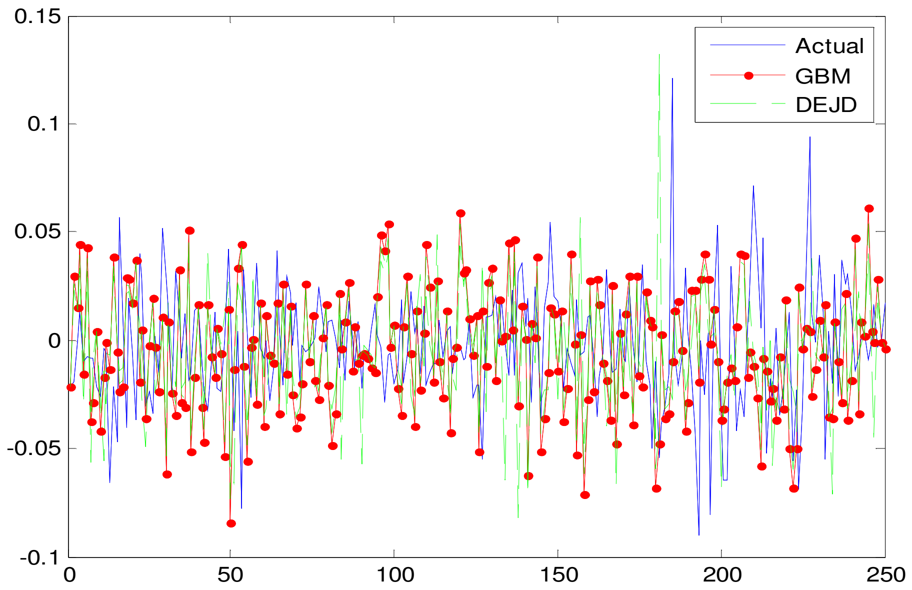

Figure 3 Actual and simulated log-returns in crisis period

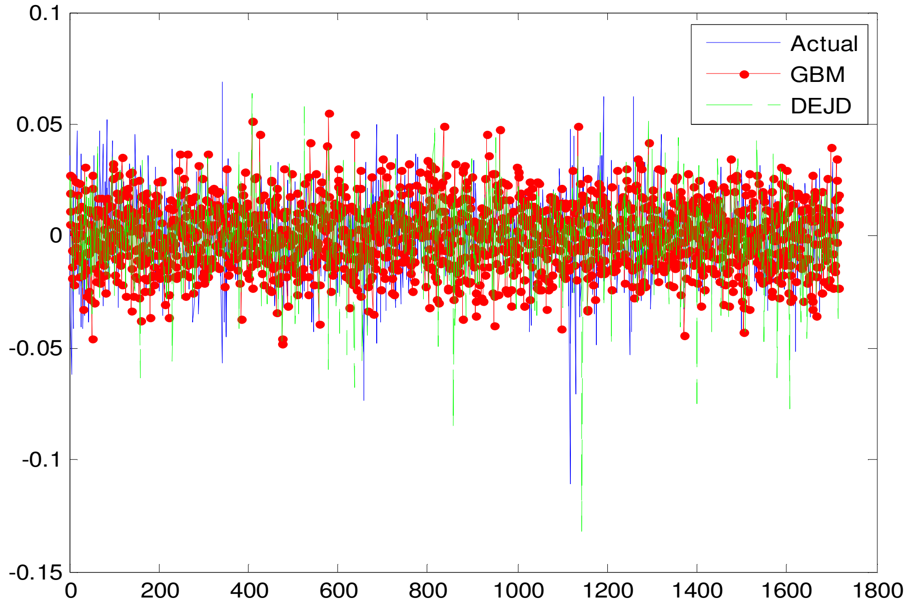

Figure 4 Actual and simulated log-returns in post-crisis period

In addition to the figures, the mean square errors (MSE) of both models that show the average deviation from actual returns are estimated for each period. The MSE results are nearly the same for both models. In pre-crisis and crisis period the MSE of DEJD model is a little bit smaller than the GBM but in post-crisis period the MSE of GBM is smaller. For each period the average deviations from actual returns are small enough.

Table 4 MSE results for GBM and DEJD for each periods

	Pre-Crisis		**Crisis**		**Post-Crisis**	
MSE	**GBM**	**DEJD**	**GBM**	**DEJD**	**GBM**	**DEJD**
	0.00062	0.00061	0.00170	0.00150	0.00045	0.00046

To evaluate the distributions of actual and simulated logarithmic returns the histograms are also plotted. Figure 5, Figure 6 and Figure 7 show the histogram of actual and simulated returns for each periods respectively. As seen from the histograms DEJD model perform to capture the leptokurtic property of logarithmic returns. In continuous time, option pricing models and portfolio optimization problems a diffusion model assumption is needed. For Turkish stock market DEJD model seem to be a better choice than the GBM.

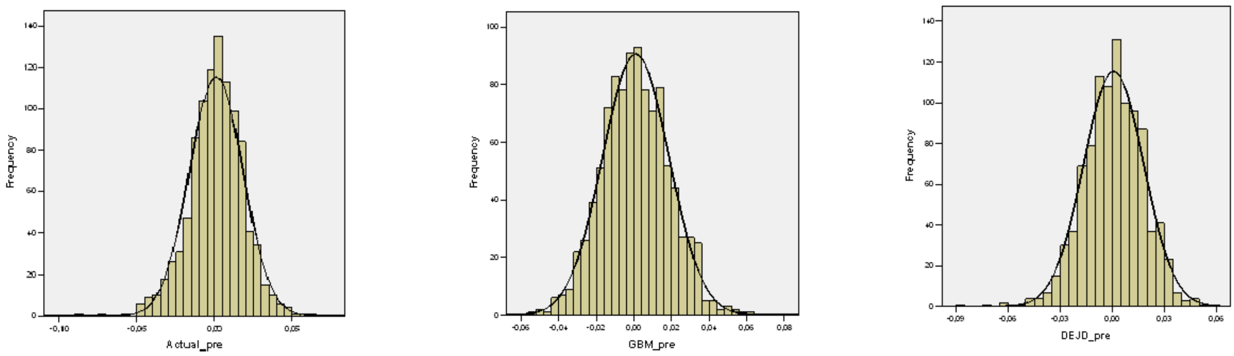

Figure 5 Histograms of actual and simulated log-returns in pre-crisis period

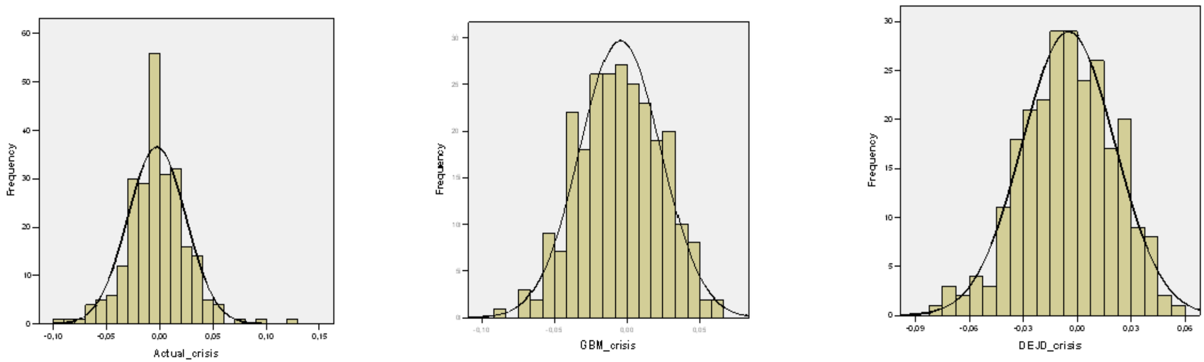

Figure 6 Histograms of actual and simulated log-returns in crisis period

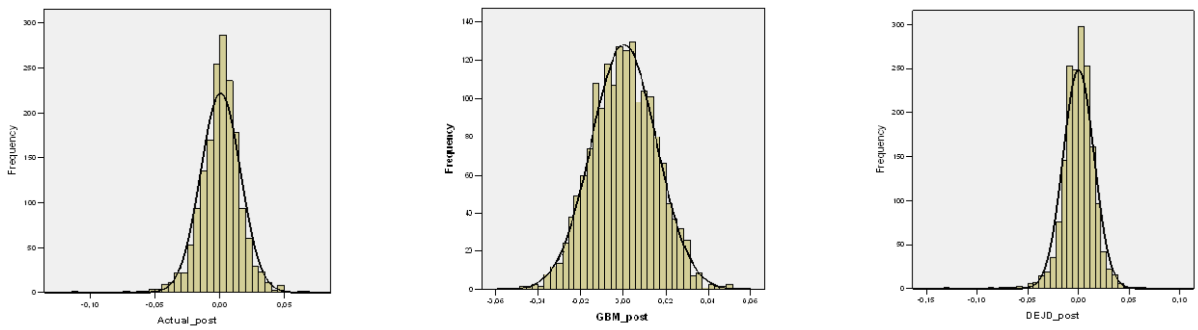

Figure 7 Histograms of actual and simulated log-returns in post-crisis period

4. Conclusion

This paper presented an empirical assessment of the GBM and DEJD stochastic models for the Turkish stock market ISE100 index. The parameter estimates are provided via MLE and ECF methods. With the estimated parameters the simulation study is conducted under both models. The performances are assessed with returns graphs, MSE results and histograms.

As an emerging market, in actual return data there are upward and downward jumps. DEJD is successful to model the jumps caused by the overreaction or under reaction to outside news. The MSE results are nearly the same for both models but DEJD model was performed to capture the leptokurtic property of logarithmic returns. As, a result, it is found that DEJD was superior to GBM.

There are a number of other interesting directions for future extensions of this work. The performance of DEJD model can be evaluated for the individual stock returns. In this paper, constant parameters are used for simplicity in parameter estimation procedure, though other extensions of DEJD model can also be applied for the emerging stock markets. In addition, the European Call option pricing formula using the DEJD model as an underlying model can be evaluated for the Turkish derivative market.

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