Variance–Covariance (Delta Normal) Approach of VaR Models: An Example From Istanbul Stock Exchange

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Abstract
Many investors desire to know how much money they can lose for example in a day or in a ten days. In this study, variance-covariance approach of the VaR models is introduced to the reader. It estimates maximum potential loss for a given probability and time horizon. It shows money type one loss value. In a calculation process, firstly, portfolios are created. Then, returns distribution is identified. And lastly, VaR values of portfolios are measured. Daily loss is calculated with using 252 days historical data belonging to the year 2015. Stocks are chosen from Istanbul Stock Exchange (BIST 100 Index). Calculation is made for both 95 % and 99 % confidence level and one day and ten days holding periods.
Keywords: Risk Measurement, VaR, Variance-Covariance approach, correlation, portfolio risk

1. Introduction

Risk and return are the main parameters of all investment process. Finance theory accepts that investors are risk averse and utility maximizer. In that sense, risk management appears as the most critical field of investment evaluations. Many financial institutions and regulatory bodies also give more importance to risk measurement after global financial crisis.

There are many ways of calculating market risk consisting of interest rate, exchange rate and etc. Value at Risk models (VaR) have been applied since 1994. VaR models only estimate quantifiable risk and they are not appropriate to measure of political and regulatory risks. Models measure probable maximum loss of portfolio for given confidence level and holding periods. In other ways, investors have a chance to know their potential loss in a reasonable bound to use VaR models. Different VaR models show different results. They have many pros and cons both. They can be classified as parametric and non–parametric models. As a parametric one, variance–covariance approach (also known as delta normal) is widely used in financial world. It is very practical and easy to use. It depends on correlation and covariance matrices to estimate variance and standard devaiaton of risky asset portfolio. However, returns using in the analysis should have a characteristics of normal distribution.

The aim of this paper is briefly to describe Variance-Covariance approach of VaR methods and to measure maximum loss portfolios consisting of different stocks. The rest of the paper is organised as follows. In section 2, VaR models are briefly introduced. In section 3, variance-covariance approach is used in empirical studies to measure loss value. Many statistical techniques are used in this section. Finally, section 4 concludes the paper.

2. Variance-Covariance Approach of VaR Methods

All investors desire to minimize their risks on investments while maximizing returns. Thus, prediction of risk is a key input in all investment decisions. The financier will invest in any kind of asset only if the expected return will be higher than the perceived cost. Generally, investors face a trade off situation in which a large but bad investment may result in huge loss while a good but small investment may result in opportunuity cost. In that sense, risk management is a necessary effort to maximize the portfolios return and to minimize losses. Value at Risk (VaR) is a widely used as a risk measurement method calculating the worst case losses over a predetermined time period and at a predefined confidence level (Johansson, 2013: 1). In 1994, VaR is firstly applied by J. P. Morgan creating CreditMetrics methodology, RiskMetrics and RAROC models. The model was to be appropriated and applied by many companies (Aniunas et al, 2009: 19). While regulatory groups have been widely promoting it as a basis for setting regulatory minimum capital standards, many financial institutions have been developed its derivates internally as a way of monitoring and managing market risk (Darbha, 2001: 2). Basel Committee of Banking Supervision, USA Federal Reserve System and USA Stock Committee in 1995, European Union Capital Requirements Directive in 1996 proposed to use value at risk method as one for market risk management (Aniunas, 2009: 19).

There are three main assumptions of VaR models. One of them is stationary requirement meaning that daily fluctuations of returns are independent from yesterday's or tomorrow's return. It is related with the random walk theory of finance. The second assumption is known as non-negativity requirement meaning that financial assets do not have negative values. The third assumption is related with distribution of financial data. VaR model assumes that financial historical data are distributed normally (Allen et all, 2004: 8 – 9). VaR methods are generally classified two main groups as parametric which is also called variance-covariance (or delta normal) approach and non-parametric method consisting of two simulation methods which are called historical simulation and Monte

Carlo simulation. There are both pros and cons of these methodologies (Bozkaya, 2013: 22). In this study, variance–covariance approach is preferred to calculate portfolio loss. Speed and simplicity are the main two advantages of this method. Moreover, distribution of returns need not be assumed to be stationary through time, since volatility updating is incorporated into the parameter estimation (Bohdalova, 2007: 2–3). However, In this approach, only linear risk is measured and correlations are assumed as stable. As an other drawbacks, it heavily relies on normal distribution and and returns in the market are widely believed to have “fatter tails” than a true to normal distribution[1].

3. Empirical Analysis

3.1. Data and Formulas

Historical data is usually used by VaR models to calculate maximum (worst case) losses over a certain holding period at a given confidence interval. In that sense, (holding) time period and a confidence level are the main parameters of measurement. Models express losses as one term and dollar values. The result shows us that losses will not be exceeded by the end of the time period with the specified confidence level (Darbha, 2001: 2).

In this study, two hypothetical portfolios are created at first. They have same three companies’ stocks but different weights. These three companies have traded in Istanbul Stock Exchange (BIST). They are operating in a petroleum industry. While TUPRS is operating in refinery field and PETKIM is in petrochemistry field, AYGAZ is an LPG company. Moreover, TUPRS and AYGAZ depend on same owner but they are managed separately.

Table 1. Name and Activities of Companies

	Stocks	Company Name	Business Activity
1	AYGAZ	Aygaz A.Ş.	LPG
2	PETKIM	Petkim PetroKimya Holding A.Ş.	Petrochemistry
3	TUPRS	Türkiye Petrol Rafineleri A.Ş.	Refinery

In this study, market return and beta of companies are estimated by using daily returns (adjusted price for US dollar). They are achieved from the Isyatirim database[2]. One year period (252 working days) data is used. It belongs to a year of 2015. Morever, BIST 100 index is used as a market index.

To calculate for stocks daily return; the formula is applied as follows:

$$R_i = \frac{\mathrm{R_{it}} - R_{it-1}}{\mathrm{R_{it-1}}} \qquad (1)$$

where “Ri” is a daily return of share i, “Rit” is a closing price of share i in t date and “Rit-1” is a closing price of share i in t - 1 date

To calculate the Index (BIST 100) daily return; the formula is applied as follows:

$$R_{Bist100} = \frac{\mathrm{Bist100_t} - \mathrm{Bist100_{t-1}}}{\mathrm{Bist100_{t-1}}} \qquad (2)$$

Where “$R_{Bist100}$” is a average return for market, “$Bist100_t$” is a market return in t date, “$Bist100_{t-1}$” is a market return in t-1 date.

To calculate variance of stocks daily return and index return, I used the following historical volatility formula:

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Ri - Raverage)^2 \qquad (3)$$

Where “σ^2” is a variance of daily share return, “R_i” is a daily return of share i, “$R_{average}$” is average daily return, “n” is a sample size (252 days)

To measure how stocks vary together, standard formula for covariance can be used:

$$\text{Cov (X,Y)} = \frac{1}{n-1}\sum_{i=1}^{n}[(X_i - \bar{X}).(Y_i - \bar{Y})] \qquad (4)$$

where the sum of the distance of each value X and Y from the mean is divided by the number of observations minus one. The covariance calculation enables us to calculate the correlation coefficient, shown as:

$$\text{Correlation Coefficient} = \frac{Cov(X,Y)}{\sigma_X . \sigma_Y} \qquad (5)$$

where σ the standard deviation of each asset. However, if there are more than two financial assets in the portfolio, then correlation and covariance matrices are needed to solve equations.

To calculate standard deviation of portfolio (position), the following formula is applied:

[1] http://www.yieldcurve.com/mktresearch/learningcurve/learningcurve3.pdf
[2] http://www.isyatirim.com.tr/LT_isadata2.aspx. (01.02.2016)

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(w_i^2.\sigma_i^2) + 2(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i.\sigma_i.w_j.\sigma_J.\rho ij)} \quad (6)$$

Where "σ_p" is a standard deviation of portfolio, "σ_i" is a standard deviation of stocks, "w_i" is a weight of stocks in a portfolio and "ρ_{ij}" is a correlation coefficient between stocks i and j.

3.2. Empirical Results

In this study, excel functions and data solver are used for all calculation. The calculation of variance- covariance model involves the following steps:

Step 1 – Determining Holding Period and Confidence Level (table 2)
Step 2 – Determining Portfolio (table 3 and table 4)
Step 3 – Creating a Probability Distribution (table 5)
Step 4 – Determining Correlations between Assets (table 6 and table 7)
Step 5 - Calculating the Volatility of the Portfolio (table 8)
Step 6 - Calculating the VaR Estimate (table 9)

Table 2. Main Parameters of Calculations

Parameter	Value
Confidence level	%95 and 99 %
Time Horizon	1 day and 10 days
Size of historical data	252 days
Testing period	02.01.2015 – 31.12.2015

A number and closing price of stocks in Porfolio 1 and Portfolio 2 are given below. They have same stocks but different weights. (31.12.2005):

Table 3. Distribution of stocks in Portfolio 1

Stocks	Number of Stocks (1)	Closing Price (USD) (2)	Market Value of Stocks (USD) (3) (3) = (1) x (2)	Weights (%)
AYGAZ	10.350	3,45	35.707	33,3
PETKIM	22.770	1,57	35.750	33,3
TUPRS	1.500	23,81	35.715	33,3
Market Value of Portfolio = 107.172				

Table 4. Distribution of stocks in Portfolio 2

Stocks	Number of Stocks (1)	Closing Price (USD) (2)	Market Value of Stocks (USD) (3) (3) = (1) x (2)	Weights (%)
AYGAZ	15.532	3,45	53.586	50
PETKIM	20.478	1,57	32.151	30
TUPRS	858	23,81	21.435	20
Market Value of Portfolio = 107.172				

As seen from Table 2 and Table 3, total market value of portfolio equals to the sum of stocks' values. Statistical features of stocks and market index (BIST 100) are given below.

Table 5. Statistical Features of Returns

Stocks	BIST 100	AYGAZ	PETKIM	TUPRS
Standard Deviation	0,013962	0,01931703	0,019480289	0,021
Variance	0,000195	0,000373148	0,000379482	0,000441
Skewness	0,153806	0,045004118	-0,005816705	-0,02924
Kurtosis	1,392946	0,749816601	1,639189483	0,487461
Average	0,000792	0,000994821	0,000481375	0,000218
Minimum	-0,0512	-0,07178218	-0,086956522	-0,065
Maximum	0,053184	0,061971831	0,068181818	0,062305

Asymmetry is statistically measured by skewness evaluating how and which way returns are distributed around mean. It is used to determine whether data is normally distributed or not. For normal distribution, skewness takes zero value and returns are distributed around mean equally. It means that 50% percentage lies below and above the mean. Kurtosis quantifies how peaked is the distribution. Tails of distribution can be expressed by kurtosis results. In normal distribution, kurtosis must be three therefore if data is not normally distributed then a kurtosis values must be more than three (Bozkaya, 2013: 18). VaR models assume probability distribution is normal distribution however financial returns are not normally distributed but very close to normal distribution. As seen from Table 5, skewness and kurtosis results are reasonable to accept that returns are distributed normally. In that sense, VaR model's main assumption is provided by historical data.

Variance-covariance approach uses matrices giving chance to measure VaR value for a portfolio consisting of hundreds of assets. As seen from the formula (6), standard deviation of portfolio measurement requires correlations of each asset and also covariance between them. Using of variance covariance matrice is practical way of calculating standard deviation of portfolio. In this study, it is demonstrated how the parametric methodology uses variance and correlation matrices to calculate the variance, and hence standard deviation, of a portfolio[1].

Table 6: Corelation matrix of share's return

	AYGAZ	PETKIM	TUPRS
AYGAZ	1	0,619106	0,587354
PETKIM	0,619106	1	0,567169
TUPRS	0,587354	0,567169	1

The degree of dependence between two variables is measured by correlation identifying what percentage and direction two variables move together. Portfolio risk (volatility) is smaller than its individual assets' risks. In that sense, it is necessary to know the relation between assets to a portfolio variance. Correlation takes value between -1 and + 1 (Bozkaya, 2013: 16). As seen from Table 6, correlation between AYGAZ and PETKIM is higher than AYGAZ–TUPRS and PETKIM–TUPRS. However, both correlations are positive and high enough. This is not good for diversified portfolios.

The correlation coefficient can be calculated as using the covariance between the assets measuring how average value of two financial assets move together, how they vary together. Covariance helps financial manager to decide which assets are similar and move together and which move inverse (Bozkaya, 2013:16).

Table 7: Covariation matrix of share's return

	BIST 100	AYGAZ	PETKIM	TUPRS
BIST 100	0,000194	0,000181	0,000185	0,000203
AYGAZ	0,000181	0,000372	0,000232	0,000237
PETKIM	0,000185	0,000232	0,000378	0,000231
TUPRS	0,000203	0,000237	0,000231	0,000439

Covariance matrice helps us to calculate volatility of portfolios. Covariance values between stocks are multiplied by each shares' weights and then collected to find portfolio volatility.

Table 8. Standard Deviation and Variance of Portfolio 1 and Portfolio 2

	Portfolio 1	Portfolio 2
Standard Deviation	0,01694378	0,01691294
Variance	0,00028709	0,00028605

VaR is calculated as using the following formula:

$$VaR = P * \alpha * \sigma * \sqrt{t} \qquad (7)$$

Where "P" is value of portfolio (or position), "α" is confidence level, "σ" is a volatility of portfolio and "t" is a holding period. For 95 % confidence level, α is 1,65 and for 99 % confidence level, α is 2,33. VaR maybe calculated for different time length. In this study, both one day and ten days holding periods are taken account. If anyone tries to find VaR values more than one day such as 10 days, it needs to multiply daily volatility results by $\sqrt{10}$.

Table 9. VaR Values of Portfolio 1 and Portfolio 2

	Portfolio 1		Portfolio 2	
	95 %	99%	95 %	99%
VaR (one day-USD)	2978	4231	2990	4247
VaR (ten days-USD)	9410	13370	9447	13421

Market risk of any portfolio can be measured by VaR models. As seen from Table 9, confidence level 95 % and one day holding period, maximum loss of Portfolio 1 will not exceeded 2978 USD and maximum loss of Portfolio 2 will not exceeded 2990 USD. It means that there is only 5 % chance that the loss of next day will be greater than 2978 USD for portfolio 1 and 2990 USD for portfolio 2. With confidence level 95 % and ten days holding period, maximum loss of Portfolio 1 will not exceeded 9410 USD and maximum loss of Portfolio 2 will not exceeded 9447 USD. With confidence level 99 % and one day holding period, maximum loss of Portfolio 1

[1] http://www.yieldcurve.com/mktresearch/learningcurve/learningcurve3.pdf

will not exceeded 4231 USD and maximum loss of Portfolio 2 will not exceeded 4247 USD. It means that there is only 1 % chance that the loss of next day will be greater than 4231 USD for Portfolio 1 and 4247 USD for Portfolio 2.With confidence level 99 % and ten days holding period, maximum loss of Portfolio 1 will not exceeded 13370 USD and maximum loss of Portfolio 2 will not exceeded 13421 USD.

4. Conclusion

It is possible for investors to estimate probable loss value of their portfolios for different holding periods and confidence level. Variance–covariance approach helps us to measure portfolio risk if returns are distributed normally. In this study, two hypothetical portfolio to calculate potential loss with both 95% and 99% confidence level as well one day and ten days holding periods are created. As a main conclusion, there is no huge difference between Portfolio 1 and Portfolio 2 results. It is thought that the portfolio was not diversified well. There were only three stocks in the portfolio but importantly their correlations were not low enough to decrease risk adequately. Stocks have equal weights in Portfolio1 as 33,3 %. Stocks have different weights in Portfolio 2, consisting of 50% AYGAZ, 30 % PETKIM and 20 % TUPRS. AYGAZ and PETKIM have lower standart deviation than TUPRS. Although Portfolio 2 has a lower standart deviation than Portfolio 1, it gets greater VaR values for all categories. Because, AYGAZ has greater correlation coefficients comparing with other two stocks. Thus, while risk evaluation of one stock is related with especially volatility characteristics, risk evaluation of portfolio is related with correlation between risky assets inside the portfolio.

References

Allen, Linda, Jacob Boudoukh and Anthony Saunders (2004). *Understanding Market, Credit, and Operational Risk The Value At Risk Approach*, Blackwell Publishing.

Aniunas, Povilas , Jonas Nedzveckas , Rytis Krušinskas (2009). Variance – Covariance Risk Value Model for Currency Market, *Engineering Economics*, No 1 (61)
http://eis.ktu.lt/index.php/EE/article/viewFile/11572/6259

Bozkaya, Muhammet (2013). Comparison of Value At Risk Models and Forecasting Realized Volatility By Using Intraday Data An Empirical Study On American Stock Exchanges, Neoma Business School, Master Of Science In Finance, December, http:/www.jeanmonnet.org.tr/Portals/0/scholars_database_thesis/muhammet_bozkaya_tez.pd

Bohdalová, Mária (2007). A comparison of Value–at–Risk methods for measurement of the financial risk, *E-Leader*, Prague. http://www.g-casa.com/ /Bohdalova.

Darbha, Gangadhar (2001). Value-at-Risk for Fixed Income portfolios –A comparison of alternative models, *National Stock Exchange*, Mumbai, India December.
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.145.2623&rep=rep1&type=

Johansson, Andreas and Victor Sowa (2013). A comparison of GARCH models for VaR estimation in three different markets., Uppsala University Department of Statistics.
https://www.diva-portal.org/smash/get/diva2:630568/FULLTEXT01.