Determination of Optimal Portfolio by Using Tangency Portfolio and Sharpe Ratio

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Abstract
In this paper, I tested workability of mean- variance approach and Sharpe ratio on Istanbul Stock Exchange (BIST). 12 months of data belonging a year of 2015 are analyzed. My initial portfolio involves ten stocks with equal weights. They are chosen from different industries for diversification issue. Then, I followed Markowitz model to determine optimal portfolios. I created many portfolios for given expected return with minimum variance. They all are efficient portfolios. They are located over efficient frontier which shows maximum return with minimum variance. Which one is the best? The optimal one is selected by using tangency portfolio analysis and Sharpe ratio. It provides nearly three times more return comparing with a portfolio with equal shares of ten stocks.
Keywords: Sharpe ratio, mean-variance approach, tangency portfolio, optimal portfolio

1. Introduction

Investors are mainly investigating different financial assets to earn more money. However, probable earning is not independent from risk which is measured by volatility of returns. In that sense, individuals create a portfolio involving many investment tools with different weights. This strategy can be called diversification. Generally, small variance assets are chosen to create a portfolio. However, this traditional method did not take into account a relation between assets invested in portfolio. It is not enough to select small variance asset to decrease portfolio risk entirely because stocks may move together if we don't know covariance values among themselves.

Markowitz was the first man explaining an importance of relation between financial assets invested in portfolio. This new model is known as mean – variance approach. Under this model, portfolio selection is based by maximizing return for a given level of risk or minimizing risk for a given level of expected return. While all possible target return and risk coupling creates efficiency set, all minimum risk and maximum return combinations are called efficient frontier which shapes a curve. It is an out boundaries of efficiency set. An investors can choose any one of these portfolios to his/her risk preference. They are all efficient portfolios. However, the optimal portfolio is the one which gets maximum return for one unit of risk. It is an interception point of tangency portfolio and efficient frontier. This point is calculated by dividing a difference of expected return and risk free rate to standard deviation of portfolio. It is called Sharpe ratio and tangency portfolio maximize to it.

The aim of this paper is to show how optimal portfolio is selected by using mean-variance model and Sharpe ratio. The rest of the paper is proceed as follows. Markowitz model and Sharpe ratio are briefly introduced in section 2. In section 3, empiricial analysis is explained. Mean-variance model and Sharpe ratio are testing on Istanbul Stock Exchange (BIST). Finally, section 4 concludes the paper.

2. Optimal Portfolio Selection and Sharpe Ratio

A portfolio is simply a set of investment tools consisting of financial assets such as bonds, foreign exchange, stocks, gold and etc (Boamah, 2012: 14). An ultimate aim of most investors is to create optimal portfolio in order to satisfy their investment goals. However, the problem is what kind of portfolio must be choosen to get maximum return with related level of risk. Or conversely, which combination of assets would results a minimum risk given a level of return (Boamah, 2012: 17).

The selection of portfolios with the lowest risk (volatility) for a given level of return, or the highest return for a given level of risk was initiated firstly by Markowitz (1952, 1959). He dwelled on mean and variance of returns (Estrada, 2010: 2). In that sense, the model is called as the mean-variance efficient portfolio. It proposes many statistical methodologies used to create the optimal portfolio for maximizing return and minimizing risk (Lee and Su, 2014: 70).

There are many assumptions of Markowitz model summarized as follows (Vanini and Vignola, 2001: 6): (1) There are n risky assets and there is no risk free asset. Prices are not affected by individual investors accepted as price takers (2) Investment horizon is limited by a single time period (3) There is probability space (4) Transaction cost does not exist (5) Markets are liquid for all assets. (6) Assets are divided infinitely. (7) Full investment and no borrowing hold (8) The mean-variance criterion is only one factor for portfolios selection

Sharpe (1964), Lintner (1965), and Mossin (1966) complemented mean-variance model by arguing that, "given a risk-free rate, the optimal combination of risky assets is given by the market (or tangency) portfolio, which is the one that maximizes returns in excess of the risk-free rate per unit of volatility risk". This new insight is known as Sharpe ratio which has been used ever since as nearly standard criteria from investors and portfolio managers trying to maximize it for selecting the portfolio of risky assets. (Estrada, 2010: 2). This ratio can be

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calculated as expected return minus the return on the riskless asset to the standard deviation. The tangency portfolio is the portfolio that maximizes the Sharpe ratio (Elton and Gruber, 1997: 1746). Many authors have advocated this model for a long time. In that sense, the mean and variance of returns are used as the main tools of portfolio optimization. Investors and portfolio managers have applied this model and have tried to achieve the maximization of risk adjusted returns as the main rule for portfolio creation. Optimal portfolio is the one that provides the highest (excess) return per unit of risk measured by the Sharpe ratio (Estrada, 2010: 1). It shows the expected return per unit of standard deviation, so the portfolio with maximum Sharpe ratio gives the highest expected return per unit of risk, and is thus the most "risk-efficient" portfolio (Engels, 2004: 10).

3. Empirical Analysis

3.1. Data and Formulas

According to Markowitz, it is not adequate to invest in many securities which have small variance. Diversification is not achieved only by adding many assets in portfolios. Investors must avoid assets which have high covariances among themselves as well. Portfolio manager should seek different industries because generally if they have different economic characteristics, they have also lower covariances than firms within an industry (Markowitz, 1952: 89). In this study to create a hypothetical portfolio, ten securities are selected from Istanbul Stock Exchange (BIST) arbitrarily. However they are chosen from different industries for diversification issue. They are operating from cement, to tourism.

Table 1. Company Information

	Assets	Trade Name of Companies	Industry
1	BOLUC	Bolu Çimento A.Ş.	Cement
2	BMEKS	Bimeks Bilgi İşlem ve Dış Ticaret A.Ş.	Techno-Market
3	FENER	Fenerbahçe Sportif Hizmetler Sanayi ve Ticaret Anonim Şirketi	Sport
4	GEDIZ	Gediz Elektrik Dağıtım A.Ş.	Electricity
5	IDAS	İdaş İstanbul Döşeme Sanayi A.Ş.	Furniture
6	KERVT	Kerevitaş Gida Sanayi Ve Ticaret Anonim Şirketi	Food
7	ORGE	ORGE Enerji Sistemleri İnşaat Metal Ticaret ve Taahhüt ...Şirket	Energy systems
8	RAYSG	Ray Sigorta A.Ş.	Insurance
9	TBORG	Türk Tuborg Bira ve Malt San. A.Ş.	Beer
10	UTYPA	Utopya Turizm İnşaat İşletmecilik Ticaret A.Ş	Tourism

In this study, market return of companies are estimated by using monthly returns (adjusted price for US dollar). They are achieved from the Isyatirim database[1]. One year period (12 months) data is used. It belongs to a year of 2015. All formulas which are used in this study are belowed:

- daily return

$$R_i = \frac{R_{it} - R_{it-1}}{R_{it-1}} \qquad (1)$$

where "R_i" is a daily return of stock i, "R_{it}" is a closing price of stock i in t date and "R_{it-1}" is a closing price of stock i in t - 1 date

- average return

$$E(R_i) = \frac{1}{N} i \sum_{t=1}^{N} R_{it} \qquad (2)$$

Where "E (R_i)" is a average return for stock i, "R_{it}" is a market return in t date, "N" is a number of dates.

- expected return

$$E(r_P) = \sum_{i=1}^{n} w_i E(r_i) \qquad (3)$$

Where "$\Sigma w_i = 1$", "n" is the number of stocks, "w_i" is the proportion of the funds invested in stock *i*, "r_i, r_p" is the return on *i*th stock and portfolio *p*, and "E (R_p)"the expectation of the variable in the parentheses

- historical volatility

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (R_i - R_{average})^2 \qquad (4)$$

Where "σ^2" is a variance of daily stock return, "R_i" is a daily return of stock i, "$R_{average}$" is average daily return, "n" is a sample size (252 days)

- covariance

$$\text{Cov (X,Y)} = \frac{1}{n-1} \sum_{i=1}^{n} [(X_i - \bar{X}).(Y_i - \bar{Y})] \qquad (5)$$

[1] http://www.isyatirim.com.tr/LT_isadata2.aspx. (05.01.2016)

where "$\bar{x}$" is a mean of stock x and "$\bar{y}$" is a mean of stock Y and n is the number of observations

- correlation coefficient

$$\text{Correlation Coefficient} = \frac{Cov(X,Y)}{\sigma_X . \sigma_Y} \qquad (6)$$

where σ the standard deviation of each asset.

- standard deviation of portfolio

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(w_i^2.\sigma_i^2) + 2(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i.\sigma_i.w_j.\sigma_J.\rho ij)} \qquad (7)$$

Where "σ_p" is a standard deviation of portfolio, "σ_i" is a standard deviation of stocks, "w_i" is a weight of stocks in a portfolio and "ρ_{ij}" is a correlation coefficient between stocks i and j.

- Monthly risk free rate

R_f = [(1 + annual risk free rate /100) ^ (1/12) – 1]* 100 (8)

3.2. Empirical Results

The Markowitz mean-variance model could be summarized as follows: to calculate the expected return rates and the variance or standard deviation risk for each stock to be included in the portfolio first, then to calculate the covariance or correlation coefficients for all stocks, treating them as pairs (Boamah, 2012: 15). In this paper, all calculations are made by excel functions and data solver. Table 2, Table 3 and Table 4 are achieved by using formulas between 1 – 7. I used excel data solver for correlation and covariance matrices.

Table 2. Risk and return of stocks (montly-2015)

	BOLUC	BMEKS	FENER	GEDIZ	IDAS	KERVT	ORGE	RAYSG	TBORG	UTYPA
December	0,027473	0,060606	0,047133	0,277778	-0,5	-0,05489	0,210526	0,125	0,034188	0,138889
November	0,010695	0,014286	-0,02776	0,086957	0,166667	0,056354	-0,45963	-0,11111	-0,11157	0
October	-0,09524	-0,02817	-0,02623	0,12	-0,14286	-0,18998	-0,24138	-0,0625	-0,07442	-0,04878
September	0,02924	0,043478	0,049128	0,142857	0	0	0,030303	0,066667	0,065327	0,025641
August	0,039773	0,027778	0,126888	-0,125	0	0,033602	-0,17647	0,125	0,089623	0,05
July	0	0,054054	-0,00469	-0,03571	0,166667	0,522757	0,089286	0,055556	-0,14719	0,095238
June	0,147541	0,025641	-0,1367	0,222222	0,142857	-0,37489	0,081967	0,052632	0,005076	0,043478
May	-0,00476	-0,0375	0,198908	0,090909	0,125	0,161885	0,075758	0,1	-0,17677	-0,04167
April	0,019139	0,012987	-0,11646	0,027778	-0,11111	-0,08818	-0,08451	-0,04545	-0,08589	-0,02174
Marc	0,070423	0,166667	0,357143	0,162162	0,25	0,132818	0,169231	0,142857	0,020134	0,133333
February	0,114035	0,010989	-0,23169	0,162791	-0,1	0,319294	0,065789	0,125	0,032895	0,078431
January	-0,08661	-0,02174	0,016243	0,26	0	-0,0522	0,938272	0,185185	0,082803	0,090909
Average	0,022642	0,027423	0,020993	0,116062	-0,00023	0,03888	0,058262	0,063236	-0,02215	0,045311
Variance	0,004895	0,002894	0,024567	0,013964	0,040622	0,054126	0,113278	0,008383	0,008566	0,004141
Std Dev	0,069966	0,053796	0,156738	0,118169	0,201549	0,232651	0,336568	0,091557	0,092552	0,064352

Table 3. Correlation matrice

	BOLUC	BMEKS	FENER	GEDIZ	IDAS	KERVT	ORGE	RAYSG	TBORG	UTYPA
BOLUC	1									
BMEKS	0,976239	1								
FENER	0,84313	0,909543	1							
GEDIZ	0,916447	0,925564	0,808553	1						
IDAS	0,821832	0,825376	0,814708	0,702346	1					
KERVT	0,774108	0,802015	0,752102	0,65681	0,749999	1				
ORGE	0,630742	0,667304	0,651165	0,761295	0,542585	0,558972	1			
RAYSG	0,944863	0,957084	0,893162	0,927016	0,773263	0,796632	0,800959	1		
TBORG	0,940474	0,94744	0,848251	0,916623	0,74124	0,707428	0,747284	0,957703	1	
UTYPA	0,96699	0,98788	0,882764	0,941016	0,785637	0,810296	0,738288	0,978096	0,959307	1

Table 4. Covariation matrice

	BOLUC	BMEKS	FENER	GEDIZ	IDAS	KERVT	ORGE	RAYSG	TBORG	UTYPA
BOLUC	0,004487	0,001542	-0,00181	0,000429	0,002189	3,96E-05	-0,00415	0,001053	0,001356	0,001295
BMEKS	0,001542	0,002653	0,00382	0,00026	0,001941	0,002431	0,000146	0,001207	0,001233	0,00217
FENER	-0,00181	0,00382	0,02252	-0,00179	0,008797	0,004932	0,005596	0,004718	0,000215	0,001899
GEDIZ	0,000429	0,00026	-0,00179	0,0128	-0,00691	-0,01011	0,019705	0,002716	0,00316	0,002244
IDAS	0,002189	0,001941	0,008797	-0,00691	0,037237	0,010747	-0,0055	-0,0012	-0,00498	-0,00146
KERVT	3,96E-05	0,002431	0,004932	-0,01011	0,010747	0,049616	0,000455	0,004348	-0,00598	0,004071
ORGE	-0,00415	0,000146	0,005596	0,019705	-0,0055	0,000455	0,103838	0,021256	0,011875	0,010421
RAYSG	0,001053	0,001207	0,004718	0,002716	-0,0012	0,004348	0,021256	0,007684	0,004527	0,003673
TBORG	0,001356	0,001233	0,000215	0,00316	-0,00498	-0,00598	0,011875	0,004527	0,007852	0,0029
UTYPA	0,001295	0,00217	0,001899	0,002244	-0,00146	0,004071	0,010421	0,003673	0,0029	0,003796

As seen from Table 5, my original portfolio is compromised of 10 stocks with equal weights. I calculated return of this original portfolio is as % 3,7 and risk of this portfolio is as % 6,8.

Table 5. Variance-covariation matrice with equal weight

A	B	C	D	E	E	G	H	I	J	K	L
1		BOLUC	BMEKS	FENER	GEDIZ	IDAS	KERVT	ORGE	RAYSG	TBORG	UTYPA
2	1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
3	0,1	0,004487	0,001542	-0,00181	0,000429	0,002189	3,96E-05	-0,00415	0,001053	0,001356	0,001295
4	0,1	0,001542	0,002653	0,00382	0,00026	0,001941	0,002431	0,000146	0,001207	0,001233	0,00217
5	0,1	-0,00181	0,00382	0,02252	-0,00179	0,008797	0,004932	0,005596	0,004718	0,000215	0,001899
6	0,1	0,000429	0,00026	-0,00179	0,0128	-0,00691	-0,01011	0,019705	0,002716	0,00316	0,002244
7	0,1	0,002189	0,001941	0,008797	-0,00691	0,037237	0,010747	-0,0055	-0,0012	-0,00498	-0,00146
8	0,1	3,96E-05	0,002431	0,004932	-0,01011	0,010747	0,049616	0,000455	0,004348	-0,00598	0,004071
9	0,1	-0,00415	0,000146	0,005596	0,019705	-0,0055	0,000455	0,103838	0,021256	0,011875	0,010421
10	0,1	0,001053	0,001207	0,004718	0,002716	-0,0012	0,004348	0,021256	0,007684	0,004527	0,003673
11	0,1	0,001356	0,001233	0,000215	0,00316	-0,00498	-0,00598	0,011875	0,004527	0,007852	0,0029
12	0,1	0,001295	0,00217	0,001899	0,002244	-0,00146	0,004071	0,010421	0,003673	0,0029	0,003796
13		0,004487	0,001542	-0,00181	0,000429	0,002189	3,96E-05	-0,00415	0,001053	0,001356	0,001295
14		Monthly									
15	Variance of portfolio = 0,004634427										
16	Standard deviation of portfolio = 0,0680766										
17	Return of portfolio = 0,03704										

In Markowitz portfolio analysis, two main aims are tried to achieve by maximizing return at given risk or minimizing risk at given return. In this paper, I calculated tangency portfolio at first using formula 9 as objection function. Then I calculated minimum variance portfolios which are shown in Figure 1 and Table 10. The minimum variance portfolios are achieved by using subject function in formula 10.

The subject function of tangency portfolio can be showed as belowed:

$$\text{Max } \alpha = (E_p - R_f) / \sigma_p \qquad (9)$$

Where "Ep" is expected return of portfolio and "R_f" is risk free rate and $\boldsymbol{\sigma}_p$ is standard deviation of portfolio.

Markowitz minimum variance subject function formula is as followed;

$$\text{Min } \sigma^2 \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j Cov_{ij} \qquad (10)$$

Where "W_i" and "W_j" are weights of stocks in the portfolio and "Cov_{ij}" is covariance value between stocks i and j.

I used three main constraints in my work which are nearly standard in Markowitz mean-variance model. They are written as

$$\sum_{i=1}^{n} W_i \, E\,(R_i) \geq E^* \qquad (11)$$

where E^* is the target expected return, E (R_i) is an expected return and "W_i" is a weight of each stock.

$$\sum_{i=1}^{n} W_i = 1.0 \qquad (12)$$

$$W_i \geq 0, \qquad i = 1, \ldots\ldots\ldots\ldots, N \qquad (13)$$

While the first constraint (formula 11) is meaning equality between expected return and target return, the second one (formula 12) is showing sum of all stocks weight must be one. The third constraint (formula 13) involves short sale condition which means that none of stocks can not be sale if it doesn't exist in portfolio.

Table 6. Parameters of Excel Solver

Target cell	F 18 (tangency portfolio)
Equal to	Maximum (tangency portfolio)
By changing cells	\$B\$3 : \$B\$12
Constraints	\$C\$2 : \$L\$2 ≥ 0 (short sale restrictions)
	\$B\$2 = 1
	\$F\$17 = 0,04 (target return)

In this study, all these three constraints are identified by an excel data solver. Tangency portfolio is achieved by excel data solver functions. For minimization problem, target cell must be F 15 (variation of portfolios) and "equal to" cell must be minimum. In that sense, maximization and minimization problem must be solved by separately.

Table 7. Variance-covariation matrice with different weights

A	B	C	D	E	F	G	H	I	J	K	L
1		BOLUC	BMEKS	FENER	GEDIZ	IDAS	KERVT	ORGE	RAYSG	TBORG	UTYP
2	1	0,191	0,397	0,010	0,205	0,043	0,059	0,000	0,007	0,088	0,0
3	0,191	0,00016	0,00012	0,00000	0,00002	0,00002	0,00000	0,0	0,00	0,00002	0,00
4	0,397	0,00012	0,00042	0,00002	0,00002	0,00003	0,00006	0,0	0,00	0,00004	0,00
5	0,010	0,00000	0,00002	0,00000	0,00000	0,00000	0,00000	0,0	0,00	0,00000	0,00
6	0,205	0,00002	0,00002	0,00000	0,00054	-0,00006	-0,00012	0,0	0,00	0,00006	0,00
7	0,043	0,00002	0,00003	0,00000	-0,00006	0,00007	0,00003	0,0	0,00	-0,00002	0,00
8	0,059	0,00000	0,00006	0,00000	-0,00012	0,00003	0,00017	0,0	0,00	-0,00003	0,00
9	0,000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,0	0,00	0,00000	0,00
10	0,007	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,0	0,00	0,00000	0,00
11	0,088	0,00002	0,00004	0,00000	0,00006	-0,00002	-0,00003	0,0	0,00	0,00006	0,00
12	0,000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,0	0,00	0,00000	0,00
13			Monthly								
14			Risk free rate = 0,019[1]								
15	Variance of portfolio = 0,001842355										
16	Standard deviation of portfolio = 0,0429227										
17	Return of portfolio = 0,04										
18	Tangency Portfolio =0,4892520										

Tangency portfolio is achieved by using maximum subject function [(F 17 – F14) / F16]. It is possible to create many portfolio for a given target return. My new portfolio contains only eight securities with different weights % 19 BOLUC, % 39,7 BMEKS, % 1 FENER, % 20,5 GEDIZ, %4,3 IDAS, %5,9 KERVT, %0,7 RAYSG and %8,8 TBORG stocks. Under different target return scenarios, I created five portfolios which is shown in Table 8:

Table 8. New portfolios

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5
BOLUC	0,186330077	0,192446873	0,190973232	0,170240779	0,12370756
BMEKS	0,377536545	0,379676914	0,396513261	0,423807478	0,32829172
FENER	0,006520379	0,010725405	0,009982215	0	0
GEDIZ	0,076448227	0,143177896	0,204956261	0,237539518	0,34186755
IDAS	0,057845203	0,050168118	0,043136243	0,039228699	0,04332652
KERVT	0,048710056	0,055002632	0,058957911	0,049510316	0,07253435
ORGE	0	0	0	0	0
RAYSG	0	0	0,007469362	0,079673212	0,0902723
TBORG	0,246609514	0,168802162	0,088011515	0	0
UTYPA	0	0	0	0	0
Std dev	0,044	0,043	0,04292	0,044	0,04828
Return	0,02	0,03	0,04	0,05	0,06
Tangency port	0,022756794	0,2581364	0,489252037	0,6970782	0,84913006

The out boundaries of efficiency set is called efficient frontier which shapes as a curve. An efficient portfolio is defined as the portfolio that maximizes the expected return for a given amount of risk (standard deviation), or the portfolio that minimizes the risk subject to a given expected return (Engels, 2004: 5). An investor can choose any one of portfolios located over efficient frontier under his/her risk preferences. However, optimal portfolio is the one which is an interception point of tangency portfolio and efficient frontier. It is a maximum return portfolio for one unit of risk. Table 9 shows all efficient portfolios. They are achieved by using minimization subject function in formula 10. To find optimal portfolio, I used Sharpe ratio. As seen from Table 9, Portfolio 8 is determined as optimal portfolio involving maximum Sharpe ratio. It is achieved by dividing risk premium to standard deviation of portfolio.

The formula of Sharpe Ratio is as belowed:

$$Sp = (R_p - R_f) / \boldsymbol{\sigma}_p \qquad (14)$$

Where "R_p" is an expected return of portfolio and "R_f" is risk free rate and $\boldsymbol{\sigma}_p$ is standard deviation of portfolio.

[1] Monthly risk free rate is calculated by using formula 8. Annual risk free rate was taken from a website: http://www.mahfiegilmez.com/2015/05/ekonomi-politikasyla-ilgili-faizler.html (05.01.2016).

Table 9. Determination of Optimal Portfolio

Portfolios	Standard Deviation	Return	Risk Free Rate	Risk Premium	Sharpe Ratio
Portfolio 1	0,0439416	0,02	0,019	-0,014	0,022757433
Portfolio 2	0,0426131	0,03	0,019	-0,009	0,258136366
Portfolio 3	0,0429223	0,04	0,019	0,001	0,489256596
Portfolio 4	0,0444713	0,05	0,019	0,011	0,69707879
Portfolio 5	0,0482847	0,06	0,019	0,021	0,849130263
Portfolio 6	0,0545331	0,07	0,019	0,031	0,935211675
Portfolio 7	0,0624903	0,08	0,019	0,041	0,976151243
Portfolio 8	***0,0715957***	***0,09***	***0,019***	***0,051***	***0,991678915***
Portfolio 9	0,0823403	0,1	0,019	0,061	0,983721274
Portfolio 10	0,0488366	0,005	0,019	0,071	-0,28666996
Portfolio 11	0,0467726	0,01	0,019	0,081	-0,19242004

"Graphically, the portfolio with maximum Sharpe ratio is the point where a line through the origin is tangent to the efficient frontier. That is why we call this the tangency portfolio" (Engels, 2004: 10). Optimal portfolio is seen from Figure 1. It is an interception point of efficient frontier and tangency portfolio line.

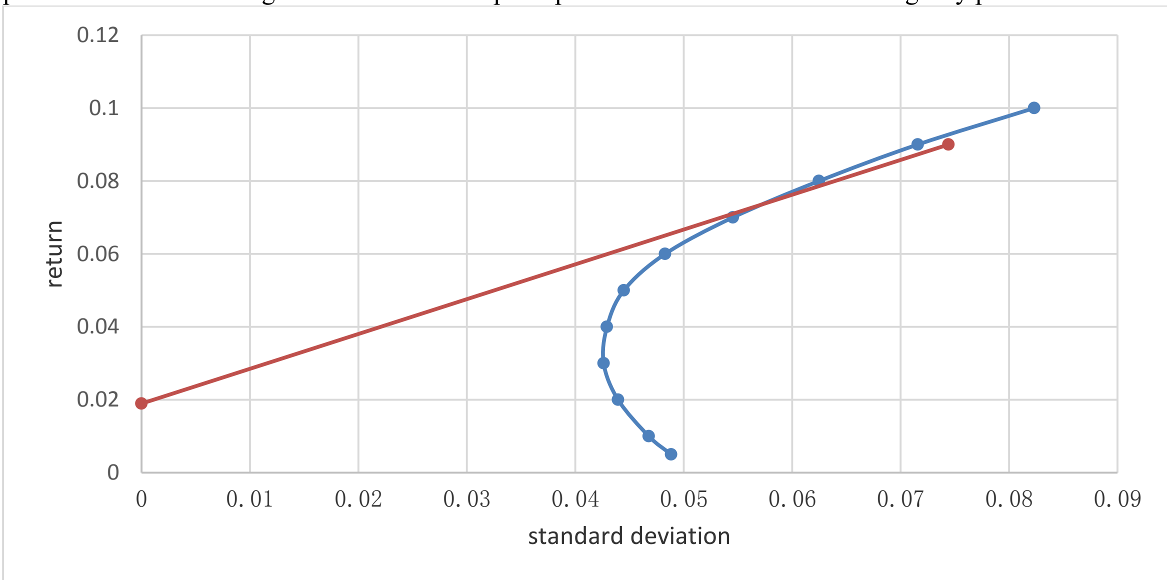

Figure 1. Efficient frontier and tangency portfolio

After determining optimal portfolio, it is very easy to compare an original (initial) and optimal portfolio values. As seen from Table 10, optimal portfolio monthly return is nearly three times more than the original one.

Table 10. Comparison of portfolio return

Portfolios	Monthly Return (%)	Annual Return (%)
Original Portfolio	0,03704	0,44448
Optimal Portfolio 8	0,09000	1,08

4. Conclusion

In this study, I examined mean – variance approach of Markowitz for optimal portfolio selection. First, I created hypothetical portfolio involving ten stocks with equal weights. They have been traded on Istanbul Stock Exchange (BIST). They are all from different industries for diversification issues. However it is not adequate to minimize risk of portfolio entirely. I followed Markowitz model to select optimal portfolio. I used both minimization and maximization subject functions separately. My new portfolio has eight stocks with different weights. I created many portfolios for a given expected return with minimum variance. However, which one of them is optimal ? I used Sharpe ratio and tangency portfolio analysis to determine it. They showed me the one which involves a highest return for one unit of risk. This optimal portfolio has nearly three times more return than the original one. It is an interception point of efficient frontier and tangency portfolio.

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