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Determining Whether the Geometric Brownian Motion Model is An Appropriate Model for Forecasting Stock Prices on the Ghana Stock Exchange

Dr. Isaac Junior Damptey (PhD)

Abstract
This study examined the appropriateness of the Geometric Brownian Motion model in forecasting stock prices on the Ghana stock exchange (GSE). Quantitative approach was employed to analyze and forecast secondary data sourced from the Ghana stock exchange (GSE). The target population was thirty-six (36) listed companies out of which top 10 market performers were purposely sampled for the study. The study used closing offer market price to analyze data from January 2008 to July 2015.The chi-square test was used to test the research hypothesis. Furthermore, the stock prices were simulated using the model in Microsoft excel, and R software. The findings showed that after simulating weekly stock prices, the true values of seven (7) listed companies out of the ten (10) does not lie in the confidence interval of the simulated values. The study also failed to accept the null hypothesis tested. Additionally, since all the MAPE values discovered were between 0% and 10%, it implies that the GBM model is a highly accurate model for forecasting stock prices on the Ghana Stock Exchange. Therefore, concludes based on the several tests conducted that the Geometric Brownian Motion (GBM) model is an appropriate model for forecasting stock prices on the Ghana stock exchange.
Key words: Geometric Brownian Motion, Forecast, Stock Prices

1. Introduction

The Geometric Brownian Motion (GBM) model has, since its development, been widely recognized as a fundamental model for the valuation of various financial assets such as stock prices years (Zhijun, 2015). Stock prices have been proved by the work of Bachelier in 1900 to have an erratic and random behavior, and move like a Brownian motion. Stock prices change over non-overlapping time intervals which are independent and identically distributed, with the variance of each price proportional to the length of time involved (Marathe & Ryan, 2005).

The Geometric Brownian motion, sometimes referred to as an exponential Brownian motion, is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion with a drift (Ross, 2014). This model, is relatively simple to comprehend and does not allow for negative values since it applies logarithm in finding the stock price.

A stochastic (random) process, S_t is said to follow a GBM if it satisfies the following stochastic differential equation (SDE):

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t \tag{1.1}$$

where W_t is a Brownian motion, dS_t is the change in the stock price in an infinitesimal time, dt is an infinitesimal change in time, μ is the percentage drift and σ is the percentage volatility. μ and σ are constants.

Recent findings have shown that the GBM is unable to capture some features including long range correlations and heavy-tailed distributions (Brigo et. Al, 2007). That notwithstanding, it remains the single most crucial model in financial modelling as no alternative has been proposed (Gadja & Wylomanska, 2012).

Ghana has a young stock market with immense possibility for growth. The Ghana stock exchange is the principal stock exchange of Ghana. Therefore the problem statement is, as a young market, Ghana's exposure to options trading can consequently lead to a boost in the economy. The Geometric Brownian motion is the underlying assumption for pricing options under the Black-Scholes framework and as such this paper aims at laying the foundation for pricing of such contracts in the economy.

The general objective of this study is to determine the appropriateness of the Geometric Brownian motion model in forecasting stock prices on the Ghana stock exchange market. The specific objectives are to stimulate stock values with 95% confidence interval and compare with real stock data.

Findings of this research will go a long way on assisting investors and analysts in obtaining a benchmark for possible future stock prices. Furthermore, this research will lay the foundation for the pricing of derivative products on the Ghanaian market, be a source of reference for students and other researches and also set the pace for further research in the area of derivatives and the pricing of financial assets in the Ghanaian economy.

Research Hypothesis

The hypotheses to be tested in this study are as follows:

$\mathbf{H_{1o}}$: The Geometric Brownian motion is not an appropriate model for forecasting stock prices on the Ghana stock exchange.

H_{1a}: The Geometric Brownian motion is an appropriate model for forecasting stock prices on the Ghana stock exchange.

The rest of the study is organized as follows: section 2 evaluates literature review on the concept of the Geometric Brownian Motion, its parameters and applications, as well as the relationship between the GBM and Black Scholes, the Ghana Stock Exchange and Stock pricing on the exchange. Section 3 details the methodology of this research work, followed by the analysis and findings in section 4, and then section 5 captures the conclusion.

2. Literature Review

Concept and Applications of the Geometric Brownian Motion (GBM) model

The Geometric Brownian motion is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion with a drift (Ross, 2014). Various researchers have used the GBM model as an underlying assumption to reach conclusions in various field. For instance, Marathe (2005) noted Thorsen (1965) in his research that the demand for services in rapidly growing industries followed a GBM process. Marathe also stated that Thorsen (1998) assumed that the future net prices of round wood, where the real options theory is applied to decisions of establishing a new forest stand, followed a GBM process.

This model, however, has been widely accepted as valid for modelling the growth of financial assets over time, specifically, stock prices such that, Hull (2000) refers to it as "the model for stock prices". Hull explains that aside the GBM process being relatively easy to comprehend, the expected returns of the GBM are not dependent on the value of the process, it assumes only non-negative values and shows an erratic path which reflect real stock prices. That notwithstanding, the GBM is not considered a completely realistic model. This is because in reality, stock price show jumps in its path and volatility changes over time but the model assumes otherwise.

Black-Scholes Model (BSM) and the GBM model

The primary model that was used for financial modelling was invented by Merton and Scholes in 1997. This model which led to a boom in options trading is widely used and adapted by many but often with adjustments by option market participants (Zvi, Kane, & Marcus, 2008). This model is known as the Black-Sholes model which proposes that the price of stock follow a stochastic process, implying that, the price of stock at any future time t is independent of stock price at time $t-1$.

The basic assumptions of the Black-Scholes model as below (Botoş & Ciumaş, 2012; DanJie et al.,2011);

- Stock prices follow a Geometric Brownian Motion with constant drift and volatility
- An underlying asset does not pay dividend
- There exists a frictionless market (thus, trading in the market does not incur any prices)
- The interest rate of an underlying asset is constant
- There is no arbitrage opportunity (that is, there is no way to make a riskless profit)

Ray (2012) explains the relationship between the BSM and the GBM as follows:

Consider a general derivative V, whose value is a function of the underlying security S. Considering the first assumption of the BSM model, assume that the asset follows a GBM process. Then S follows equation (1.1). Applying It$\bar{o}$'s lemma to (1.1), the equation results in

$$\partial \mathrm{V} = \frac{\partial V}{\partial S} dS + \left(\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}\right) dt$$
$$= \left(\mu S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}\right) dt + \sigma S W \frac{\partial V}{\partial S}\, dt$$

The above cannot be valued directly as it is a stochastic term. Hence, to eliminate the stochastic term, consider a portfolio $\pi = V - \frac{\partial V}{\partial S} S$. Then:

$$d\pi = dV - \frac{\partial V}{\partial S} dS = \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}\right) dt = r\pi dt = r\left(V - \frac{\partial V}{\partial S} S\right) dt$$

The above is not a stochastic term, π is a risk-free investment and therefore has the same return as any other risk-free investment. This follows from the no arbitrage condition. A simplification of the equation therefore yields the Black-Scholes equation as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \qquad (2.1)$$

The price of European put or call options can therefore be calculated from above by solving the equation for the corresponding terminal and boundary conditions.

Other Forecasting Methods for Stock Prices

Several researchers have used other methods to forecast stock prices. Majhi and Ansiah (2015), Sharma (2015), Deng et al. (2011) and Abbasi and Abouec (2008), all used an approach which involved several modifications to the basic ARIMA model in time series as well as other claims and varying assumptions in a quest to forecast stock prices. Kao et al. (2013), developed a novel model which first uses a nonlinear independent component analysis (NLICA) as pre-processing to extract the features from forecasting variables. These features were then used to build the forecasting model. Enke and Thawornwon (2005) used the neural network technique, a model which considers the non-linearity on the financial market movements in order to produce predictions of future stock prices.

Al-Radaideh et al. (2013) also made use of data mining through the use of decision tree to graphically represent all possible outcomes and paths by which future stock prices can be reached. They claimed that the data mining method will be able to point out associations and correlations, trend analysis as well as outlier and dviation analysis. Ying et al. (2005) also used an innovative statistical approach, known as the Hierarchail Bayesian approach to forecast stock prices.

Operational Framework of Stock Market

Capital market is classified into primary and secondary market, however most capital market transactions happen on the securities market. Normally, transactions on the securities market indirectly makes it easier for corporations to raise finance on the primary market, as investors understand if they need to get their money back urgently, they can easily re-sell their securities. On the primary market, each security is sold just one, and therefore the method of forming batches of new shares or bonds is usually prolonged as a result of regulative necessities whiles on the secondary markets, there is no limit on the amount of times a security will be listed, and therefore the method is normally fast (Spaulding, 2009, 2011; Chisholm, 2011).

A stock market may be a regulated market that provides services for stock brokers and traders to trade stocks, bonds, and different securities. More significantly, stock exchanges are part of a worldwide marketplace for securities. A stock market is the most vital element of a securities market but supply and demand in stock markets are driven by varied factors that, that, as in all free market affect the value of stocks as altogether free markets, have an effect on the value of stocks. The initial stock or bond offering to investors is by description done in the primary market and subsequent trading is done in the secondary market. There is normally no compulsion to issue stock through stock exchange but rather trading can also be done through off exchange or over the counter.

The stocks are listed and traded on stock exchanges which are entities of a corporation or mutual organization specialized in the business of bringing buyers and sellers of the organizations to a listing of stocks and securities together. Regular individuals account for a small proportion of trading, though their shares have slightly increased; only a few wealthy people could afford an account with the broker during the 20th century, but accounts are now much cost-effective and available over the internet. Several small traders are also able to use platforms provided by brokers via internet to buy and sell on the secondary market. Such trades involve a two-stage transaction: Firstly, the trader places an order with the broker, after which the broker carries out the trade. The trade process is automated if it can be done on an exchange, however if there is the need for a dealer to intervene then a larger fee is expected. Traders in investment banks often make deals on behalf of their banks and also execute trade for their clients. Staffs in the Capital markets department of investment banks usually keep themselves abreast with the various opportunities in the primary and secondary markets in order to duly advise clients. Investing in the capital market is not limited to the buying share or bonds; one can invest in mutual funds or exchange traded funds or buy and sell derivatives based on the secondary market (Chisholm, 2009; Spaulding, 2011).

The Ghana Stock Exchange is a physical exchange, also known as a listed exchange and only stocks listed with the exchange are traded, with a hybrid market for placing orders both electronically and manually on the trading floor. Orders executed on the trading floor enter by way of exchange members and flow down to a floor broker, who goes to the floor trading post specialist for that stock to trade the order. The job of the specialist is to match buy and sell orders using open outcry but however if a spread exists, no trade immediately takes place and in this case the specialist uses his/her own resources (thus, money or stock) to close the difference after his/her judged time. Once a trade has been made, the details are reported on the "tape" and sent back to the brokerage firm, which then informs the investor who placed the order.

Market participants include institutional investors such as banks, insurance companies, mutual funds, and hedge funds, individual retail investors and also publicly traded corporations trading in their own shares. Some studies have suggested that institutional investors and corporations trading in their own shares generally receive higher risk-adjusted returns than retail investors (Cesari, Espenlaub, Khurshed, & Simkovic, 2010).

Structure of the Ghana stock exchange

The idea of building a stock market in Ghana dates back to 1968 and continued with the promulgation of securities market Act of 1971. This laid the foundation for the establishment of the Accra Stock Market Limited (ASML) in 1971. However, political instability, lack of government support, as well as unfavorable macroeconomic environment, challenged the take-off of the ASML and as a result the idea of an exchange remained an illusion. Despite these setbacks, two stock brokerage firms, namely National Trust Holding Company Limited (NTHC) and National Stockbrokers Limited, currently Merban Stockbrokers, prior to the establishment of the Ghana Stock Exchange in November 1990, did over-the counter (OTC) trading in shares of some foreign-owned companies (Osei, 2005). Under the supervision of the International Monetary Fund and World Bank, Ghana went through structural and financial reforms such as deregulation of interest rates, removal of credit controls, and floating of exchange rates, among many others in 1983 so as to eradicate distortions in the economy. The need for stock market in Ghana however, became unavoidable after the financial liberalization and the divestiture of a host of state owned enterprises (Boateng, 2010; Anokye and Tweneboah, 2008).

The Ghana Stock Exchange (GSE) was incorporated in July 1989 and began trading in 1990, gradually facilitating the enhancement of the Ghanaian capital market as it served as the only stock exchange in the country. Currently, it contains a list of 38 equities and 2 corporate bonds from mining, manufacturing, brewery, oil and banking corporations, all of which represent the major sectors of Ghana's economy. The GSE has paved the way for various businesses and governments to raise long-term capital and for investors to obtain liquidity, fair capital safety and diversity of investments.

The Exchange is governed by, a Council with representation from Licensed Dealing Members, Listed Companies, the banks, Insurance Companies, Money Market and the public (Ghana stock exchange, 2015). This Council sets the policies of the Exchange and its functions include maintaining good order among members, preventing fraud and malpractices, regulating stock market business and granting listing.

Operation and trading of the GSE

The GSE uses the GSE Automated Trading System (GATS) to govern electronic trading undertaken by brokers and this application accommodates the needs of investors for a relatively dynamic trading system. The use of the GATS is to automate the GSE's objective of improving efficiency, increasing liquidity and enhance GSE's competitiveness to attract investors and issuers.

Trading takes place on every working day, that is, every Monday to Friday, starting between 9:30 to 10:00 hrs (GMT) and remains opened to 15:00 hrs (GMT). Settlement (exchange of the shares with cash) of trades is done electronically and occur three business days after the trade date (Ghana Securities Exchange, 2015).

Stock prices on the Ghana stock exchange (GSE)

In Ghana, investment bankers study the past and present performance of various companies which inform their decision on setting a price for the company's shares. The GSE then becomes the platform on which these stock / share prices are featured for exchange, where brokers facilitate this exchange.

The various stock prices present on the GSE are listed under opening price, closing price, previous closing price, closing bid price, closing offer price and last transaction price. The opening price relates to all securities on the Regular market whereas the closing price is the Volume Weighted Average Price (VWAP), which is the ratio of total value traded to total volume traded for all trades executed during the day (GSE, 2010).

3. Methodology of the Study

This study was an attempt to check the appropriateness of the Geometric Brownian motion model in forecasting stock prices. The case study area was the Ghana stock exchange (GSE) and the quantitative research approach was employed. The data from the GSE was used to estimate the model parameters and consequently to study the appropriateness of the model. This study also made use of secondary data of stock prices readily available on the Ghana Stock Exchange webpage.

Among the thirty-six (36) listed companies on the GSE, this study sampled the top 10 performers on the market and use specifically their closing offer price for analysis. The companies included HFC Bank (HFC), Enterprise Group Limited (EGL), CAL Bank (CAL), PZ Cussons (PZC), Societe Generale (SOGEGH), Guinness Ghana Brewery Limited (GGBL), Ghana Commercial Bank (GCB), Benso Oil Palm Plantation (BOPP), Total Ghana Limited (TOTAL) and Ghana Oil Company Limited (GOIL). Due to availability of data, this study performed all its computations and analysis using secondary data from January 2008 to July 2015.

Explanation of variables and parameter estimation

A stochastic process, S_t follows a GBM if it satisfies the stochastic differential equation (SDE):

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t \qquad \text{where } dW_t \sim N(0, dt) \quad (3.1)$$

The price of stock in nearest possible time dt can therefore be decomposed into two parts; a predictable and deterministic part, which is the expected rate of return on the stock at the time dt, $\mu S_t dt$ and a stochastic and unexpected part which is as a result of the random changes in the stock price over the time dt, $\sigma S_t dW_t$; where the σ is a constant and dW_t is a Brownian motion or Wiener process.

The above leads to the differential equation (1.1) which is a stochastic differential equation (Dmouj, 2006).

Applying separation of variables and integrating both sides of equation (3.1) yields:

$$\int \frac{dS_t}{S_t} dt = \int (\mu dt + \sigma dW_t) dt$$

$$\ln\left(\frac{dS_t}{S_t}\right) = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t \quad (3.1)$$

$$S_t = S_0 \exp\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t \quad (3.2)$$

$$E\,(S_t) = S_0 \exp\left(\mu + \frac{\sigma^2}{2}\right) t \quad (3.3)$$

$$Var\,(S_t) = S_0^2\, e^{(2\mu t + \sigma^2 t)}\,(e^{\sigma^2 t} - 1) \quad (3.4)$$

Equations (3.3) and (3.4) represent the expected and variance of the process at time t.

Considering equation (3.1), the variable $\ln S_t \sim N\,(lnS_0 + \left(\mu - \frac{\sigma^2}{2}\right) t, \sigma^2 t)$. Therefore, at 95% confidence interval, the confidence interval of S_t is given by:

$$e^{lnS_0 + \left(\mu - \frac{\sigma^2}{2}\right)t - 1.96(\sigma\sqrt{t})} \leq S_t \leq e^{lnS_0 + \left(\mu - \frac{\sigma^2}{2}\right)t + 1.96(\sigma\sqrt{t})} \quad (3.5)$$

Test of Hypothesis

The chi-square test would be used to test the hypothesis. This checks the "goodness-of-fit" between the observed stock price values and the forecasted ones. This test helps to determine if there is a significant difference between the expected (forecasted) and the observed (real) stock prices. The equation is given by;

$$\chi^2 = \frac{(\,observed - expected)^2}{expected}$$

A p-value (the probability that the deviation of the observed value from the expected value is by chance alone) will be computed against 0.05. If the p-value is less than 0.05, the null hypothesis will be rejected. Otherwise, it is accepted.

Data analysis and forecast measure

The data was analyzed using the R software and the Microsoft Excel data analysis packages. This study assumed constant parameters as assumed in Geometric Brownian motion models. Also, stock prices with 95% confidence interval was simulated and compared with the value of real stock prices. The forecast accuracy of the model was then determined by using the Mean Absolute Percentage Error. This measures the deviation between the forecasted values and the real stock values.

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|$$

F_t and A_t are the forecasted values and actual values and *n* is the number of observations.

4. Analysis and Discussion of Findings

This section shows a comparison of the weekly real and simulated stock prices for the year 2010. A confidence level of 95% was used to compare the real and simulated prices. The actual stock prices (Actual S_t), were an average of the weekly stock prices for the year 2010. The simulated prices (SimulatedS_t), corresponded to an average of 1000 simulated values after one trading week. The results for each company was tabulated below. UCL and LCL represent the upper and lower confidence level respectively. The parameters volatility and drift as well as the value of the stock S_0 were known at the beginning of each week. The volatility and drift were assumed constant throughout the year.

The tables below illustrate the results of simulating the actual stock value of HFC, SIC, ETI, GOIL, UTB, GCB, GGBL, BOPP, TBL and TRANSOL respectively at 95% confidence level.

Table 4.1.1: Weekly simulated values of the stock price for HFC for the year 2010

HFC				
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.47	0.48	0.45	0.46
51	0.49	0.51	0.48	0.49
50	0.48	0.49	0.46	0.49
…				
3	0.49	0.5	0.47	0.5
2	0.48	0.49	0.46	0.51
1	0.46	0.48	0.45	0.45

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Table 4.1.2: Weekly simulated values of the stock price for SIC for the year 2010

	SIC			
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.31	0.3	0.32	0.30
51	0.32	0.31	0.33	0.32
50	0.33	0.32	0.34	0.31
…				
3	0.3	0.29	0.31	0.29
2	0.29	0.28	0.3	0.29
1	0.29	0.28	0.3	0.29

Table 4.1.3: Weekly simulated values of the stock price for ETI for the year 2010

	ETI			
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.17	0.15	0.35	0.16
51	0.17	0.15	0.35	0.17
50	0.17	0.15	0.35	0.19
…				
3	0.18	0.16	0.36	0.18
2	0.16	0.14	0.35	0.18
1	0.16	0.14	0.35	0.18

Table 4.1.4: Weekly simulated values of the stock price for GOIL for the year 2010

	GOIL			
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.29	0.30	0.29	0.29
51	0.29	0.29	0.29	0.29
50	0.29	0.31	0.28	0.29
…				
3	0.29	0.31	0.28	0.29
2	0.29	0.29	0.29	0.29
1	0.28	0.28	0.29	0.29

Table 4.1.5: Weekly simulated values of the stock price for UTB for the year 2010

	UTB			
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.33	0.36	0.31	0.31
51	0.32	0.35	0.30	0.28
50	0.32	0.34	0.30	0.31
…				
3	0.32	0.34	0.29	0.34
2	0.30	0.32	0.28	0.30
1	0.29	0.32	0.27	0.31

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Table 4.1.6: Weekly simulated values of the stock price for GCB for the year 2010

GCB				
Week	Actual S_t	UCL	LCL	Simulated S_t
52	1.06	1.17	0.91	1.03
51	1.02	1.21	0.95	1.03
50	1.09	1.07	0.82	1.08
…				
3	0.94	0.66	0.40	1.14
2	1.01	0.69	0.43	1.11
1	1.05	0.69	0.43	1.05

Table 4.1.7: Weekly simulated values of the stock price for GGBL for the year 2010

GGBL				
Week	Actual S_t	UCL	LCL	Simulated S_t
52	2.26	1.96	1.88	2.49
51	1.68	1.88	1.88	2.18
50	1.63	1.82	1.82	2.20
…				
3	1.55	1.13	1.13	0.59
2	1.25	1.28	1.28	0.57
1	1.11	1.19	1.19	0.55

Table 4.1.8: Weekly simulated values of the stock price for BOPP for the year 2010

BOPP				
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.80	0.87	0.83	0.86
51	0.84	0.93	0.93	0.79
50	0.91	0.94	0.94	0.68
…				
3	0.57	0.58	0.58	1.21
2	0.62	0.57	0.57	1.20
1	0.67	0.46	0.46	1.26

Table 4.1.9: Weekly simulated values of the stock price for TBL for the year 2010

TBL				
Week	Actual S_t	UCL	LCL	Simulated S_t
52	1.31	1.61	1.34	1.32
51	1.21	1.38	1.38	1.23
50	1.20	1.48	1.48	1.17
…				
3	0.85	1.75	1.75	0.71
2	0.82	1.69	1.69	0.72
1	0.79	1.74	1.74	0.64

Table 4.1.10: Weekly simulated values of the stock price for TRANSOL for the year 2010

TRANSOL				
Week	Actual S_t	UCL	LCL	Simulated S_t
52	0.11	0.11	0.10	0.11
51	0.11	0.10	0.10	0.11
50	0.11	0.10	0.10	0.12
…				
3	0.10	0.12	0.12	0.10
2	0.09	0.12	0.12	0.11
1	0.09	0.12	0.12	0.11

The results in Tables 4.1.1, 4.1.2 and 4.1.3 showed that most of the true stock value of HFC, SIC and ETI respectively, lie in the 95% confidence interval of the simulated value.

Tables 4.1.4 and 4.1.5 showed that only a few of the true value of the GOIL and UTB stock respectively, lie within the 95% confidence interval of the simulated value.

Tables 4.1.6 and 4.1.7 demonstrated that very few true values of the GCB, GBL, stocks respectively, lie within the 95% confidence level interval. This was the same with that of BOPP, TBL and TRANSOL true stock values in Tables 4.1.8, 4.1.9 and 4.1.10 respectively.

Generally, the confidence intervals were observed to be very close to the actual simulated values and this confirms the findings of Dmouji (2006). Dmouji (2006) discovered that the confidence intervals are small if the simulation is done for short time intervals such as days or weeks as in this study, rather than when it is done in years.

Forecast results and Accuracy measure

In order to forecast the stock prices for our chosen listed companies, data was extracted from the GSE website from January 1, 2008 to January 1, 2015. From the 2008 and 2009 data, the annual volatility for each listed company was estimated using R software. Due to the bulkiness of the daily data, this paper used the averages of the week to forecast weekly stock prices.

A geometric random motion model was built in microsoft excel software for forecasting these stock prices. Using this model and the weekly volatility for each listed company, stock prices from the year 2010 was forecasted over a 156 weeks (3 years) period.

The diagrams below compared the forecasted stock prices with the actual prices retrieved from GSE. The company stock price forecasts were reviewed in turn and the discussion was done using the ability of the GBM to forecast in each of the three years (that is, first year comprised weeks 1-52, second year comprised of weeks 53-102 and the third year was weeks 103-156).

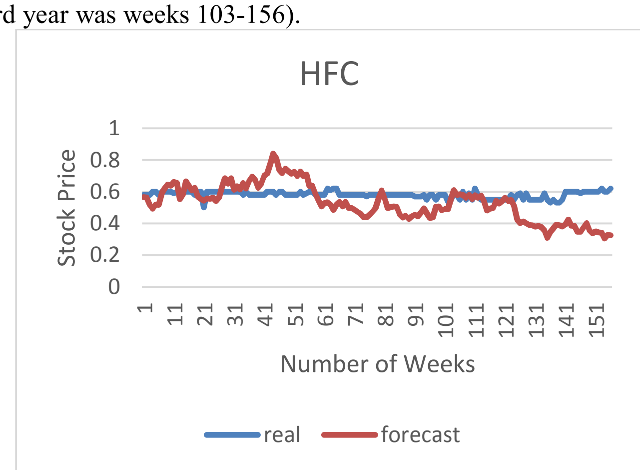

Figure 1 Real and forecasted stock prices for HFC

Figure 1 reveals a very close relationship between the real and forecasted of HFC stock prices especially in the first and first half of second years. The GBM model however, seems to underestimate the stock prices for the second year. The third year also shows a downward trend in the forecast values.

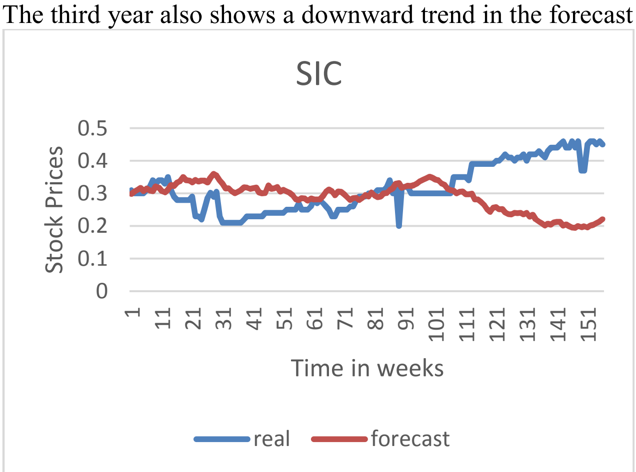

Figure 2 Real and forecasted stock prices for SIC

Figure 2 illustrates the real and forecasted HFC stock prices. This reveals a close relationship between the real and forecasted stock prices for the first two years. However, the third year begins to show a funnel shape even though by very small difference, as the GBM tends to underestimate the real stock values.

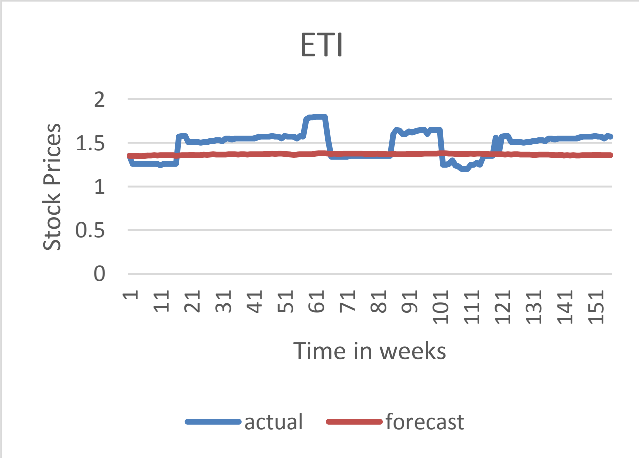

Figure 3 Real and forecasted stock prices for ETI

The real and forecasted ETI stock prices are demonstrated in Figure 3. This shows a very close relationship between the real and forecasted stock price values. The model however seems to underestimate most of the stock prices in the last part of the first year and the first part of the second year, as well as in the last year. But, in the middle of the second year, the model seems to predict almost the same stock prices as the real ones.

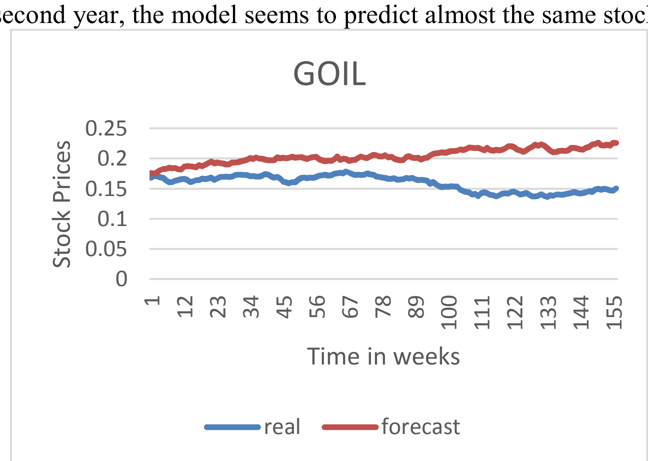

Figure 4 Real and forecasted stock prices for GOIL

Figure 4 shows an almost funnel shaped difference between the real and forecasted GOIL stock prices. The GBM seems to be deviating from the real stock prices with time but the deviation is not too large.

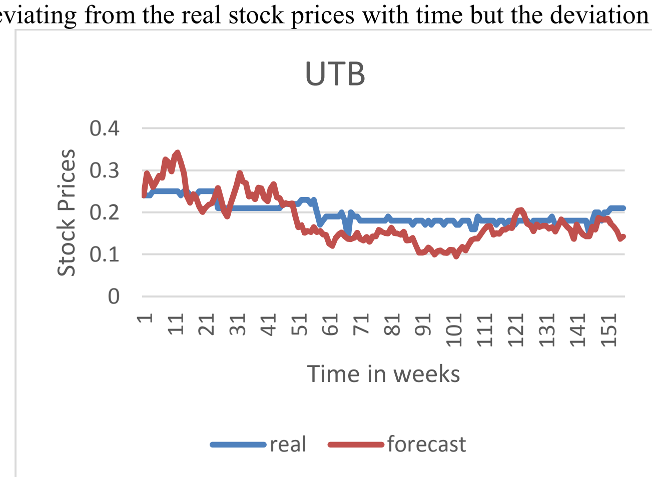

Figure 5 Real and forecasted values for UTB

Figure 5 illustrates a very close relationship between the real and the forecasted stock prices of UTB, especially during the first and third years. That notwithstanding, the GBM model seems to underestimate the stock prices in the second year.

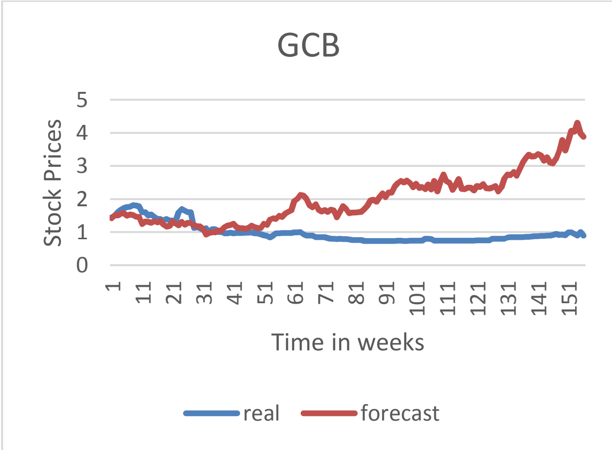

Figure 6 Real and forecasted stock prices for GCB

In Figure 6 the GBM model seems to forecast the stock prices of GCB quite well but tends to overestimate the prices after the first year. It is also worth noting that the model seems to show an upward trend with time after the first year, which begins to bend downward just before the start of the fourth year.

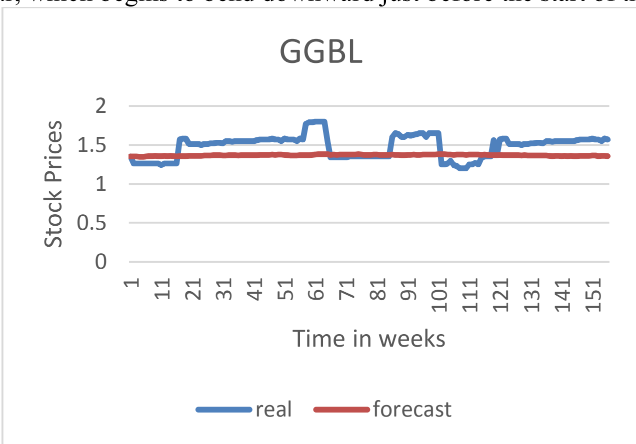

Figure 7 Real and forecasted stock prices for GGBL

Figure 7 demonstrates an overall fairly smooth forecasted GGBL stock prices. However, the GBM seems to underestimate most of the stock prices in the last part of the first year and the first part of the second year, as well as in the last year. But, the model seems to predict almost the same stock prices in the middle of the second year.

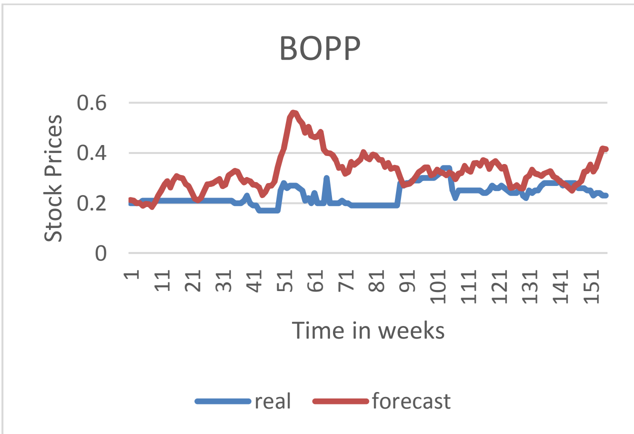

Figure 8 Real and forecasted stock prices for BOPP

Figure 8 reveals a close relationship between the forecasted and real values in the third year of forecast in BOPP stock prices. There is however large deviation from the real stock prices in the second years forecast results.

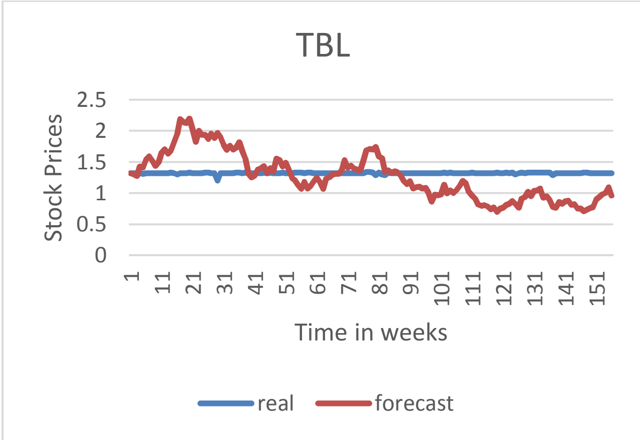

Figure 9 Real and forecasted stock prices for TBL

Figure 9 reveals a close relationship between the real and forecasted TBL stock prices. The model seems to overestimate the prices in the first half of the first year and underestimate that of the third year. The difference in the price difference is however not too large.

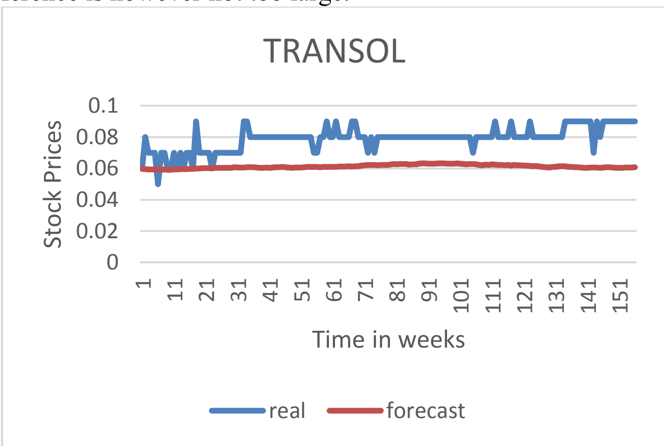

Figure 10 Real and forecasted stock prices for TRANSOL

In Figure 10 it is observed that the forecasted TRANSOL stock prices follow quite a smooth path. The real values however, shows a relatively smooth path except at the first twenty-six weeks, between the fifty-sixth and seventy-sixth week, and during the third year. It is worth noting that the model seems to underestimate the TRANSOL stock prices.

From the graphs, it is observed that there is not much deviation between the real and the forecasted stock price values among most of the companies. For instance, considering GCB, HFC, GOIL, BOPP, GGBL, and UTB it is observed that there is little or no variation of the forecasted values from the actual values for the first year. However these show little or more deviation after the first year.

Forecast Accuracy

In this section, the accuracy of the GBM model to forecast the stock price is checked using the Mean Absolute Percentage Error (MAPE). This measure shows the percentage of error or deviation of the forecasted values from the actual stock values and is calculated as:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|$$

Where A_t is the actual stock value, F_t is the forecasted value n is the number of observations.

The model is then judged as follows:

MAPE %	Model Accuracy
0-10%	Highly accurate
11-20%	Good forecast
21-50%	Reasonable forecast
>51%	Inaccurate forecast

Source: Lawrence et. al (2009)

Below are the MAPE results of the companies under consideration.

Table 4.2.1: MAPE results of each company

Listed Company	MAPE (%)
HFC	0.3258
SIC	0.1354
ETI	0.1356
GOIL	0.5004
UTB	0.3250
GCB	3.3097
GGBL	0.1355
BOPP	0.8043
TBL	0.2738
TRANSOL	0.3258

Since all the MAPE values are between 0% and 10%, it implies that the GBM model is a highly accurate model for forecasting these stock prices on the Ghana Stock Exchange.

Hypothesis Testing

A further test is carried out using a chi-square test of independence to test the hypothesis as of this study under section 1.1. This will check the "goodness-of-fit" between the observed stock price values and the forecasted ones. The test statistic is:

$$\chi^2 = \frac{(\,observed - expected)^2}{expected}$$

Decision rule: Reject null hypothesis if p-value is less than 0.05

Fail to reject null hypothesis if p-value is greater than 0.05

The p-value for each of the companies under consideration are observed as below:

Table 4.2.2: p-values of the ten (10) companies under consideration

Company	p-value
HFC	0.25
SIC	0.01
ETI	0.30
GOIL	0.06
UTB	0.01
GCB	0.07
GGBL	0.50
BOPP	0.14
TBL	0.30
TRANSOL	0.24

From Table 4.2.2, we accept the null hypothesis for HFC, ETI, GOIL, GCB, GGBL, BOPP, TBL and TRANSOL stock prices. This is because the variation that exist between the real stock prices and the forecasted stock prices are due to chance and hence can be ignored.

However, we reject null hypothesis for SIC and UTB stock prices as the test reveals that the deviations between the real stock prices and the forecasted values may be due to other factors other than just chance.

5. Conclusion

The objective striking optimal investment decisions have informed the building of models and methods as well as the use of various means such as analyst prediction in an attempt to forecast stock prices. The Geometric

Brownian motion model has served as a fundamental model in undertaking such actions.

The study found out that, simulating weekly stock prices, the true values of almost all ten (10) companies do not lie in the confidence interval of the simulated values, and the confidence intervals are relatively small. After forecasting and checking the appropriateness of the forecasted values and model, this study fails to accept the null hypothesis being tested and therefore concludes based on the several tests conducted that the Geometric Brownian Motion (GBM) model is appropriate for forecasting stock prices on the Ghana stock exchange.

As a form of recommendation, the model should be used to forecast daily stock prices over short intervals as that is likely to give a more accurate forecast. Additionally, the model should be used to compare different sample sizes to further check its' accuracy with respect to time.

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Dr. Isaac Junior Damptey holds a PhD Finance from the Central University of Nicaragua, Managua, Nicaragua and Doctor of Finance degree from the Swiss Management Center University, Vorstadt 26a, 6300 Zug, Switzerland (2016). He also holds a Master of financial management from Amity University (2012), Uttar Pradesh, India and a Bachelor Degree in Economics with Sociology (2008) from the University of Ghana, Legon, Ghana.