Convertible Bonds and the Price Discriminating Monopolist Firm

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Abstract
This paper examines whether a price discriminating monopolist will issue convertible bonds with similar features as a perfectly competitive firm. The paper finds that, ceteris paribus, the discriminating monopolist uses relatively more convertible bonds. Also, it designs its convertibles differently with a relatively lower conversion ratio and conversion value as well as neutral hedge ratio. On the other hand, it has a higher breakeven yield and conversion premium. At conversion, the discriminating monopolist has a higher dilution factor. However, after complete market adjustment following conversion, the discriminating monopolist has less outstanding shares.

Keywords: Convertible bonds; Price discriminating monopolist; Market structure.

1. Introduction
Convertible bonds, hereafter referred to as convertibles, are a significant portion of outstanding financial instruments. Convertibles range from basic to complex designs and viewed as distinct asset classes or part of a portfolio. They are also used for varied purposes. Since their initial issue, a plethora of research has been undertaken on such bonds.

The first group of such studies evaluate how issuing convertibles impacts capital structure and other related variables. These include: Carayannopoulos and Kalimpalli (2003), Coxe (2000), Epstein (2006), Gottesman (2004), Korkeamaki and Moore (2004) and Ranaldo and Ackmann (2004). Research of this sort highlight concerns such as dilution and financial risk. Related to this, Arak and Martin (2005), Marquadt and Wiedman (2005) further detail the relationship between such financial instruments and common stock or equity.


Ayache, Forsyth and Vetzal (2003) evaluate the relation between convertibles and risk. These examine risk within different dimensions. Some of such research analyze how convertibles can be used as part of risk management strategies. This includes business and financial risks. Hung and Yan Jr. (2002), Tan and Cai (2007) and Takahashi, Kobayahashi and Nakagawa (2001) extend theoretical and empirical literature by examining how convertibles affect financial and operating leverage. The risk dimension is expanded by including the information or signaling content of conversion (Hung and Yan Jr. 2002; Loncarski, ter Horst and Veld, 2006). The relationship between arbitrage and convertibles is another facet of risk research on such financial instruments. Studies in this category include: Agarwal, Fung, Yee and Naik. (2011), Calamos (2003), Darwin, Getmansky and Tookes (2009 and 2010), Henderson (2005) and Loncarski, ter Horst and Veld (2007).

Convertibles are primarily a source of funds. Consequently, corporate or project financing and convertibles is another field of research. Studies in this area include: Cornelli and Yoshia (2003), Hellmann (2000), Krishnaswami and Yaman (2007) and Schmidt (2000).

Moreover, another very important facet of research on convertibles is the relationship between convertibles and options. Such research include Butler (2002), Ederington and Goh (2001), Lau and Kwok (2004). However, none of the afore-mentioned research examines the use and design of convertibles by a price discriminating monopolist. The raison d’être for this research emanates from this gap in the body of research on convertible bonds. Specifically, this research compares and contrasts the use and design of convertibles by a price discriminating monopolist relative to a perfectly competitive firm. As such, this research considers the effect of the industrial structure of the firm on the optimal use and design of convertibles as part of its capital structure. It focuses on only convertible bonds and excludes other types of convertibles such as exchangeable and convertible preferred stock.

For this intent, the paper firstly outlines the salient features of a price discriminating monopolist. The succeeding section examines the equilibrium conditions and optimal use of convertible bonds by the afore-mentioned firm. In the ensuing section, the paper analyzes whether the discriminating monopolist will issue
similar convertibles as those issued by a perfectly competitive firm. The conclusion highlights the pertinent findings and issues raised by this research. Hereafter, the price discriminating monopolist may be referred to as monopolist, price discriminator or simply discriminator.

2. Price Discriminating Monopolist

Assume that firm \( j \) is a price discriminating monopolist producing commodity \( x \). It can perfectly price discriminate. Due to its monopoly power, it has a higher markup per unit of its product it sells. Let \( c_j \) denote the marginal cost of production. Further suppose that \( Q_{j,1}^{d} \) and \( Q_{j,0}^{d} \) are quantity demanded at the new price and old price respectively. Additionally, \( \eta_j \) is an indicator of the price elasticity of demand (Mas-Colell, Whinston and Green, 1995). More relevant to this study, it is an indicator of the degree of market power of the monopolist. As in Afful (2004). The price discriminator’s unit product price, \( p_j \), is postulated as:

\[
p_j = c_i \left[ 1 + \left( \frac{1}{\eta_j} \cdot \frac{Q_{j,1}^{d} - Q_{j,0}^{d}}{Q_{j,0}^{d}} \right) \right] \tag{1}
\]

On the other hand, let industry \( i \) be comprised of individual perfectly competitive firms. As an amalgamated whole, the perfectly competitive industry produces the same quantity of output as the price discriminating monopolist. However, industry \( i \) cannot price discriminate. Therefore, \( \eta_i = \infty \). According to this, one implication of equation (1) is that \( p_j > p_i \).

It is further assumed that there are only two sources of capital, namely: convertibles and equity. For ease of analysis, suppose that only plain vanilla non-callable convertibles can be issued (Andersen and Buffum, 2004). Moreover, the bond market is characterized by a flat term structure. Let \( N \) be the total outstanding stock prior to conversion, while \( n \) is the additional stock resulting from conversion. As in Ingersoll (1977), the dilution factor, \( y \), is:

\[
y = \frac{n}{N + n}.
\]

Presume that the terms of conversion are determined prior to purchasing the convertible (Ayache, Forsyth and Vetzal, 2002). It cannot be changed thereafter. There is also no call notice during the conversion process. If \( \Psi \) is the face value of a convertible and the conversion price is \( \vartheta \), then the conversion ratio, \( \Omega \), is defined as:

\[
\Omega = \frac{\Psi}{\vartheta}.
\]

Additionally, \( V_c \), the total market value of the firm, is the sum of the market value of the convertibles, \( V_{c,b} \) and equity, \( V_e \). Analogously, \( V \) is the market value of capital. It is worth noting that \( V_{c,b} \) is negatively related to the convertible yield to maturity, \( r_{c,b} \). In addition, \( p_{c,b,j} \) and \( p_{c,b,d} \) are the respective price of convertibles for the monopolist and competitive industry.

Financial markets determine the cost of debt or equity, interest rate as well as price of financial instruments (Buhler and Koziol, 2002). Neither the perfect competitor nor the monopolist can alter market conditions or prices. They may, however, determine the design of financial instruments they issue. If \( V_j \) and \( V_i \) are the value of capital for the perfectly competitive industry and the monopolist respectively, then under a no-arbitrage assumption: \( V_i = V_j \).

For the monopolist, the costs of convertible bonds and equity are \( r_{c,b,j} \) and \( r_{c,j} \) respectively. The same variables for the perfect industry are denoted as: \( r_{c,b,i} \) and \( r_{c,i} \). Additionally, the corresponding weights of debt and equity for the monopolist are \( w_{c,b,j} \) and \( w_{c,j} \). Similar weights are constructed for the competitive industry.

Financial markets are perfect and frictionless. There are no agency, bankruptcy or transaction costs. In addition, there are no taxes. Investors are rational, preferring more wealth. Neither type of firm issues straight bonds (Darwin, Getmansky and Tookes, 2010). For ease of tractability, suppose that the sustainable growth rate
of both firms is zero. All abnormal profits are paid out as dividends and / or interest charges. Furthermore, purchased convertibles are converted immediately or at the next allowable time for conversion. The only market impact or cost of conversion is dilution. Note hereafter that any variable with a subscript or superscript of \( i \) represents that of the perfect competitor. Alternatively, a subscript or superscript of subscript \( j \) denotes the discriminating monopolist.

3. Equilibrium Conditions

**Proposition 1:** The price discriminating monopolist will use relatively more convertibles in its capital structure.

Let \( \Pi_j \) and \( \varphi_j \) denote the net income and gross interest expense of the discriminating monopolist. Similar variables are defined for the perfectly competitive industry, \( i \). However, note that \( \Pi_j > \Pi_i \). The monopolist will select a capital structure where its optimal return on total capital, \( \Phi_j \) is such that:

\[
\Phi_j = \frac{\varphi_j + \Pi_j}{V_j} = \xi_j
\]

\( w_{c,b,j} \) can be alternatively constructed as:

\[
w_{c,b,j} = \Pi_j \left( \frac{r_{c,b,j} - r_{e,j}}{r_{c,b,j}} + \frac{r_{e,j}}{r_{c,b,j}} \right) \left[ \frac{V_j + V_{j}}{w_{c,b,j}} - 1 \right]^{-1} - V_j * r_{c,b,j}\]

Equation (3) implies that \( w_{c,b,j} > w_{c,b,i} \). As the price discriminator has the same risk as the perfect competitor, the former minimizes dilution from raising more equity by issuing more convertibles (Coxe, 2000). Furthermore, it can afford the additional interest charges because \( \Pi_j > \Pi_i \). The perfect competitor cannot adopt the same financing strategy because \( \Pi_i = 0 \).

To find the optimal capital structure for the monopolist, the following Lagrangian function is developed:

\[
V_{c,b,j} + V_{e,j} + \lambda \left[ k^* \left( \frac{V_{c,b,j}}{V_j} \right) * r_{c,b,j} \right] - \left( \frac{V_{e,j}}{V_j} \right) * r_{e,j} \]

In equation (4), \( \lambda^* \) and \( k^* \) are the Lagrange multiplier and constrained optimization variable respectively. The function is based on the premise that the optimal capital structure is derived when the value of the firm is maximized as well as the cost of capital is minimized (Epstein, 2006). The optimal value of convertibles used by the monopolist, \( V_{c,b,j}^* \), is obtained as:

\[
V_{c,b,j}^* = r_{c,j} \left[ \frac{1}{k_j - (r_{c,b,j} + r_{e,j})} \frac{1}{r_{c,b,j}} \right] \]

According to (5), \( V_{c,b,j}^* \) is determined by the cost of equity, cost of debt, optimal cost of capital and margin between the aforementioned costs of the two sources of capital. As such, for the discriminating monopolist the optimal capital structure has a debt ratio, \( w_{c,b,j}^* \), such that:

\[
w_{c,b,j}^* = 1 - \left( \frac{n_j^*}{y_j^* N_j} \right)\]

From equation (6), the optimal weight of equity is: \( p_{s,j} \left[ \frac{n_j^*}{y_j^* N_j} \right] \), where \( p_{s,j} \) is the price per share of the monopolist. \( \left[ \frac{n_j^*}{y_j^* N_j} \right] \) is the number of outstanding shares. It may be noted that this is affected by the optimal dilution factor, \( y_j^* \). A priori, \( w_{c,b,j}^* \) is determined by \( r_{c,b,j} \) and \( r_{e,j} \) (Korkeamaki and Moore, 2004). However,
equation (6) illustrates that \( w_{c,b,j}^* \) is now a function of additional variables, namely: \( n_j^* \), \( N_j^* \) and \( y_j^* \). It is worth noting that equation (6) has only one unique solution.

Let \( p_{b,j} \) and \( p_{swap,j} \) be the price of a straight bond and its swap equivalent for the monopolist. \( c_j \) and \( p_{s,j} \) denote a call and put for a unit of its stock. According to Bhattacharya and Zhu (2006), a convertible may be denoted in a put-call parity association, as: \( p_{b,j} + c_j = p_{s,j}^* + put_j + p_{swap,j} \). The left-hand of the parity relation illustrates that a convertible is akin to a long straight bond, \( p_{b,j} \), and a long call, \( c_j \). Alternatively, it may be posited as purchasing a put, share and stock swap of the same firm. The same indicators are defined for the perfectly competitive industry \( i \).

The swap exchanges the coupon interest of the bond with the dividend of the underlying share it may be converted into (Epstein, Haber and Wilmott, 2000). The put-call parity implies that: \( p_{c,b,j} - p_{c,b,i} = c_j - c_i \). This parity association may be used to develop different synthetic positions in a variety of financial instruments. In line with the prior no-arbitrage assumption:

\[
\Pi_j - \Pi_i = D_j - D_i + \varphi_j - \varphi_i = (1-\delta_j)^{-1}\left[p_{b,j}(r_{c,b,j} n_j - r_{c,b,i} n_i) + (c_j r_{c,b,j} n_j - c_i r_{c,b,i} n_i)\right]
\tag{7}
\]

where \( D_j \) and \( D_i \) are the dividend per share for the discriminating monopolist and competitive industry respectively. Equation (7) indicates that additional income of the discriminating monopolist can be modeled synthetically by having generic long straight bond, \( p_{c,b,j} \), and a long stock call of the discriminating monopolist with a value of: \((1-\delta_j)^{-1}\left[(c_j r_{c,b,j} n_j - c_i r_{c,b,i} n_i)\right] \). The discriminator has a relatively longer higher premium payback duration. This may be due to its higher \( \psi_j \). Equation (11) indicates the conversion value, \( \Gamma_j \) is:

4. Convertible Bond Design for the Discriminating Monopolist

**Proposition 2:** Convertibles of the price discriminating monopolist will have relatively different optimal designs.

Let \( \Omega_j^* \) denote the optimal conversion ratio. It may be obtained as:

\[
\Omega_j^* = \frac{n_j(p_{b,j} + c_j)}{w_{c,b,j} V_j}
\tag{8}
\]

According to equations (3) and (8), the price discriminator has a lower conversion ratio. This helps it to minimize the adverse effects of dilution. Related to this, its conversion premium, \( \psi_j \), is approximated as:

\[
\psi_j = \frac{(p_{swap,j} + put_j + p_{s,j})w_{c,b,j} V_j}{n_j p_{b,j}}
\tag{9}
\]

To make up for its lower \( \Omega_j^* \), the price discriminator has a higher \( \psi_j \), as indicated by equation (9). This is related to the fact that \( D_j > D_i \) (Mas-Colell, Whinston and Green, 1995). Let \( \tau_j \) be the coupon interest income of the convertibles of the monopolist. Its premium payback period may be computed as:

\[
\frac{w_{c,b,j} V_j - p_{s,j} \Omega_j^*}{\tau_j - D_j \Omega_j^*}
\tag{10}
\]

The discriminator has a relatively longer higher premium payback duration. This may be due to its higher \( \psi_j \).
From equation (11), it may be noted that the discriminating monopolist has a relatively greater conversion value. One reason is that immediately upon conversion of the discriminator’s convertibles, the investor will own a higher valued stock and receive larger dividends (Buhler and Koziol, 2002). The higher conversion value foretells potential arbitrage opportunities between the convertibles of the monopolist and perfect competitor. Associated with the neutral hedge ratio, \( \Sigma_j \) is derived as:

\[
\Sigma_j = \frac{n_j(p_{s,j} + put_j + p_{swap,j} - c_j)}{w_{c,b,j}V_j} \times \left[ \frac{p_{c,b,j,t} - p_{c,b,j,t-1}}{p_{s,j,t} - p_{s,j,t-1}} \right] 
\]

where the last term on the right-hand side of equation (12), is the conversion delta or parity value. It measures the sensitivity of \( p_{c,b,j} \) to a change in \( p_{s,j} \). The posited \( \Sigma_j \) illustrates that for the price discriminator, fewer convertibles are required to hedge its convertibles. This conforms with equation (3). This may reduce its risk exposure. Finally, the breakeven yield, \( \pi_j \), for the monopolist is posited as:

\[
\pi_j = V_j \left[ \frac{1}{\tau_j - \delta_j} \right] * \left[ \frac{p_{c,b,j}w_{c,b,j}}{n_j(p_{s,j} + put_j + p_{swap,j} - c_j)} - 1 \right] 
\]

Equation (13) indicates that the discriminating monopolist has a higher breakeven yield than perfectly competitive firms. One reason is that it uses more convertibles. Furthermore, \( \tau_j > \tau_i \). Equation (13) implies that the spread between \( \tau_j \) and \( \delta_j \) has an inverse effect on \( \pi_j \). Additionally, combining long positions in the share, put and swap with a corresponding short call decreases \( \pi_j \). On the other hand, an increase in convertibles raises \( \pi_j \). The breakeven yield may be viewed as a measure of liquidity of the convertible’s premium.

5. Conclusion

This research paper uses comparative statics to examine the effect of the industrial structure of a firm on the design and features of plain vanilla convertibles. It contrasts these with the same security issued by a perfect competitor. The characteristics of the financial instrument belonging to the latter are the benchmark. It begins by highlighting the fundamental framework of the monopolist. This is followed by outlining critical convertible bond equilibrium conditions for the monopolist. The final section examines the pertinent convertible design for the price discriminator.

This study finds that the discriminating monopolist uses more convertibles. Consequently, it will have a higher debt ratio in its capital structure as compared to the perfect competitor. Incorporating a put-call parity framework illustrates that the net income margin between the two types of firms can be approximated by undertaking positions in derivatives and related financial instruments. It was further found that the monopolist has a lower conversion ratio. Also, it has a higher conversion premium, possibly resulting from its higher dividend. On the other hand, it has a relatively higher conversion value. In comparison with the perfect competitor, the discriminating monopolist has a lower neutral hedge ratio but higher breakeven yield. After conversion, it will have a higher dilution factor.

This study emphasizes that the industrial structure within which the firm operates is critical to convertible bond design, as suggested by Andersen and Buffum (2004). It also highlights that the ability to discriminate is pertinent. The research emphasizes that dilution should be included as one of the costs or risks of convertibles (Barone-Adesi, Bermudez and Hatgioannides, 2003). It is hoped that the findings of this study will serve as a basis for future research.

References


